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Two Person Games

- Mathematics
- Problem Solving
- Software Development

Billy has a used car for sale and is asking $2,000. Beth offers him $1,500. So Billy splits the difference and asks $1,750. If Billy and Beth continue in this manner, what common price will then settle on?

Features

- Two person
- Economic
- Fixed strategy
- Iterative
- Terminating?

Extensions

- If Beth wanted to pay $1600, what should her first offer have been?
- Generalize problem and solution (Billy asks $A, Beth offers $O)
- Program it!

General Characteristics

- Only 2 players [*Could be relaxed*]
- Only thinking skills [*Not physical*]
- Full previous information known at all times
- No luck [*Can be exceptions*]
- Finishes in a reasonable time
- Little special equipment required
  - *Adapted from 'Popularizing Mathematics', edited by A J C Begg*

Why Games? Interdisciplinary

- Sociology
- Criminal Justice
- Philosophy
- Economics
- Biology
- Evolution
- Engineering

Why Games? Mathematics

- How to play?
- Best way to play?
- Play to win …
- Strategy for winning ..
- Can always win if?
- What happens if ..
- Game is similar to …
- Game specification …

- *Understanding*
- *Strategy/Optimize*
- *Analysis/Strategy*
- *Generalization*
- *Proof*
- *Variations*
- *Isomorphism*
- *Symbols & Notation*

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Why Games? Software

- Easily understood rules
- Intellectually challenging & motivational
- Competitions (pencil & paper)
- Understanding, mathematical analysis, abstraction, reflection before programming
- Object oriented (reuse)
- Competitions (software, networks)

Prisoners Dilemma

Cooperation vs Conflict Game

Simultaneous Moves

Prisoner/Player A Prisoner/Player B

Four possibilities:

- A & B both cooperate
- A & B both defect
- A cooperates & B defects
- A defects & B cooperates

P D Punishment & Rewards

	B cooperates	B defects
A cooperates	A gets CC B gets CC	A gets CD B gets DC
A defects	A gets DC B gets CD	A gets DD B gets DD

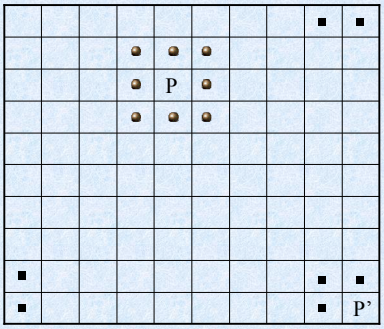
DC > CC > DD > CD

CC > (DC + CD)/2

Iterative PD - Max Rewards Strategies

- Meanie – *always defects*
- Sucker – *always cooperates*
- Spaz – *switches randomly*
- Fair play – *adjusts to count of actions of other player*
- Tit for Tat - *cooperates on the first round, every subsequent round mimics the other player's previous move*

2 D Prisoners Dilemma



O_1	O_2	O_3
O_8	P	O_4
O_7	O_6	O_5

	Opponent Cooperates	*Opponent Defects*
Player Cooperates	1, *1*	0, *b*
Player Defects	b, *0*	0, *0*

- is cooperating, did cooperate
- is defecting, did defect
- is cooperating, did defect
- is defecting, did cooperate

b: advantage for defection when opponent cooperates

p: fraction (0..1) of defectors in the first round