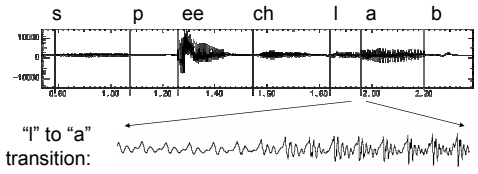
# CS 294-5: Statistical Natural Language Processing

N-Grams and Smoothing

Lecture 2: 8/31/05

## Speech in a Slide (or Three)

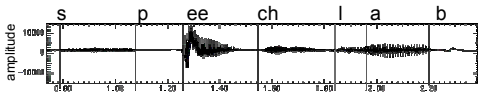
- Speech input is an acoustic wave form



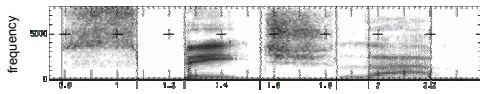

Graphs from Simon Arnfield's web tutorial on speech, Sheffield:
http://www.psyc.leeds.ac.uk/research/cogn/speech/tutorial/
Some later bits from Joshua Goodman's LM tutorial

## Spectral Analysis

- Frequency gives pitch; amplitude gives volume
  - sampling at ~8 kHz phone, ~16 kHz mic (kHz=1000 cycles/sec)

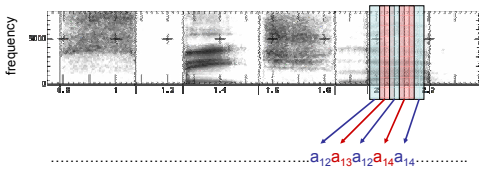



- Fourier transform of wave displayed as a spectrogram
  - darkness indicates energy at each frequency




## Acoustic Feature Sequence

- Time slices are translated into acoustic feature vectors (~15 real numbers per slice)




- Now we have to figure out a mapping from sequences of acoustic observations to words.

## The Speech Recognition Problem

- We want to predict a sentence given an acoustic sequence:

$$s^* = \arg\max_s P(s \mid A)$$

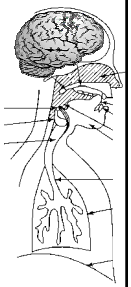
- The noisy channel approach:
  - Build a generative model of production (encoding)

$$P(A, s) = P(s)\,P(A \mid s)$$

  - To decode, we use Bayes' rule to write

$$s^* = \arg\max_s P(s \mid A)$$
$$= \arg\max_s P(s)P(A \mid s)/P(A)$$
$$= \arg\max_s P(s)P(A \mid s)$$

  - Now, we have to find a sentence maximizing this product
- Why is this progress?



## Other Noisy-Channel Processes

- Handwriting recognition

$$P(text \mid strokes) \propto P(text)P(strokes \mid text)$$

- OCR

$$P(text \mid pixels) \propto P(text)P(pixels \mid text)$$

- Spelling Correction

$$P(text \mid typos) \propto P(text)P(typos \mid text)$$

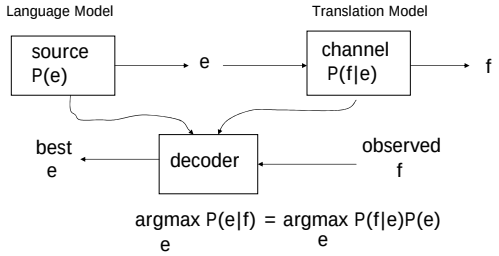
- Translation?

$$P(english \mid french) \propto P(english)P(french \mid english)$$

## Just a Code?

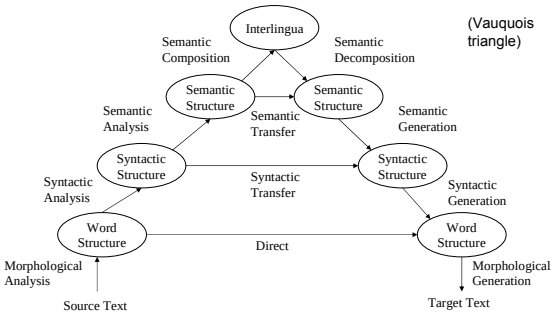
- "Also knowing nothing official about, but having guessed and inferred considerable about, the powerful new mechanized methods in cryptography—methods which I believe succeed even when one does not know what language has been coded—one naturally wonders if the problem of translation could conceivably be treated as a problem in cryptography. When I look at an article in Russian, I say: 'This is really written in English, but it has been coded in some strange symbols. I will now proceed to decode.' "
  - Warren Weaver (1955:18, quoting a letter he wrote in 1947)

