

# Materialization in Shape Analysis with Structural Invariant Checkers

**Bor-Yuh Evan Chang**

Xavier Rival

George C. Necula

University of California, Berkeley

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ITU Copenhagen

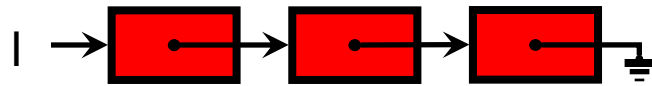
# What's shape analysis? What's special?

Shape analysis tracks **memory manipulation** in a **flow-sensitive** manner.

- **Memory manipulation**
  - Particularly important in systems code (in C)
- **Flow-sensitive**
  - Many important properties
    - E.g., Is an object freed? Is a file open?
  - Heap abstracted differently at different points
    - E.g., Not based on allocation site

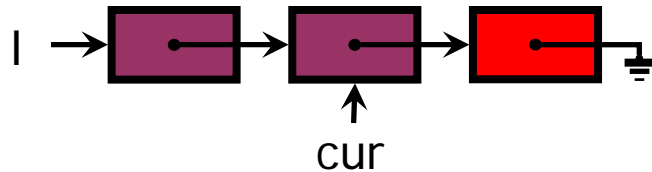
# Example: Typestate with shape analysis

## Concrete Example



```
cur = l;  
while (cur != null) {
```

```
  assert(cur is red);  
  make_purple(cur);
```



```
  cur = cur->next;
```

```
}
```

## Abstraction



program-specific predicate

flow-sensitive heap abstraction

make\_purple(.) could be

- lock(.)
- free(.)
- open(.)
- ...

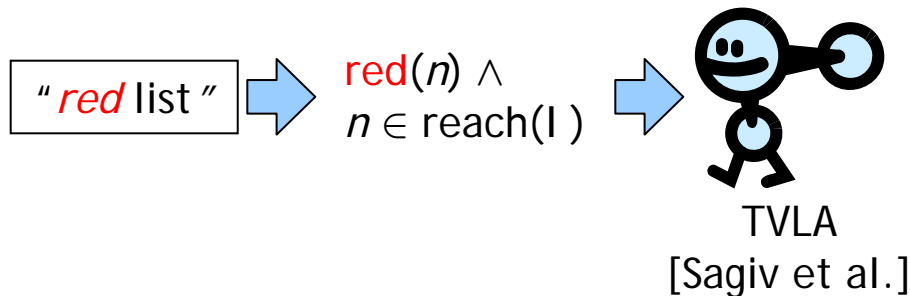
# Shape analysis is not yet practical

Usability: Choosing the heap abstraction difficult



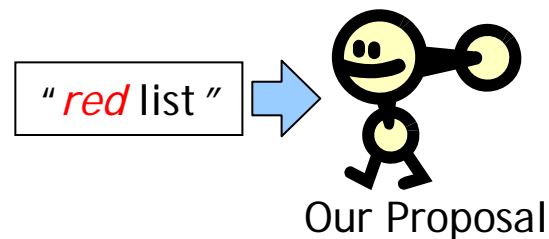
Built-in high-level predicates

- Hard to extend
- + No additional user effort



Parametric in low-level, analyzer-oriented predicates

- + Very general and expressive
- Hard for non-expert



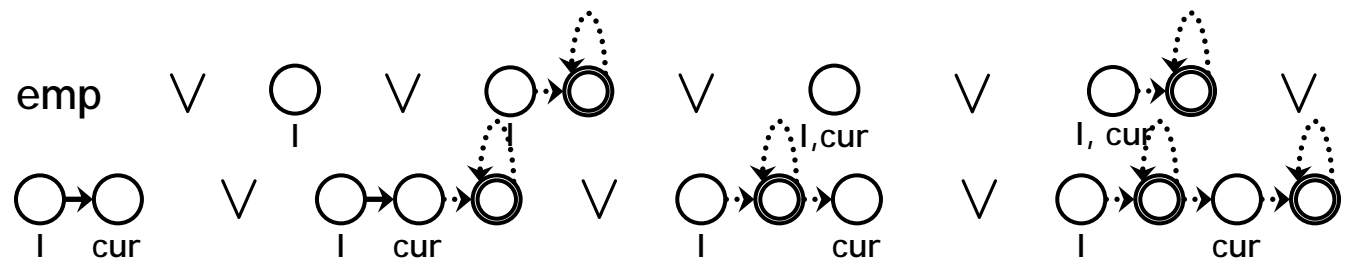
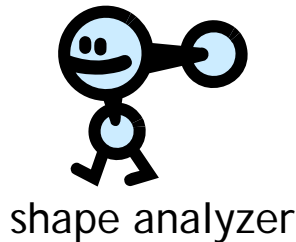
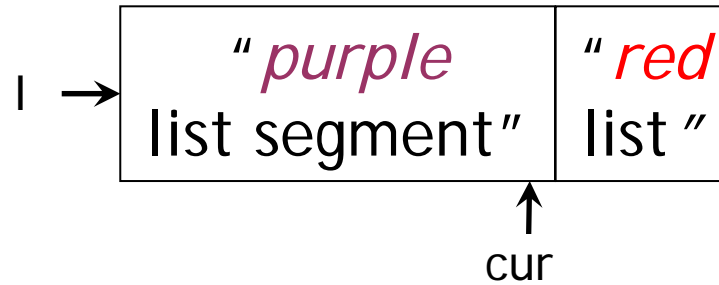
Parametric in high-level, developer-oriented predicates

- + Extensible
- + Easier for developers

# Shape analysis is not yet practical

Scalability: Finding right level of abstraction difficult

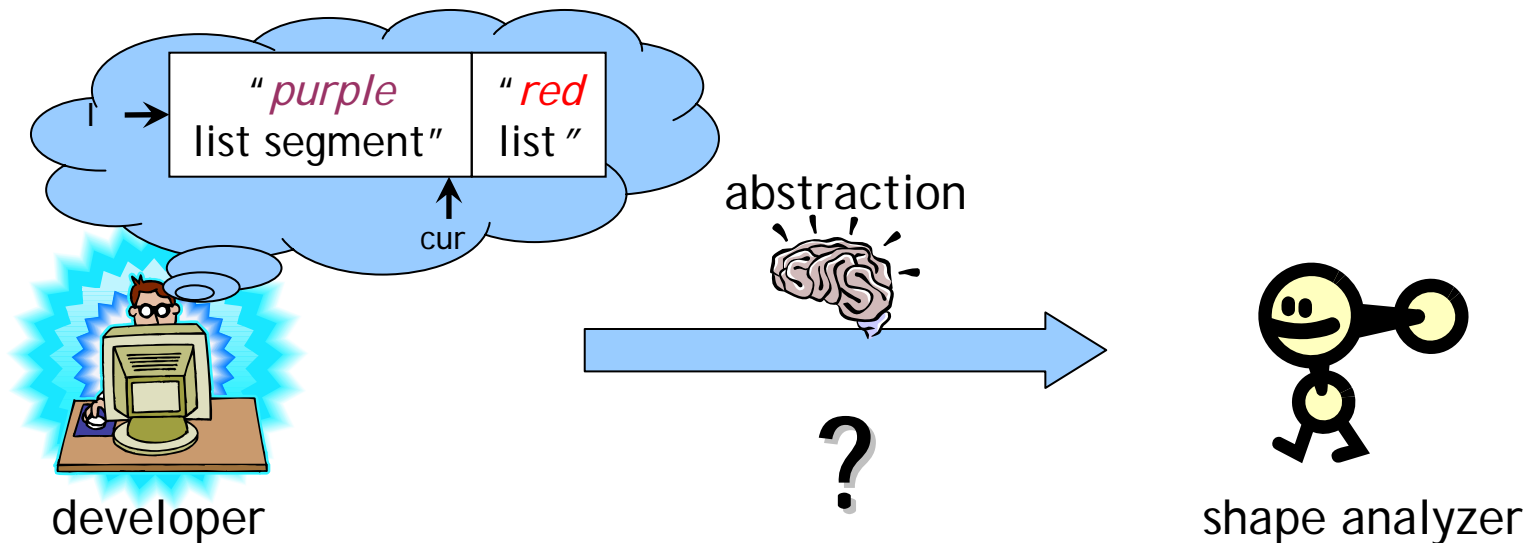
➔ Over-reliance on disjunction for precision



# Hypothesis

The **developer** can describe the memory in a **compact** manner at an abstraction level sufficient for the properties of interest (at least informally).

