CS252
Graduate Computer Architecture
Lecture 26

Quantum Computing and Quantum CAD Design
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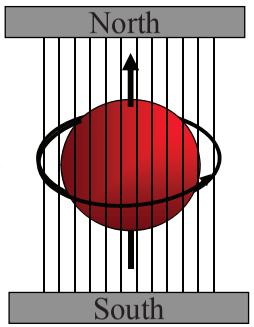
Use Quantum Mechanics to Compute?

- Weird but useful properties of quantum mechanics:
  - Quantization: Only certain values or orbits are good
    - » Remember orbitals from chemistry???
  - Superposition: Schizophrenic physical elements don't quite know whether they are one thing or another
- All existing digital abstractions try to eliminate QM
  - Transistors/Gates designed with classical behavior
  - Binary abstraction: a "1" is a "1" and a "0" is a "0"
- Quantum Computing:
  Use of Quantization and Superposition to compute.
- Interesting results:
  - Shor's algorithm: factors in polynomial time!
  - Grover's algorithm: Finds items in unsorted database in time proportional to square-root of n.
  - Materials simulation: exponential classically, linear-time QM

Quantization: Use of "Spin"

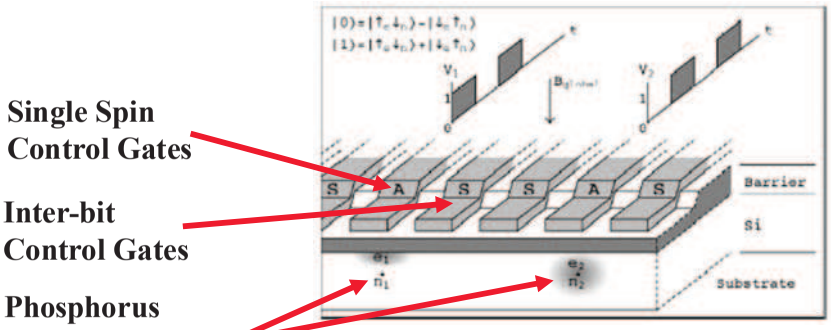


Spin ½ particle:
(Proton/Electron)

Representation:
|0> or |1>

- Particles like Protons have an intrinsic "Spin" when defined with respect to an external magnetic field
- Quantum effect gives "1" and "0":
  - Either spin is "UP" or "DOWN" nothing between

Kane Proposal II
(First one didn't quite work)



- Bits Represented by combination of proton/electron spin
- Operations performed by manipulating control gates
  - Complex sequences of pulses perform NMR-like operations
- Temperature < 1 Kelvin!

Now add Superposition!

- The bit can be in a combination of "1" and "0":
  - Written as: $\Psi = C_0|0\rangle + C_1|1\rangle$
  - The C's are *complex numbers!*
  - Important Constraint: $|C_0|^2 + |C_1|^2 = 1$
- If *measure* bit to see what looks like,
  - With probability $|C_0|^2$ we will find $|0\rangle$ (say "UP")
  - With probability $|C_1|^2$ we will find $|1\rangle$ (say "DOWN")
- Is this a real effect? Options:
  - This is just statistical – given a large number of protons, a fraction of them ($|C_0|^2$) are "UP" and the rest are down.
  - This is a real effect, and the proton is really both things until you try to look at it
- Reality: second choice!
  - There are experiments to prove it!

A register can have many values!

- Implications of superposition:
  - An *n*-bit register can have 2^n values simultaneously!
  - 3-bit example:

$$\Psi = C_{000}|000\rangle + C_{001}|001\rangle + C_{010}|010\rangle + C_{011}|011\rangle + C_{100}|100\rangle + C_{101}|101\rangle + C_{110}|110\rangle + C_{111}|111\rangle$$

- Probabilities of measuring all bits are set by coefficients:
  - So, prob of getting $|000\rangle$ is $|C_{000}|^2$, etc.
  - Suppose we measure only one bit (first):
    - » We get "0" with probability: $P_0 = |C_{000}|^2 + |C_{001}|^2 + |C_{010}|^2 + |C_{011}|^2$
      Result: $\Psi = (C_{000}|000\rangle + C_{001}|001\rangle + C_{010}|010\rangle + C_{011}|011\rangle)$
    - » We get "1" with probability: $P_1 = |C_{100}|^2 + |C_{101}|^2 + |C_{110}|^2 + |C_{111}|^2$
      Result: $\Psi = (C_{100}|100\rangle + C_{101}|101\rangle + C_{110}|110\rangle + C_{111}|111\rangle)$
- Problem: Don't want environment to *measure* before ready!
  - Solution: Quantum Error Correction Codes!