## MT System Components




$$\text{argmax}_e P(e|f) = \text{argmax}_e P(f|e)P(e)$$

## Levels of Transfer




## Probabilistic Language Models

- Want to build models which assign scores to sentences.
  - P(I saw a van) >> P(eyes awe of an)
  - Not really grammaticality: P(artichokes intimidate zippers) ≈ 0
- One option: empirical distribution over sentences?
  - Problem: doesn't generalize (at all)
- Two ways of generalizing
  - Decomposition: sentences generated in small steps which can be recombined in other ways
  - Smoothing: allow for the possibility of unseen events

## N-Gram Language Models

- No loss of generality to break sentence probability down with the chain rule

$$P(w_1 w_2 \ldots w_n) = \prod_i P(w_i \mid w_1 w_2 \ldots w_{i-1})$$

- Too many histories!
- N-gram solution: assume each word depends only on a short linear history

$$P(w_1 w_2 \ldots w_n) = \prod_i P(w_i \mid w_{i-k} \ldots w_{i-1})$$

## Regular Languages?

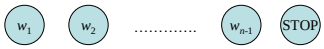
- N-gram models are (weighted) regular processes
  - Why can't we model language like this?
    - Linguists have many arguments why language can't be merely regular.
    - Long-distance effects:
      "The computer which I had just put into the machine room on the fifth floor crashed."
  - Why CAN we often get away with n-gram models?

# Unigram Models

- Simplest case: unigrams

$$P(w_1 w_2 \ldots w_n) = \prod_i P(w_i)$$

- Generative process: pick a word, pick a word, …
- As a graphical model:




- To make this a proper distribution over sentences, we have to generate a special STOP symbol last. (Why?)
- Examples:
  - [fifth, an, of, futures, the, an, incorporated, a, a, the, inflation, most, dollars, quarter, in, is, mass.]
  - [thrift, did, eighty, said, hard, 'm, july, bullish]
  - [that, or, limited, the]
  - []
  - [after, any, on, consistently, hospital, lake, of, of, other, and, factors, raised, analyst, too, allowed, mexico, never, consider, fall, bungled, davison, that, obtain, price, lines, the, to, sass, the, the, further, board, a, details, machinists, the, companies, which, rivals, an, because, longer, oakes, percent, a, they, three, edward, it, currier, an, within, in, three, wrote, is, you, s., longer, institute, dentistry, pay, however, said, possible, to, rooms, hiding, eggs, approximate, financial, canada, the, so, workers, advancers, half, between, nasdaq]

# Bigram Models

- Big problem with unigrams: P(the the the the) >> P(I like ice cream)!
- Condition on last word:

$$P(w_1 w_2 \ldots w_n) = \prod_i P(w_i \mid w_{i-1})$$


- Any better?
  - [texaco, rose, one, in, this, issue, is, pursuing, growth, in, a, boiler, house, said, mr., gurria, mexico, 's, motion, control, proposal, without, permission, from, five, hundred, fifty, five, yen]
  - [outside, new, car, parking, lot, of, the, agreement, reached]
  - [although, common, shares, rose, forty, six, point, four, hundred, dollars, from, thirty, seconds, at, the, greatest, play, disingenuous, to, be, reset, annually, the, buy, out, of, american, brands, vying, for, mr., womack, currently, sharedata, incorporated, believe, chemical, prices, undoubtedly, will, be, as, much, is, scheduled, to, conscientious, teaching]
  - [this, would, be, a, record, november]

# Is This Working?

- The game isn't to pound out fake sentences!
- What we really want to know is:
  - Will our model prefer good sentences to bad ones?
  - Bad ≠ ungrammatical!
  - Bad ≈ unlikely
  - Bad = sentences that our acoustic model really likes but aren't the correct answer

# Measuring Model Quality

- Word Error Rate (WER)

$$\frac{\text{insertions} + \text{deletions} + \text{substitutions}}{\text{true sentence size}}$$

Correct answer: Andy saw a part of the movie

Recognizer output: And he saw apart of the movie

*WER: 4/7 = 57%*

- The "right" measure:
  - Task error driven
  - For speech recognition
  - For a specific recognizer!
- For general evaluation, we want a measure which references only good text, not mistake text

# Measuring Model Quality

- The Shannon Game:
  - How well can we predict the next word?