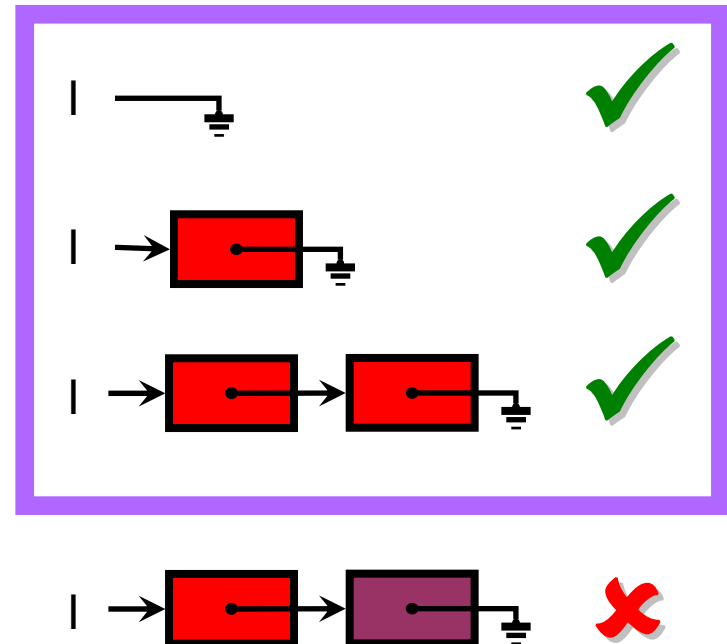
- Good abstraction is program-specific



# Observation

**Checking code** expresses a shape invariant and an intended usage pattern.

```
bool redlist(List* l) {  
  if (l == null)  
    return true;  
  else  
    return  
      l->color == red  
      && redlist(l->next);  
}
```

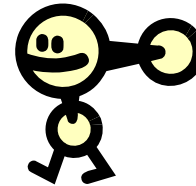
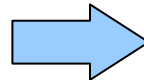


# Proposal

An automated **shape analysis** with a memory abstraction parameterized by **invariant checkers**.

```
bool redlist(List* l) {  
  if (l == null)  
    return true;  
  else  
    return  
      l->color == red  
      && redlist(l->next);  
}
```

checkers



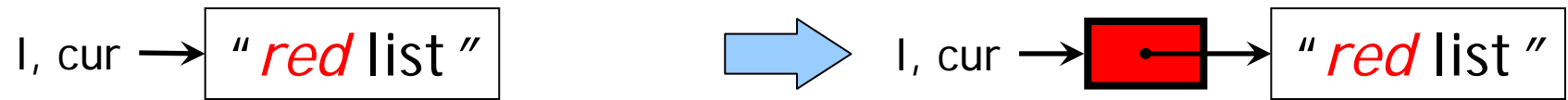
shape analyzer

- Extensible
  - Abstraction based on the developer-supplied checkers
- Targeted for Usability
  - Global data structure specification, local invariant inference
- Targeted for Scalability
  - Based on the hypothesis

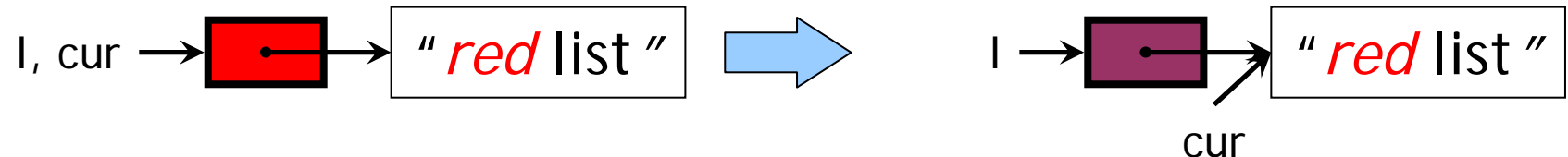


# Shape analysis is an abstract interpretation on memory states with ...

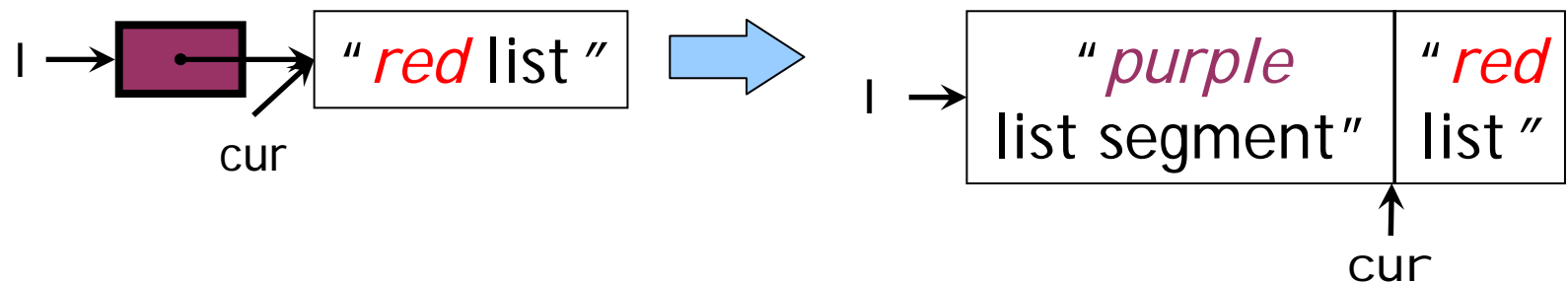
- **Materialization** (partial concretization)



- To perform strong updates



- And **widening** for termination



# Outline

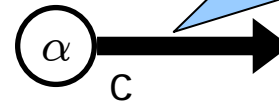
- Memory abstraction
  - Restrictions on checkers
  - Challenge: Intermediate invariants
- Materialization by forward unfolding
  - Where and how
  - Challenge: Unfolding segments
- Materialization by backward unfolding
  - Challenge: Back pointers
- Deciding where to unfold generically

# Abstract memory using checkers

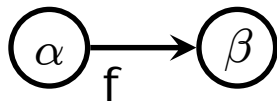
## Graphs



values  
(address or null)



checker run  
 $c(\alpha)$



points-to relation  
 $\alpha@f \mapsto \beta$

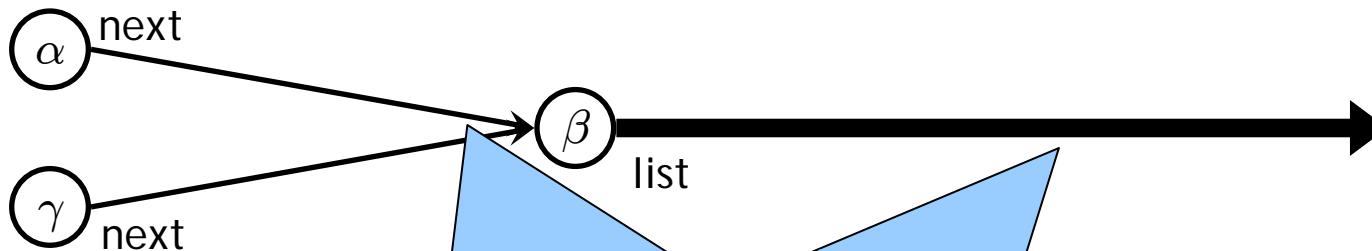


partial run  
?

"Some number of points-to edges that satisfies checker  $c$ "

## Example

"Disjointly,  $\alpha \rightarrow \text{next} = \beta$ ,  $\gamma \rightarrow \text{next} = \beta$ , and  $\beta$  is a list."