Spooky action at a distance

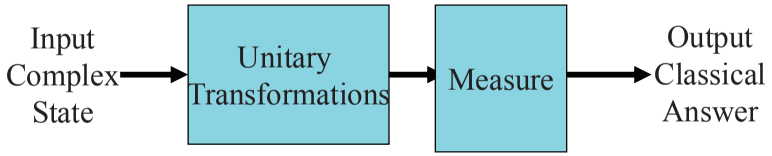
- Consider the following simple 2-bit state:

$$\Psi = C_{00}|00\rangle + C_{11}|11\rangle$$

  - Called an "EPR" pair for "Einstein, Podolsky, Rosen"
- Now, separate the two bits:

- If we measure one of them, it instantaneously sets other one!
  - Einstein called this a "spooky action at a distance"
  - In particular, if we measure a $|0\rangle$ at one side, we get a $|0\rangle$ at the other (and vice versa)
- Teleportation
  - Can "pre-transport" an EPR pair (say bits X and Y)
  - Later to transport bit A from one side to the other we:
    - » Perform operation between A and X, yielding two classical bits
    - » Send the two bits to the other side
    - » Use the two bits to operate on Y
    - » Poof! State of bit A appears in place of Y

Model: Operations on coefficients + measurements



- Basic Computing Paradigm:
  - Input is a register with superposition of many values
    - » Possibly all 2^n inputs equally probable!
  - Unitary transformations compute on coefficients
    - » Must maintain probability property (sum of squares = 1)
    - » Looks like doing computation on all 2^n inputs simultaneously!
  - Output is one result attained by measurement
- If do this poorly, just like probabilistic computation:
  - If 2^n inputs equally probable, may be 2^n outputs equally probable.
  - After measure, like picked random input to classical function!
  - All interesting results have some form of "fourier transform" computation being done in unitary transformation

Shor's Factoring Algorithm

- The Security of RSA Public-key cryptosystems depends on the difficulty of factoring a number N=pq (product of two primes)
  - Classical computer: sub-exponential time factoring
  - Quantum computer: polynomial time factoring
- Shor's Factoring Algorithm (for a quantum computer)

Easy 1) Choose random x : $2 \le x \le N-1$.
Easy 2) If $\gcd(x,N) \neq 1$, Bingo!
Hard 3) Find smallest integer r : $x^r \equiv 1 \pmod N$
Easy 4) If r is odd, GOTO 1
Easy 5) If r is even, $a \equiv x^{r/2} \pmod N \Rightarrow (a-1)\times(a+1) = kN$
Easy 6) If $a \equiv N-1 \pmod N$ GOTO 1
Easy 7) ELSE $\gcd(a \pm 1, N)$ is a non trivial factor of N.

Finding r with $x^r \equiv 1 \pmod N$

$$\sum_k |k\rangle|1\rangle \rightarrow \sum_k |k\rangle|x^k\rangle$$

$$= \sum_{w=0}^{r-1} \sum_y |w + ry\rangle|x^w\rangle$$

Quantum Fourier Transform

$$\sum_{w=0}^{r-1} \left(\int_0 \int_1 \cdots \int_k \cdots \right) |x^w\rangle$$

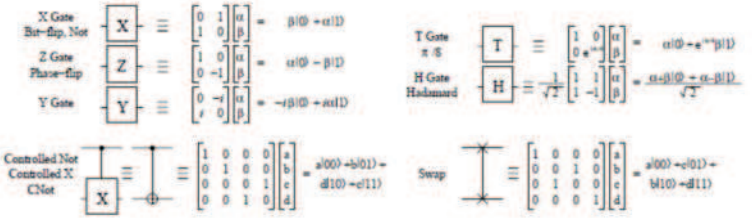
0 1 k
r r r

- Finally: Perform measurement
  - Find out r with high probability
  - Get $|y\rangle|a^{w'}\rangle$ where y is of form k/r and w' is related

