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# Two Person Games

Mathematics
Problem Solving
Software Development

Billy has a used car for sale and is asking \$2,000. Beth offers him \$1,500. So Billy splits the difference and asks \$1,750. If Billy and Beth continue in this manner, what common price will then settle on?

# Features

- Two person
- Economic
- Fixed strategy
- Iterative
- Terminating?

# Extensions

- If Beth wanted to pay \$1600, what should her first offer have been?
- Generalize problem and solution (Billy asks \$A, Beth offers \$0)
- Program it!

# General Characteristics

- Only 2 players [Could be relaxed]
- Only thinking skills [Not physical]
- Full previous information known at all times
- No luck [Can be exceptions]
- Finishes in a reasonable time
- Little special equipment required

- Adapted from 'Popularizing Mathematics', edited by A J C Begg

### Why Games? Interdisciplinary

- Sociology
- Criminal Justice
- Philosophy
- Economics
- Biology
- Evolution
- Engineering

### Why Games? Mathematics

- How to play?
- Best way to play?
- Play to win ...
- Strategy for winning ...
- Can always win if?
- What happens if .. Variations
- Game is similar to ... Isomorphism
- Game specification ... Symbols & Notation

- Understanding
- Strategy/Optimize
- Analysis/Strategy
- Generalization
- Proof

Adapted from 'Popularizing Mathematics', edited by A J C Begg

#### Why Games? Software

- Easily understood rules
- Intellectually challenging & motivational
- Competitions (pencil & paper)
- Understanding, mathematical analysis, abstraction, reflection before programming
- Object oriented (reuse)
- Competitions (software, networks)

# Prisoners Dilemma

**Cooperation vs Conflict Game** Simultaneous Moves Prisoner/Player B Prisoner/Player A Four possibilities: -A & B both cooperate - A & B both defect - A cooperates & B defects - A defects & B cooperates

### PD Punishment & Rewards

	B cooperates	B defects		
	A gets CC	A gets CD		
A cooperates	B gets CC	B gets DC		
	A gets DC	A gets DD		
A defects	B gets CD	B gets DD		

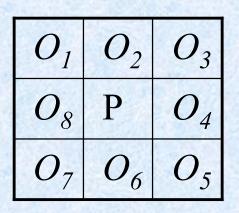
DC > CC > DD > CDCC > (DC + CD)/2

## Iterative PD - Max Rewards Strategies

- Meanie always defects
- Sucker always cooperates
- Spaz switches randomly
- Fair play adjusts to count of actions of other player
- Tit for Tat cooperates on the first round, every subsequent round mimics the other player's previous move

### 2 D Prisoners Dilemma

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Player Cooperates

**Player Defects** 

	Opponent	Opponent			
	Cooperates	Defects			
es	1, 1	0, <i>b</i>			
ts	b, 0	0, 0			

is cooperating, did cooperate
is defecting, did defect
is cooperating, did defect
is defecting, did cooperate

b: advantage for defection when opponent cooperatesp: fraction (0..1) of defectors in the first round