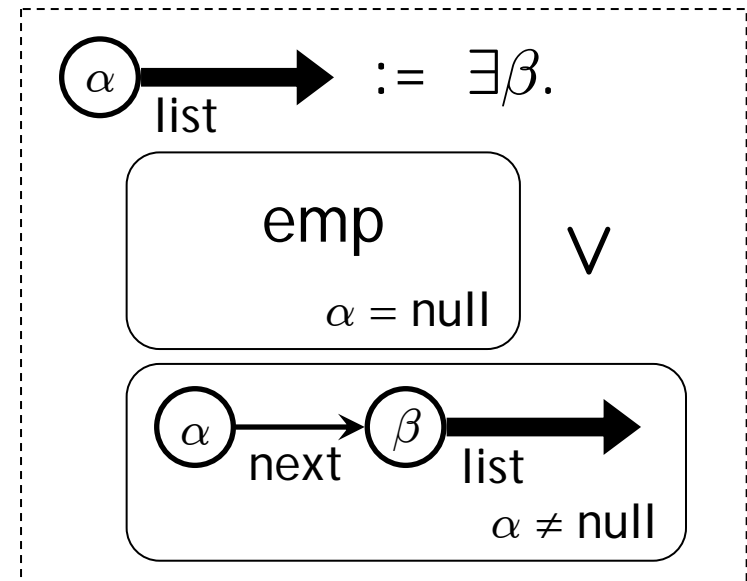
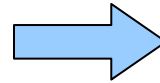


disjoint memory regions (\*)

# Checkers as inductive definitions

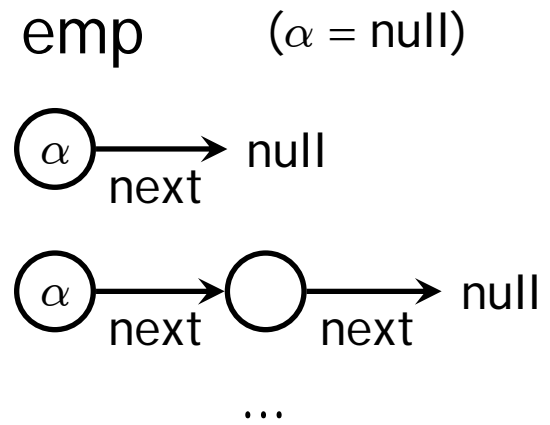
```

bool list(List* l) {
  if (l == null)
    return true;
  else
    return list(l->next);
}
    
```



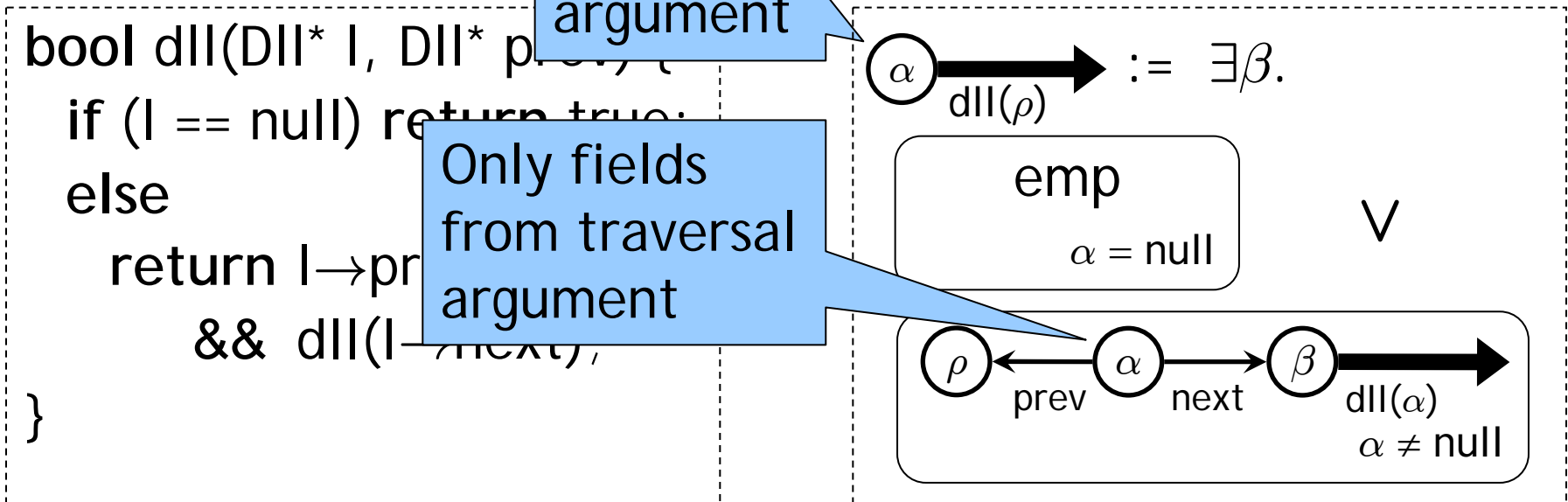
list(l)  
⋮  
list(...)  
⋮

Disjointness  
Checker run can dereference any object field only once

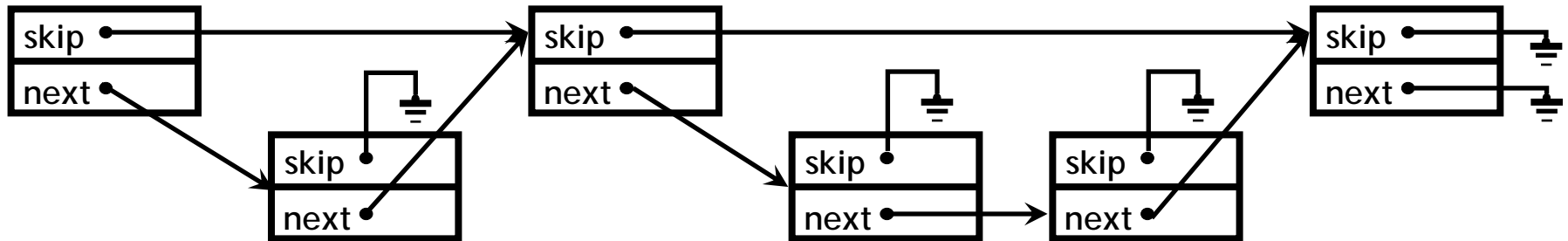
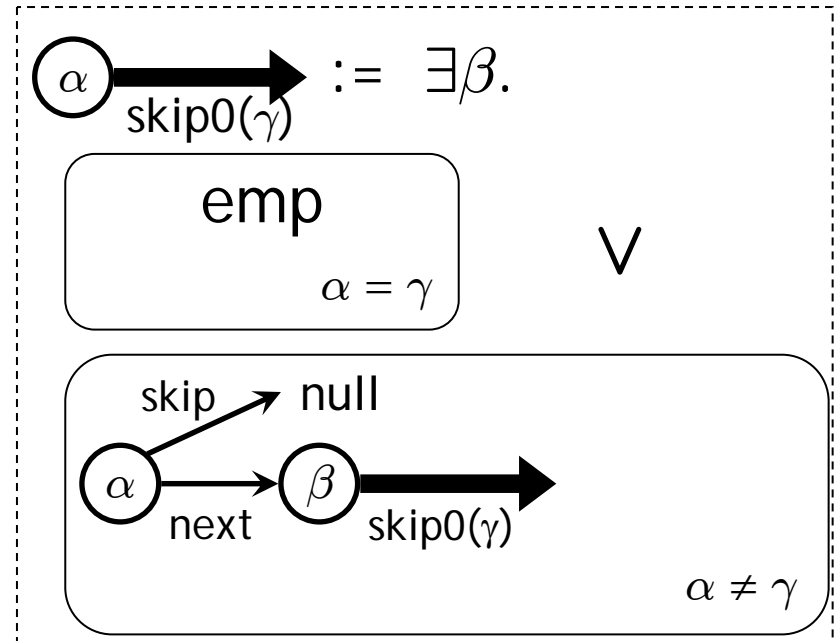
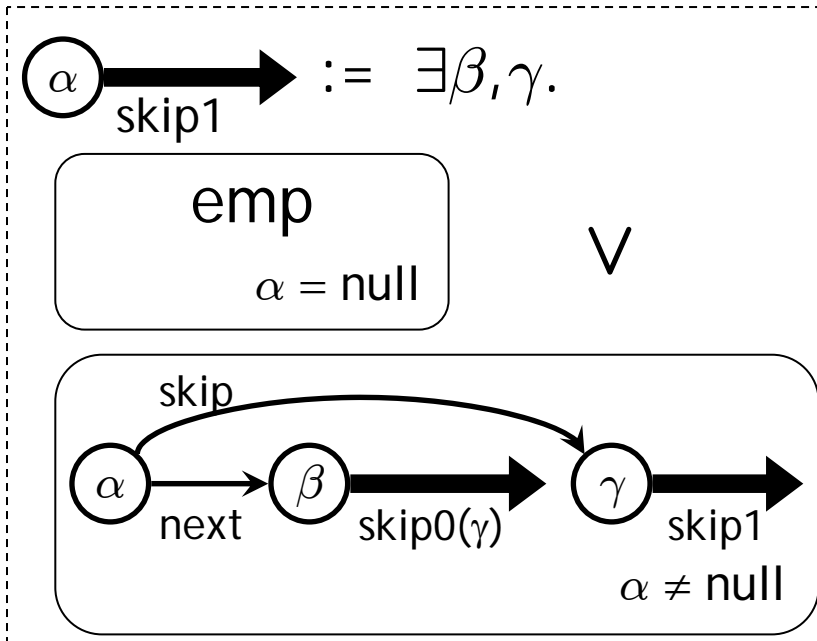