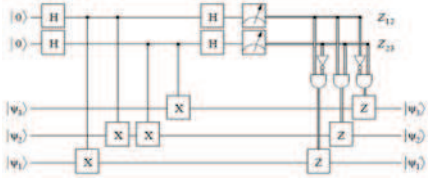
Quantum Computing Architectures

- Why study quantum computing?
  - Interesting, says something about physics
    - » Failure to build ⇒ quantum mechanics wrong?
  - Mathematical Exercise (perfectly good reason)
  - Hope that it will be practical someday:
    - » Shor's factoring, Grover's search, Design of Materials
    - » Quantum Co-processor included in your Laptop?
- To be practical, will need to hand quantum computer design off to classical designers
  - Baring Adiabatic algorithms, will probably need 100s to 1000s (millions?) of working logical Qubits ⇒ 1000s to millions of physical Qubits working together
  - Current chips: ~1 billion transistors!
- Large number of components is realm of *architecture*
  - What are optimized structures of quantum algorithms when they are mapped to a physical substrate?
  - Optimization not possible by hand
    - » Abstraction of elements to design larger circuits
    - » Lessons of last 30 years of VLSI design: USE CAD

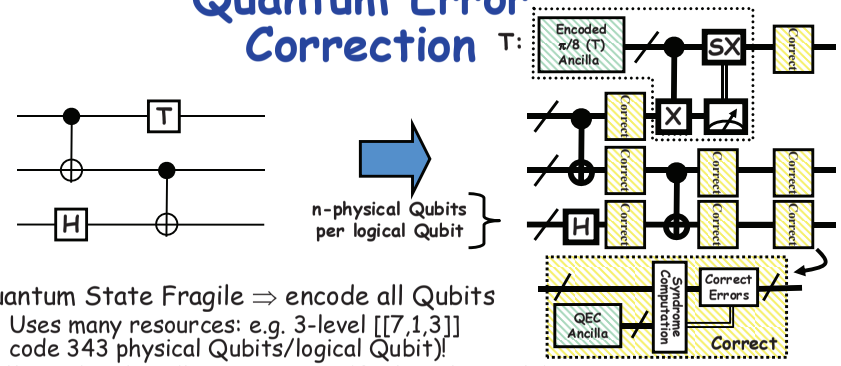
Quantum Circuit Model



- Quantum Circuit model – graphical representation
  - Time Flows from left to right
  - Single Wires: persistent Qubits, Double Wires: classical bits
    - Qubit – coherent combination of 0 and 1: $\psi = \alpha|0\rangle + \beta|1\rangle$
  - Universal gate set: Sufficient to form all unitary transformations
- Example: Syndrome Measurement (for 3-bit code)
  - Measurement (meter symbol) produces classical bits
- Quantum CAD
  - Circuit expressed as netlist
  - Computer manpulated circuits and implementations



Quantum Error Correction



- Quantum State Fragile ⇒ encode all Qubits
  - Uses many resources: e.g. 3-level [[7,1,3]] code 343 physical Qubits/logical Qubit)!
- Still need to handle operations (fault-tolerantly)
  - *Some* set of gates are simply "transversal:"
    - Perform identical gate between each physical bit of logical encoding
  - Others (like T gate for [[7,1,3]] code) cannot be handled transversally
    - Can be performed fault-tolerantly by preparing appropriate ancilla
- Finally, need to perform periodical error correction
  - Correct after every(?): Gate, Long distance movement, Long Idle Period
  - Correction reducing entropy ⇒ Consumes Ancilla bits
- Observation: ≥ 90% of QEC gates are used for ancilla production
  ≥ 70-85% of all gates are used for ancilla production