When I order pizza, I wipe off the ____

Many children are allergic to ____

I saw a ____

  - Unigrams are terrible at this game. (Why?)

grease 0.5
sauce 0.4
dust 0.05
….
mice 0.0001
….
the 1e-100

- The "Entropy" Measure
  - Really: average cross-entropy of a text according to a model

$$H(S \mid M) = \frac{\log_2 P_M(S)}{|S|} = \frac{\sum_i \log_2 P_M(s_i)}{\sum_i |s_i|}$$

$$\sum_j \log_2 P_M(w_j \mid w_{j-1})$$

# Measuring Model Quality

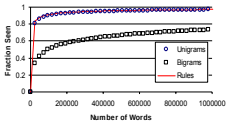
- Problem with entropy:
  - 0.1 bits of improvement doesn't sound so good
  - Solution: perplexity

$$P(S \mid M) = 2^{H(S|M)} = \sqrt[n]{\frac{1}{\prod_{i=1}^{n} P_M(w_i \mid h)}}$$

  - Note that even though our models require a stop step, we typically don't count it as a symbol when taking these averages.

# Sparsity

- Problems with n-gram models:
  - New words appear all the time:
    - Synaptitute
    - 132,701.03
    - fuzzificational
  - New bigrams: even more often
  - Trigrams or more – still worse!

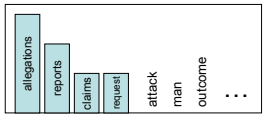



- Zipf's Law
  - Types (words) vs. tokens (word occurences)
  - Broadly: most word types are rare
  - Specifically:
    - Rank word types by token frequency
    - Frequency inversely proportional to rank
  - Not special to language: randomly generated character strings have this property

# Smoothing

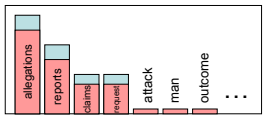
- We often want to make estimates from sparse statistics:

P(w | denied the)
3 allegations
2 reports
1 claims
1 request
7 total




- Smoothing flattens spiky distributions so they generalize better

P(w | denied the)
2.5 allegations
1.5 reports
0.5 claims
0.5 request
2 other
7 total




- Very important all over NLP, but easy to do badly!
- We'll illustrate with bigrams today (h = previous word, could be anything).

# Smoothing

- Estimating multinomials
  - We want to know what words follow some history h
  - There's some true distribution P(w | h)
  - We saw some small sample of N words from P(w | h)
  - We want to reconstruct a useful approximation of P(w | h)
  - Counts of events we didn't see are always too low (0 < N P(w | h))
  - Counts of events we did see are *in aggregate* to high
- Example:

P(w | denied the)
3 allegations
2 reports
1 claims
1 speculation
…
1 request
13 total

P(w | affirmed the)
1 award

- Two issues:
  - Discounting: how to reserve mass what we haven't seen
  - Interpolation: how to allocate that mass amongst unseen events

# Add-One Estimation

- Idea: pretend we saw every word once more than we actually did [Laplace]

$$P(w \mid h) = \frac{c(w,h)+1}{c(h)+V}$$

  - Corresponds to a uniform prior over vocabulary
  - Think of it as taking items with observed count r > 1 and treating them as having count r* < r
  - Holds out V/(N+V) for "fake" events
    - $N_{1+}$/N of which is distributed back to seen words
    - $N_0$/(N+V) actually passed on to unseen words (nearly all!)
    - Actually tells us not only how much to hold out, but where to put it
  - Works astonishingly poorly in practice
- Quick fix: add some small δ instead of 1 [Lidstone, Jefferys]
  - Slightly better, holds out less mass, still a bad idea

# How Much Mass to Withhold?

- Remember the key discounting problem:
  - What count should r* should we use for an event that occurred r times in N samples?
  - r is too big
- Idea: held-out data [Jelinek and Mercer]
  - Get another N samples
  - See what the average count of items occuring r times is (e.g. doubletons on average might occur 1.78 times)
  - Use those averages as r*
- Much better than add-one, etc.