# What can a checker do?

- In this talk, a *checker* ...
  - is a pure, recursive function
  - dereferences any object field only once during a run
  - only one argument can be dereferenced (traversal arg)
  - has only additional parameters



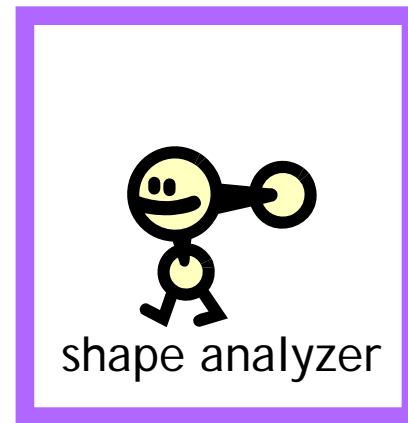
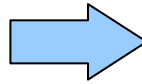
# Example checker: Two-level skip list



# back to the abstract domain ...

```
bool redlist(List* l) {  
  if (l == null)  
    return true;  
  else  
    return  
      l->color == red  
      && redlist(l->next);  
}
```

checkers



# Challenge: Intermediate invariants

```
assert(redlist(l));
```

```
cur = l;
```

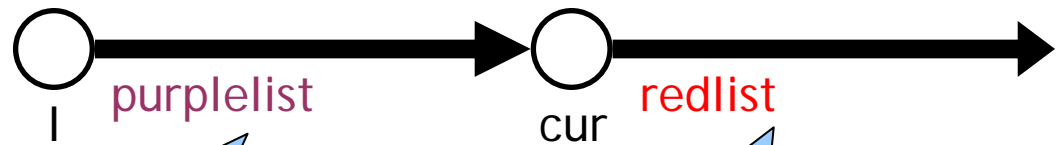
```
while (cur != null) {
```

```
    make_purple(cur);
```

```
    cur = cur→next;
```

```
}
```

```
assert(purplelist(l));
```



Prefix Segment  
Described  
by ?

Suffix  
Described  
by checkers



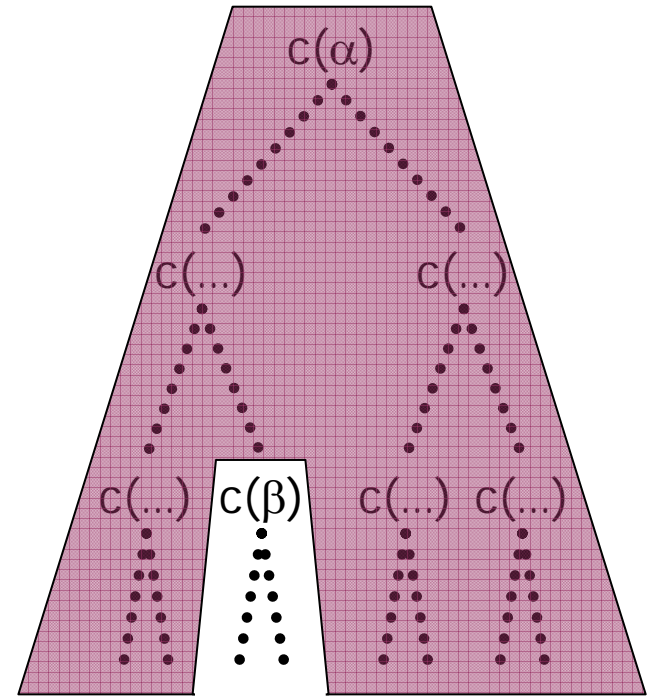
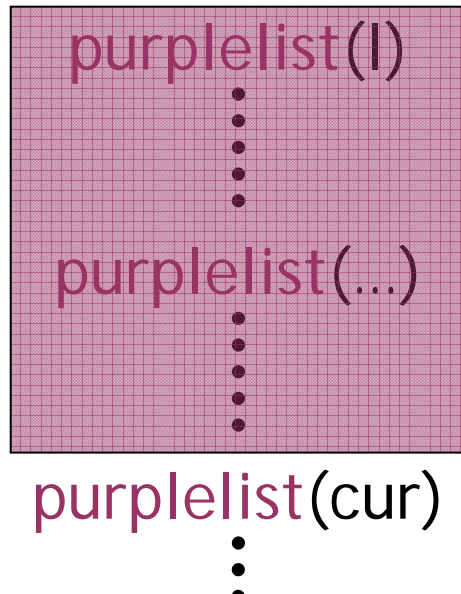


# Prefix segments as partial checker runs

Abstraction



Checker Run



Formula

Doesn't quite work because we need materialization

$$c(\alpha) * - c(\beta) \quad ?$$

# Outline

- Memory abstraction
  - Restrictions on checkers
  - Challenge: Intermediate invariants
- **Materialization by forward unfolding**
  - **Where and how**
  - **Challenge: Unfolding segments**
- Materialization by backward unfolding
  - Challenge: Back pointers
- Deciding where to unfold generically

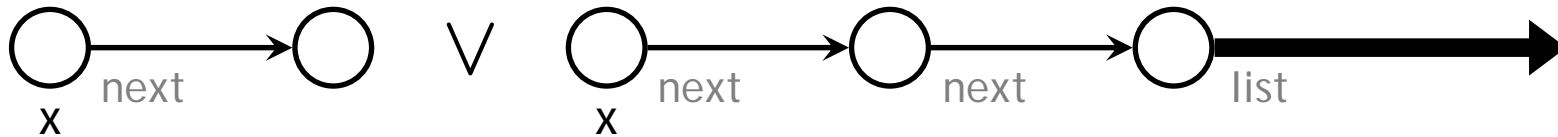
# Flow function: Unfold and update edges

$x \rightarrow \text{next} =$   
 $x \rightarrow \text{next} \rightarrow \text{next};$



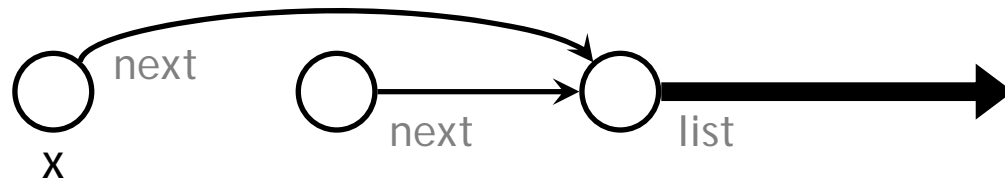
Unfold inductive definition

*materialize:*  $x \rightarrow \text{next}, x \rightarrow \text{next} \rightarrow \text{next}$



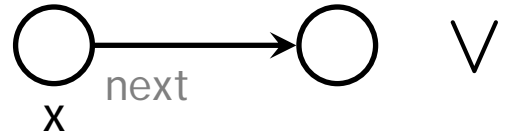
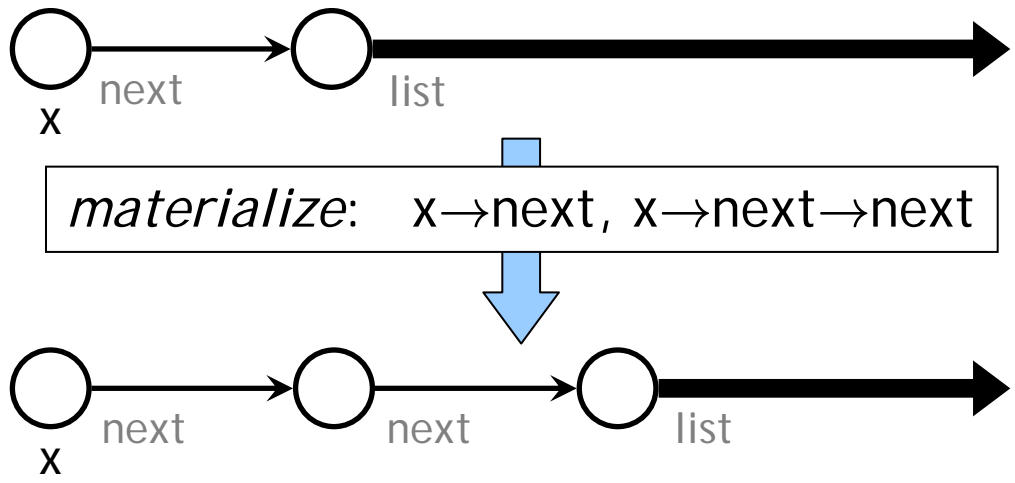
Strong updates using disjointness of regions

*update:*  $x \rightarrow \text{next} = x \rightarrow \text{next} \rightarrow \text{next}$



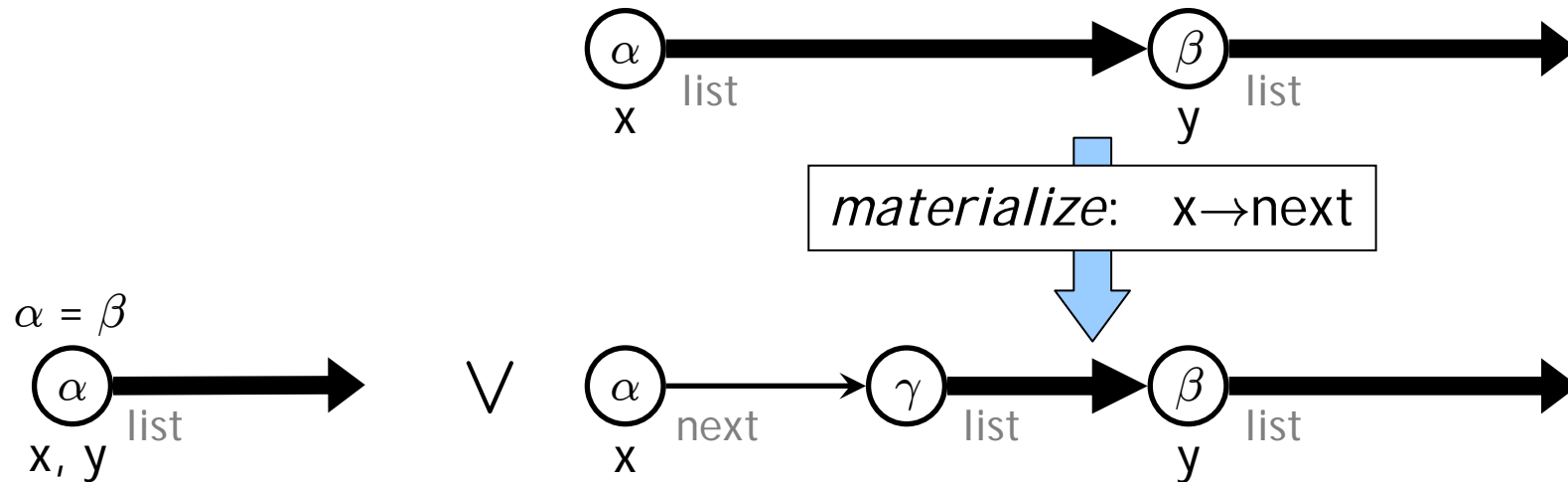
# Unfolding: where, how, and why ok

$x \rightarrow \text{next} =$   
 $x \rightarrow \text{next} \rightarrow \text{next};$

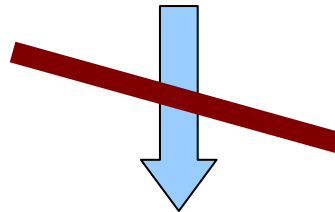


- Where
  - “Reach” a traversal argument with  $x \rightarrow \text{next}$
- How and Why Ok (concretizations same)
  - By definition

# What about unfolding segments?



$list(\alpha) * - list(\beta)$



$emp \vee \alpha @ f \mapsto \gamma * (list(\gamma) * - list(\beta))$

# Segment connector (for unfolding)

## Concrete

store  $\sigma : \text{Val} \rightarrow \text{Val}$

valuation  $\nu : \text{SymVal} \rightarrow \text{Val}$

"unfolded"  
points-to

"folded"  
recursive  
calls

pure  
formula

$c(\alpha) \doteq$   
 $\dots \vee (M^u * M^f \wedge F) \vee \dots$

$\sigma, \nu \models c(\alpha) * = c'(\alpha')$

iff there exists an  $i$  such that  $c(\alpha) * =^i c'(\alpha')$

$[\cdot], \nu \models c(\alpha) * =^0 c'(\alpha')$

iff  $\nu(\alpha) = \nu(\alpha')$

$\sigma, \nu \models c(\alpha) * =^{i+1} c'(\alpha')$

iff there exists a disjunct  $(M^u * M^f * c''(\beta) \wedge F)$  such that  
 $\nu$  satisfies [actuals/formals] $F$  and

$\sigma, \nu \models$  [actuals/formals] $(M^u * M^f * c''(\beta) * =^i c'(\alpha'))$

# Basic properties of segments

- If  $\sigma, \nu \vDash c(\alpha) *_{=} c'(\alpha')$ , then  $\sigma, \nu \vDash c(\alpha) *_{-} c'(\alpha')$ 
  - If  $\sigma, \nu \vDash (c(\alpha) *_{=} c'(\alpha')) * c'(\alpha')$ , then  $\sigma, \nu \vDash c(\alpha)$   
(elimination)
- $[\cdot], \nu \vDash c(\alpha) *_{=} c(\alpha)$  (reflexivity)
- If  $\sigma, \nu \vDash (c(\alpha) *_{=} c'(\alpha')) * (c'(\alpha') *_{=} c''(\alpha''))$ ,  
then  $\sigma, \nu \vDash c(\alpha) *_{=} c''(\alpha'')$  (transitivity)



# Outline

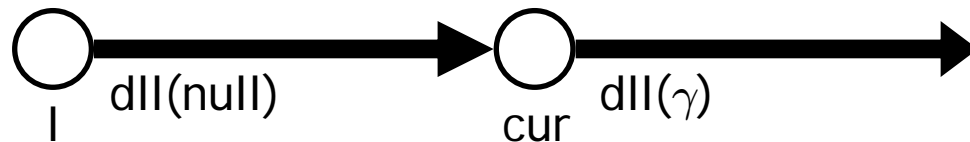
- Memory abstraction
  - Restrictions on checkers
  - Challenge: Intermediate invariants
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- **Materialization by backward unfolding**
  - **Challenge: Back pointers**
- Deciding where to unfold generically



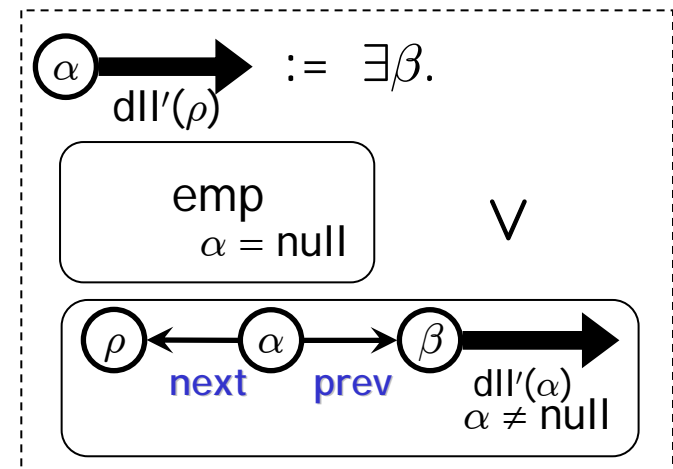
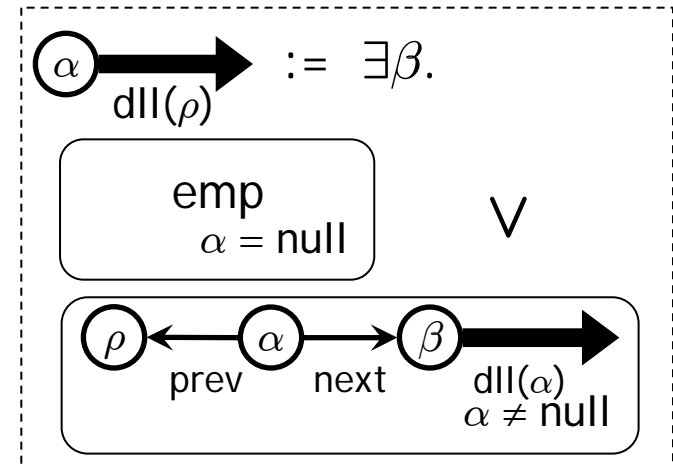
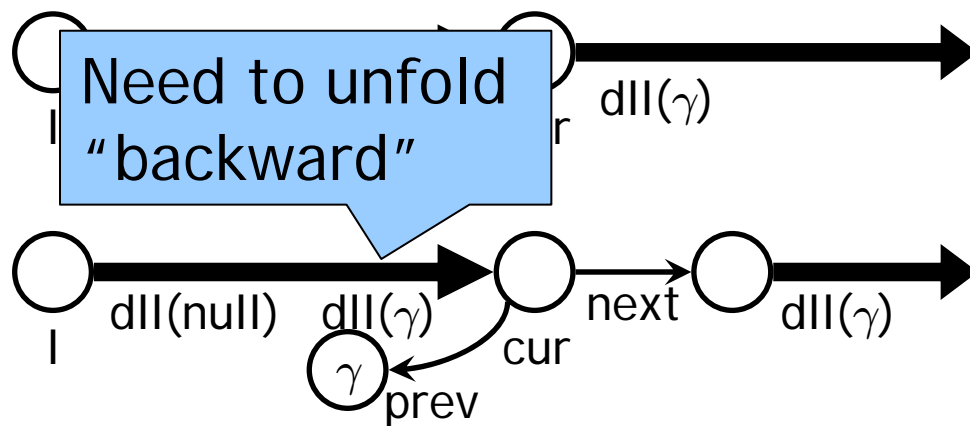
# Challenge: Back pointers