# Smoothing: Add-One, Etc.

| | |
|---|---|
| $c$ | number of word tokens in training data |
| $c(w)$ | count of word $w$ in training data |
| $c(w,w_{-1})$ | count of word $w$ following word $w_{-1}$ |
| V | total vocabulary size |
| $N_k$ | number of word types with count $k$ |

- One class of smoothing functions:
  - Add-one / delta: assumes a uniform prior

$$P_{ADD-\delta}(w \mid w_{-1}) = \frac{c(w,w_{-1})+\delta(1/V)}{c(w_{-1})+\delta}$$

  - Better to assume a unigram prior

$$P_{UNI-PRIOR}(w \mid w_{-1}) = \frac{c(w,w_{-1})+\delta\hat{P}(w)}{c(w_{-1})+\delta}$$

# Linear Interpolation

- One way to ease the sparsity problem for n-grams is to use less-sparse n-1-gram estimates
- General linear interpolation:

$$P(w \mid w_{-1}) = [1 - \lambda(w, w_{-1})]\hat{P}(w \mid w_{-1}) + [\lambda(w, w_{-1})]P(w)$$

  - Having a single global mixing constant is generally not ideal:

$$P(w \mid w_{-1}) = [1 - \lambda]\hat{P}(w \mid w_{-1}) + [\lambda]P(w)$$

  - Solution: have different constant buckets
    - Buckets by count
    - Buckets by average count (better)

# Held-Out Data

- Important tool for getting models to generalize:

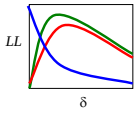

  - When we have a small number of parameters that control the degree of smoothing, we set them to maximize the (log-)likelihood of held-out data

$$LL(w_1 \ldots w_n \mid M(\lambda_1 \ldots \lambda_k)) = \sum_i \log P_{M(\lambda_1 \ldots \lambda_k)}(w_i \mid w_{i-1})$$

  - Can use any optimization technique (line search or EM usually easiest)
- Examples:

$$P_{LIN(\lambda_1,\lambda_2)}(w \mid w_{-1}) = \lambda_1 \hat{P}(w \mid w_{-1}) + \lambda_2 \hat{P}(w)$$

$$P_{UNI-PRIOR(\delta)}(w \mid w_{-1}) = \frac{c(w, w_{-1}) + \delta \hat{P}(w)}{c(w_{-1}) + \delta}$$




# Held-Out Reweighting

- What's wrong with unigram-prior smoothing?
- Let's look at some real bigram counts [Church and Gale 91]:

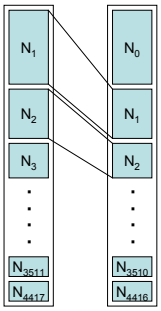
| Count in 22M Words | Actual c* (Next 22M) | Add-one's c* | Add-0.0000027's c* |
|---|---|---|---|
| 1 | 0.448 | 2/7e-10 | ~1 |
| 2 | 1.25 | 3/7e-10 | ~2 |
| 3 | 2.24 | 4/7e-10 | ~3 |
| 4 | 3.23 | 5/7e-10 | ~4 |
| 5 | 4.21 | 6/7e-10 | ~5 |

| Mass on New | 9.2% | ~100% | 9.2% |
|---|---|---|---|
| Ratio of 2/1 | 2.8 | 1.5 | ~2 |

- Big things to notice:
  - Add-one vastly overestimates the fraction of new bigrams
  - Add-0.0000027 still underestimates the ratio 2*/1*
- One solution: use held-out data to predict the map of c to c*

# Good-Turing Reweighting I

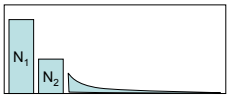
- We'd like to not need held-out data (why?)
- Idea: leave-one-out validation
  - Take each of the c training words out in turn
  - c training sets of size c-1, held-out of size 1
  - What fraction of held-out words are unseen in training?
    - $N_1/c$
  - What fraction of held-out words are seen k times in training?
    - $(k+1)N_{k+1}/c$
  - So in the future we expect $(k+1)N_{k+1}/c$ of the words to be those with training count k
  - There are $N_k$ words with training count k
  - Each should occur with probability:
    - $(k+1)N_{k+1}/c/N_k$
  - …or expected count $(k+1)N_{k+1}/N_k$

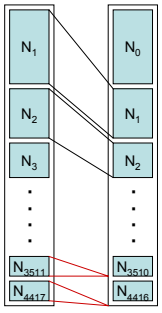



# Good-Turing Reweighting II

- Problem: what about "the"? (say c=4417)
  - For small k, $N_k > N_{k+1}$
  - For large k, too jumpy, zeros wreck estimates


  - Simple Good-Turing [Gale and Sampson]: replace empirical $N_k$ with a best-fit power law once count counts get unreliable