## Example: Removal in doubly-linked lists

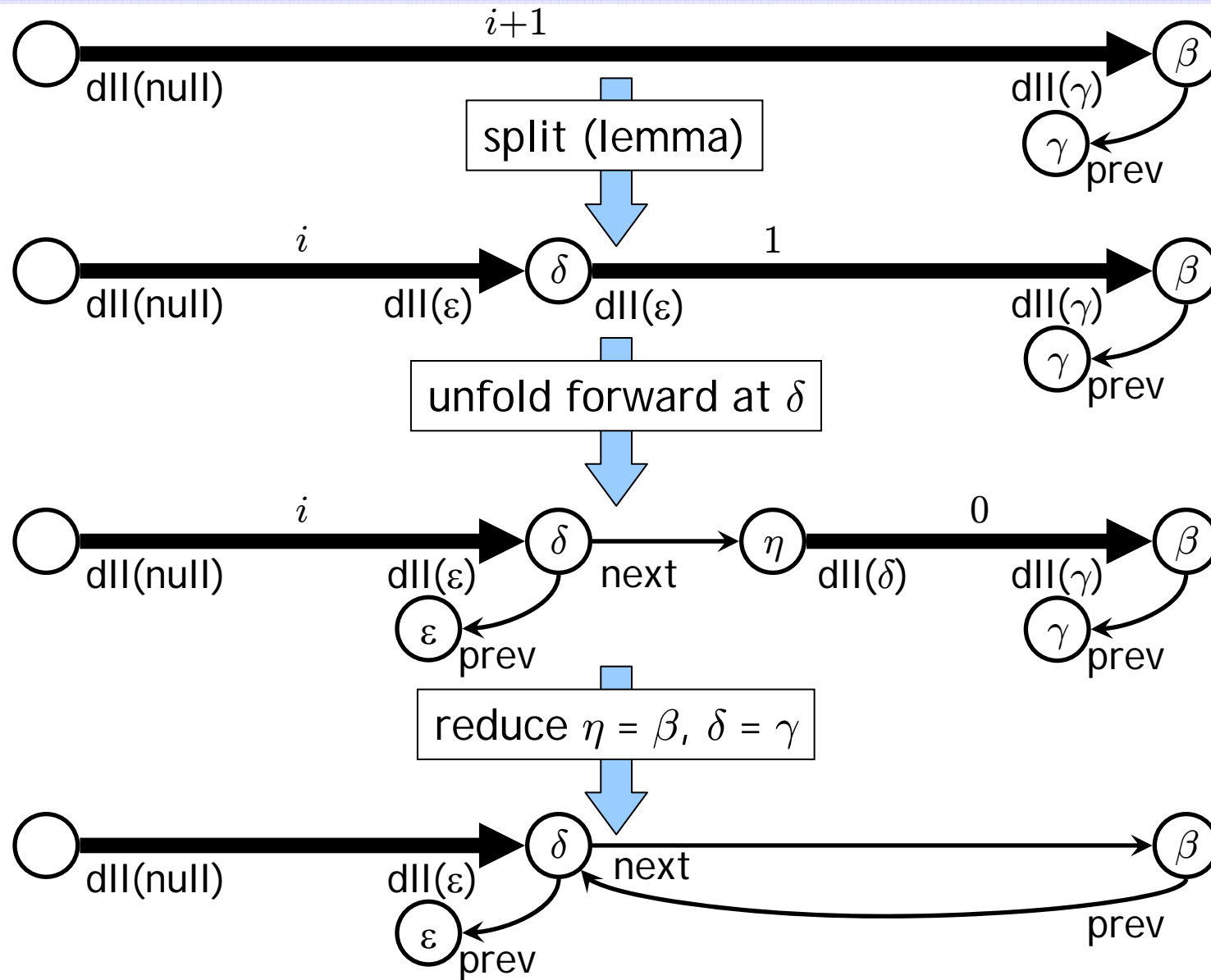
- Traversal on 'next' field to find element to remove:



- Materialize 'cur → prev' and remove 'cur':



# Backwards unfolding by forwards unfolding



# Outline

- Memory abstraction
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  - Challenge: Intermediate invariants
- Materialization by forward unfolding
  - Where and how
  - Challenge: Unfolding segments
- Materialization by backward unfolding
  - Challenge: Back pointers
- **Deciding where to unfold generically**

# Deciding where to unfold

- Observations: Callers and callees  
fields are materialized

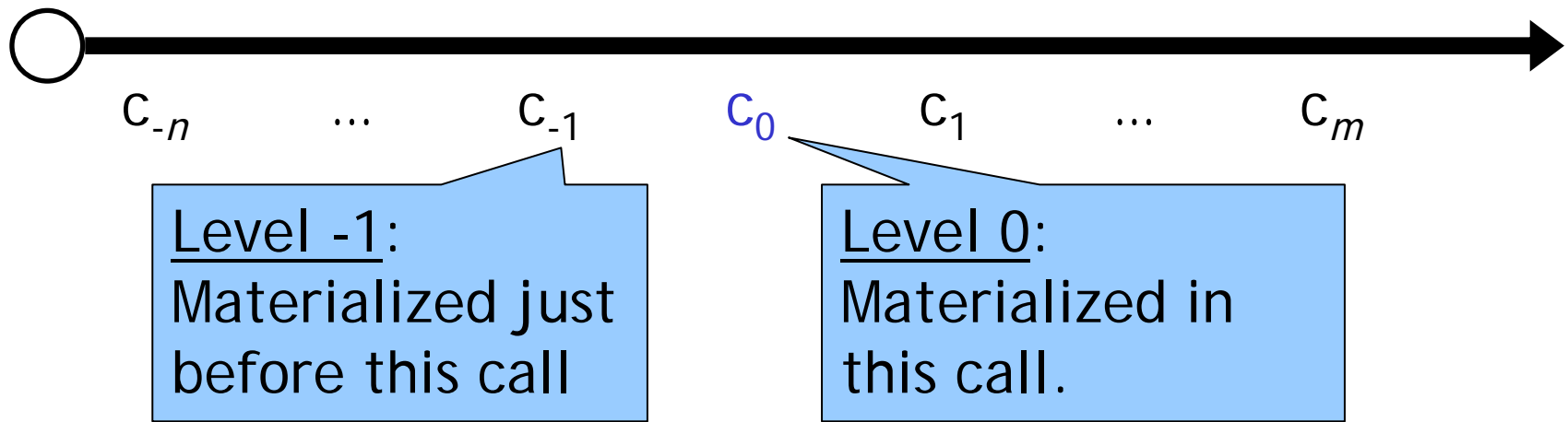
A pointer that  
may materialize  
these fields

Where in the  
traversal it may  
be materialized

types  $\tau ::= \{ f_1 \langle l_1 \rangle, \dots, f_n \langle l_n \rangle \}$

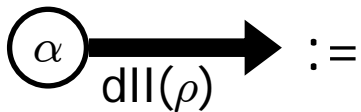
levels  $l ::= n \mid \text{unk}$

- Levels

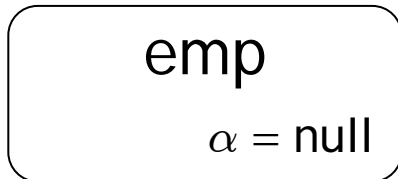


# Example: Doubly-linked lists

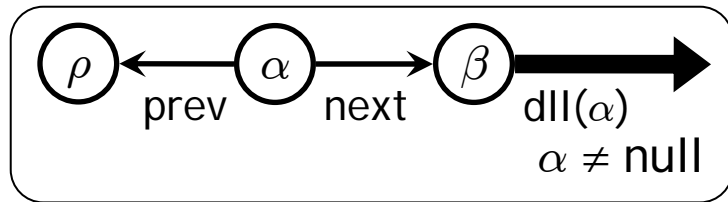
$\alpha : \{\text{next}\langle 0 \rangle, \text{prev}\langle 0 \rangle\},$   
 $\rho : \{\text{next}\langle -1 \rangle, \text{prev}\langle -1 \rangle\}$



$\exists(\beta : \{\text{next}\langle 1 \rangle, \text{prev}\langle 1 \rangle\}).$



∨



Before:

Traversal argument had level 0 fields (implicitly)

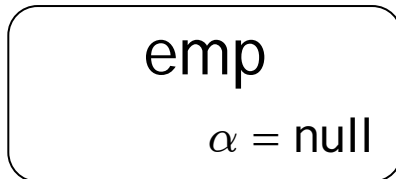
Backward unfolding parameter  $\rho$  has level -1

# Example: Alternative doubly-linked list

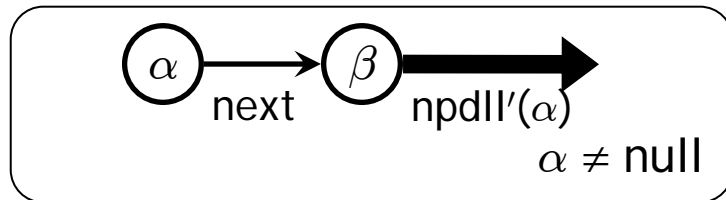
$\alpha : \{\text{next}\langle 0 \rangle, \text{prev}\langle -1 \rangle\}$



$\exists(\beta : \{\text{next}\langle 2 \rangle, \text{prev}\langle 1 \rangle\})$ .



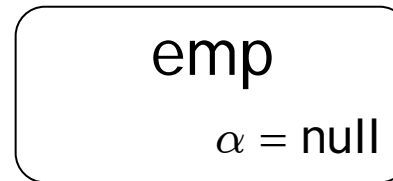
$\vee$



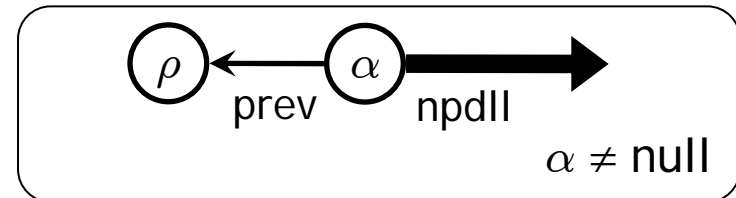
$\alpha : \{\text{next}\langle 1 \rangle, \text{prev}\langle 0 \rangle\}$ ,  
 $\rho : \{\text{next}\langle -1 \rangle, \text{prev}\langle -2 \rangle\}$



$\exists(\beta : \{\text{next}\langle 1 \rangle, \text{prev}\langle 1 \rangle\})$ .

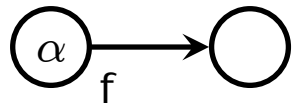


$\vee$

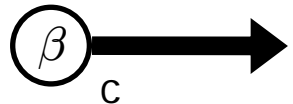


# Types can be inferred automatically

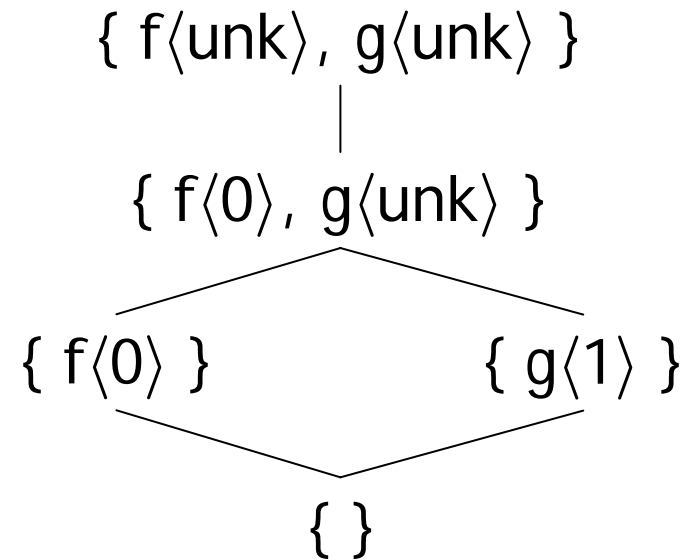
## Checking



$\{ f\langle 0 \rangle \} \leq \text{typeof}(\alpha)$



$\text{typeof}(\beta) - 1$   
 $\leq \text{declared\_typeof}(\pi)$   
(where  $c(\pi) := \dots$ )



Inference using a fixed-point computation with types initialized to  $\{\}$

## Summary:

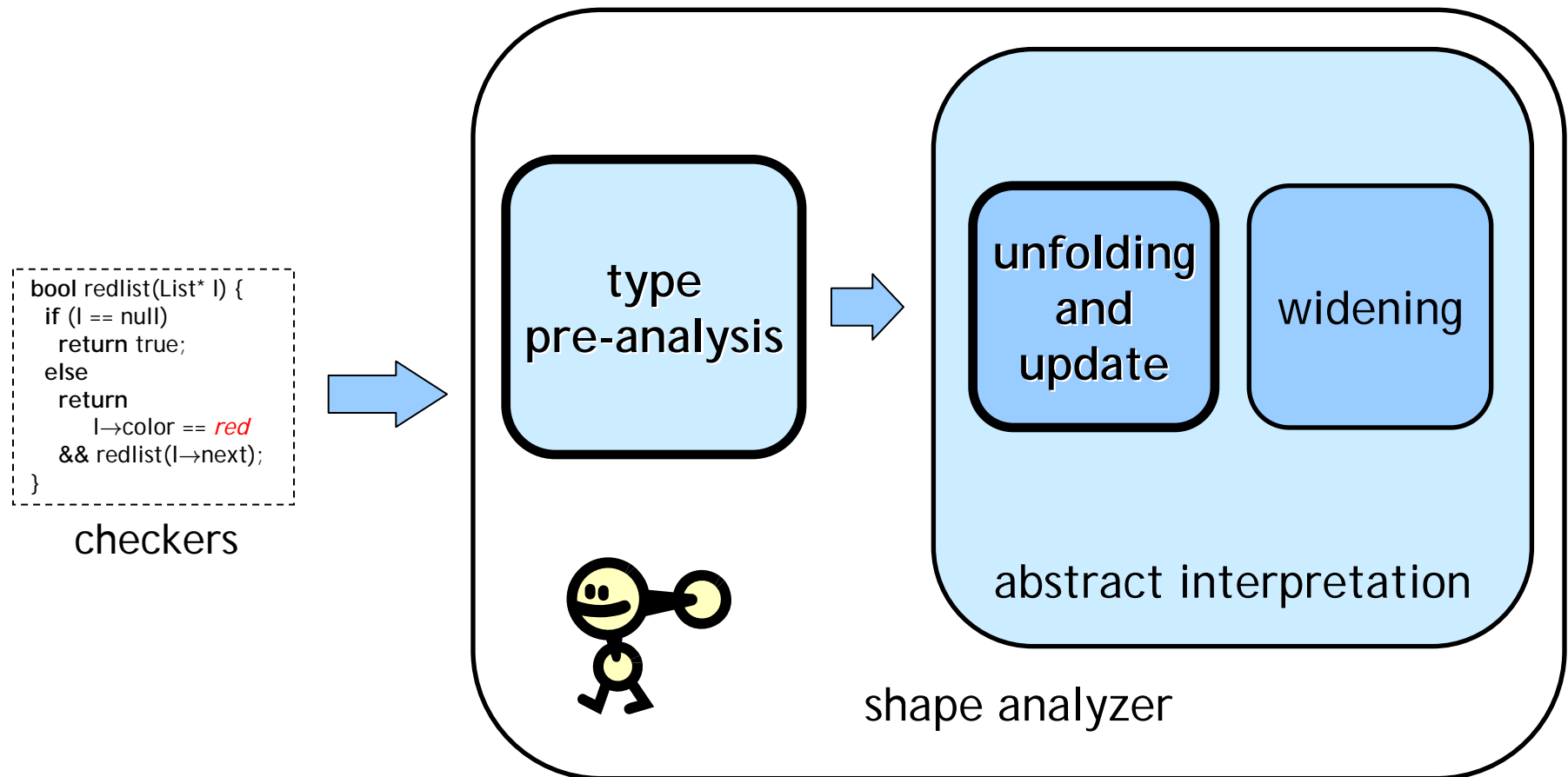
### Enabling materialization anywhere

- Defined segments as partial checker runs directly (inductively)
  - For forward unfolding
  - Backward unfolding derived from forward unfolding
- Checker parameter types with levels
  - For deciding where to unfold
  - Inferable and does not affect soundness