# Good-Turing Reweighting III

- Hypothesis: counts of k should be $k^* = (k+1)N_{k+1}/N_k$

| Count in 22M Words | Actual c* (Next 22M) | GT's c* |
|---|---|---|
| 1 | 0.448 | 0.446 |
| 2 | 1.25 | 1.26 |
| 3 | 2.24 | 2.24 |
| 4 | 3.23 | 3.24 |
| Mass on New | 9.2% | 9.2% |

- Katz Smoothing
  - Use GT discounted *bigram* counts (roughly – Katz left large counts alone)
  - Whatever mass is left goes to empirical unigram

$$P_{KATZ}(w \mid w_{-1}) = \frac{c^*(w, w_{-1})}{\sum_w c(w, w_{-1})} + \alpha(w_{-1})\hat{P}(w)$$

# Kneser-Ney Smoothing I

- Something's been very broken all this time
  - Shannon game: There was an unexpected ____?
    - delay?
    - Francisco?
  - "Francisco" is more common than "delay"
  - … but "Francisco" always follows "San"
- Solution: Kneser-Ney smoothing
  - In the back-off model, we don't want the unigram probability of w
  - Instead, probability given that we are observing a novel continuation
  - Every bigram type was a novel continuation the first time it was seen

$$P_{CONTINUATION}(w) = \frac{|\{w_{-1} : c(w, w_{-1}) > 0\}|}{|(w, w_{-1}) : c(w, w_{-1}) > 0|}$$

# Kneser-Ney Smoothing II

- One more aspect to Kneser-Ney:
  - Look at the GT counts:

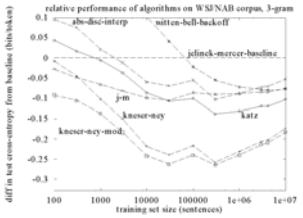
| Count in 22M Words | Actual c* (Next 22M) | GT's c* |
|---|---|---|
| 1 | 0.448 | 0.446 |
| 2 | 1.25 | 1.26 |
| 3 | 2.24 | 2.24 |
| 4 | 3.23 | 3.24 |

- Absolute Discounting
  - Save ourselves some time and just subtract 0.75 (or some d)
  - Maybe have a separate value of d for very low counts

$$P_{KN}(w \mid w_{-1}) = \frac{c(w, w_{-1}) - D}{\sum_{w'} c(w', w_{-1})} + \alpha(w_{-1}) P_{CONTINUATION}(w)$$

# What Actually Works?

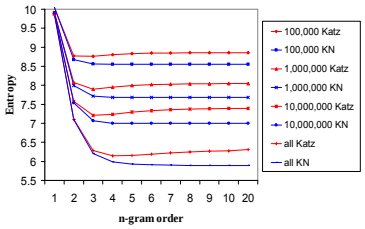
- Trigrams:
  - Unigrams, bigrams too little context
  - Trigrams much better (when there's enough data)
  - 4-, 5-grams usually not worth the cost (which is more than it seems, due to how speech recognizers are constructed)
- Good-Turing-like methods for count adjustment
  - Absolute discounting, Good-Turing, held-out estimation, Witten-Bell
- Kneser-Ney equalization for lower-order models
- See [Chen+Goodman] reading for tons of graphs!




[Graphs from Joshua Goodman]

# Data >> Method?

- Having more data is always good…




- … but so is picking a better smoothing mechanism!
- N > 3 often not worth the cost (greater than you'd think)

# Beyond N-Gram LMs

- Caching Models
  - Recent words more likely to appear again

$$P_{CACHE}(w \mid history) = \lambda P(w \mid w_{-1} w_{-2}) + (1 - \lambda) \frac{c(w \in history)}{|history|}$$

  - Can be disastrous in practice for speech (why?)
- Skipping Models

$$P_{SKIP}(w \mid w_{-1}, w_{-2}) = \lambda_1 \hat{P}(w \mid w_{-1}, w_{-2}) + \lambda_2 \hat{P}(w \mid w_{-1}, \_) + \lambda_3 \hat{P}(w \mid \_, w_{-2})$$

- Trigger Models: condition on bag of history words (e.g., maxent)
- Structured Models: use parse structure (we'll see these later)

# For Next Time

- Readings: M+S 6, J+M 6, Chen & Goodman (on web page)
- Assignment 1 is out!
- Next up: More smoothing, EM