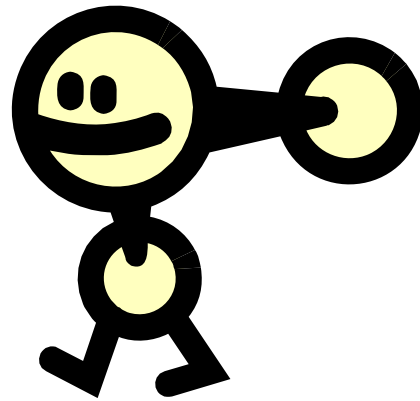
# Summary:

Given checkers, everything is automatic



# Conclusion

- Invariant checkers can form the basis of a memory abstraction that
  - Is easily extensible on a per-program basis
  - Expresses developer intent
    - Critical for usability
    - Prerequisite for scalability
- Enabling materialization anywhere
  - Inductive segments
  - Pre-analysis on checkers to decide where to unfold robustly



*What can checker-based  
shape analysis do for you?*

# Challenge: Termination and precision

```
last = l;
```

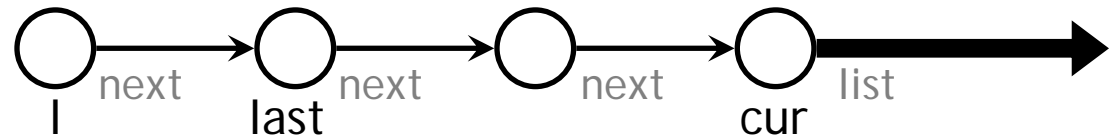
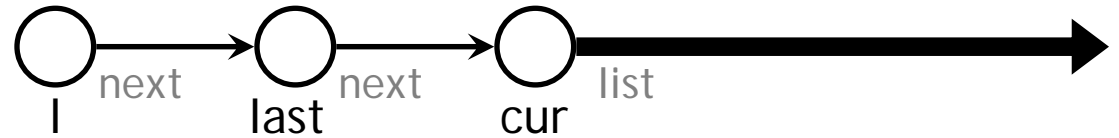
## Observation

Previous iterates are "less unfolded"

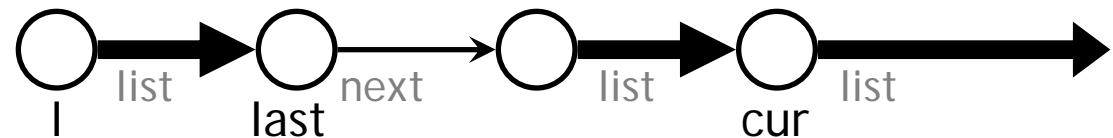
```
if (...) last = cur,  
cur = cur → next;
```

Fold into checker edges

But where and how much?



*widen (canonicalize, blur)*

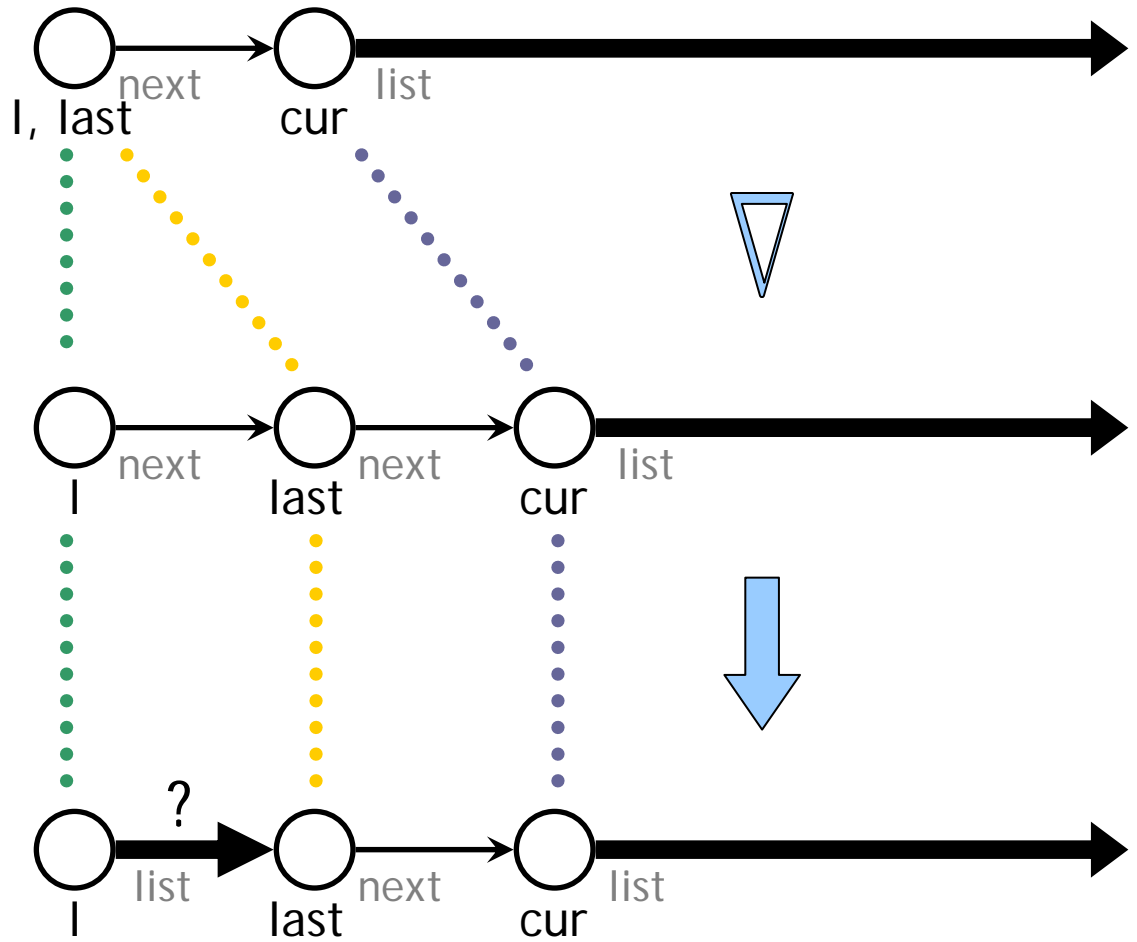
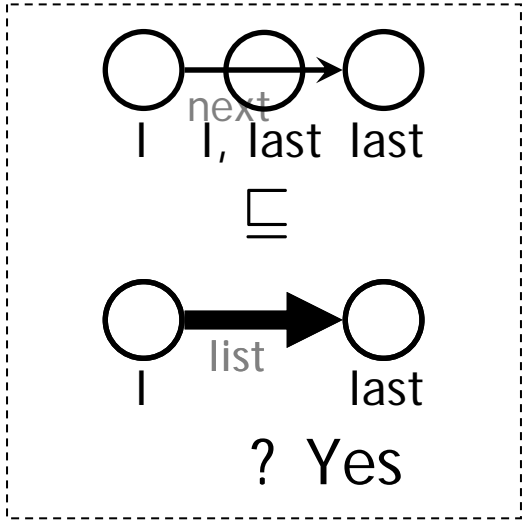


# History-guided folding

```

last = l;
cur = l → next;
while (cur != null) {
    if (...) last = cur;
    cur = cur → next;
}
    
```

- Match edges to identify where to fold
- Apply local folding rules



# Summary:

## Enabling checker-based shape analysis

- Built-in disjointness of memory regions
  - As in separation logic
  - Checkers read any object field only once in a run
- Generalized segment abstraction
  - Based on partial checker runs



- Generalized folding into inductive predicates
  - Based on iteration history (i.e., a widening operator)



# Experimental results

Benchmark	Lines of Code	Analysis Time	Max. Num. Graphs at a Program Point	Max. Num Iterations at a Program Point
list reverse	19	0.007s	1	3
list remove element	27	0.016s	4	6
list insertion sort	56	0.021s	4	7
search tree find	23	0.010s	2	4
skip list rebalance	33	0.087s	6	7
scu11 driver	894	9.710s	4	16

- Verified structural invariants as given by checkers are preserved across data structure manipulation
- Limitations (in `scu11 driver`)
  - Arrays not handled (rewrote as linked list), char arrays ignored
- Promising as far as number of disjuncts