Outline

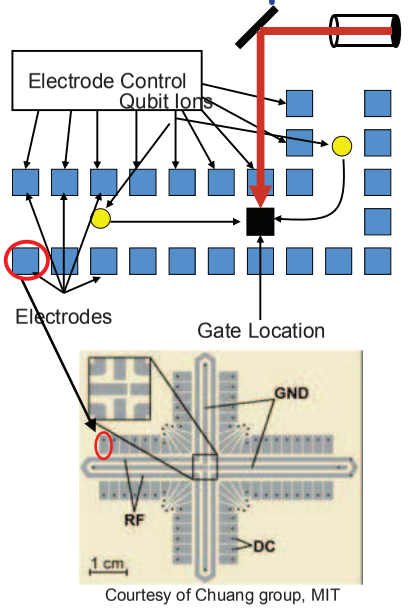
- Quantum Computing
- Ion Trap Quantum Computing
- Quantum Computer Aided Design
  - Area-Delay to Correct Result (ADCR) metric
  - Comparison of error correction codes
- Quantum Data Paths
  - QLA, CQLA, Qalypso
  - Ancilla factory and Teleportation Network Design
- Error Correction Optimization ("Recorrection")
- Shor's Factoring Circuit Layout and Design

MEMs-Based Ion Trap Devices

- Ion Traps: One of the more promising quantum computer implementation technologies
  - Built on Silicon
    - Can bootstrap the vast infrastructure that currently exists in the microchip industry
  - Seems to be on a "Moore's Law" like scaling curve
    - 12 bits exist, 30 promised soon, …
    - Many researchers working on this problem
  - Some optimistic researchers speculate about room temperature
- Properties:
  - Has a long-distance Wire
    - So-called "ballistic movement"
  - Seems to have relatively long decoherence times
  - Seems to have relatively low error rates for:
    - Memory, Gates, Movement

Quantum Computing with Ion Traps

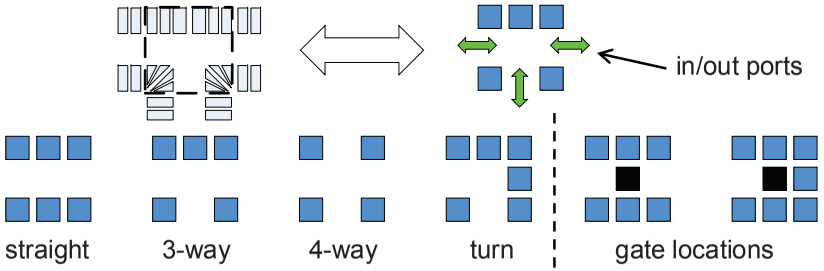
- **Qubits are atomic ions (e.g. Be^+)**
  - State is stored in hyperfine levels
  - Ions suspended in channels between electrodes
- **Quantum gates performed by lasers (either one or two bit ops)**
  - Only at certain trap locations
  - Ions move between laser sites to perform gates
- **Classical control**
  - Gate (laser) ops
  - Movement (electrode) ops
    - Complex pulse sequences to cause Ions to migrate
    - Care must be taken to avoid disturbing state
- **Demonstrations in the Lab**
  - NIST, MIT, Michigan, many others



Courtesy of Chuang group, MIT

An Abstraction of Ion Traps

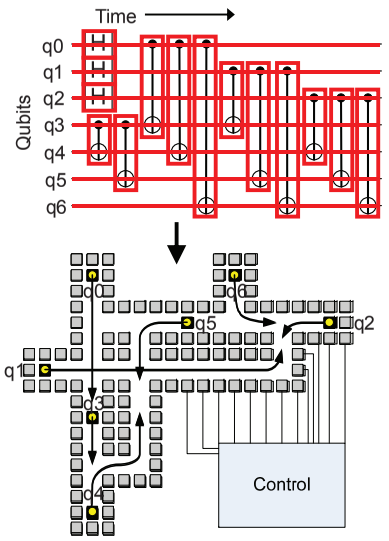
- *Basic block* abstraction: Simplify Layout



- Evaluation of layout through simulation
  - Movement of ions can be done classically
  - Yields Computation Time and Probability of Success
- Simple Error Model: Depolarizing Errors
  - Errors for every Gate Operation and Unit of Waiting
  - Ballistic Movement Error: Two error Models
    1. Every Hop/Turn has probability of error
    2. Only Accelerations cause error

Ion Trap Physical Layout

- Input: Gate level quantum circuit
  - Bit lines
  - 1-qubit gates
  - 2-qubit gates
- Output:
  - Layout of channels
  - Gate locations
  - Initial locations of ions
  - Movement/gate schedule
  - Control for schedule



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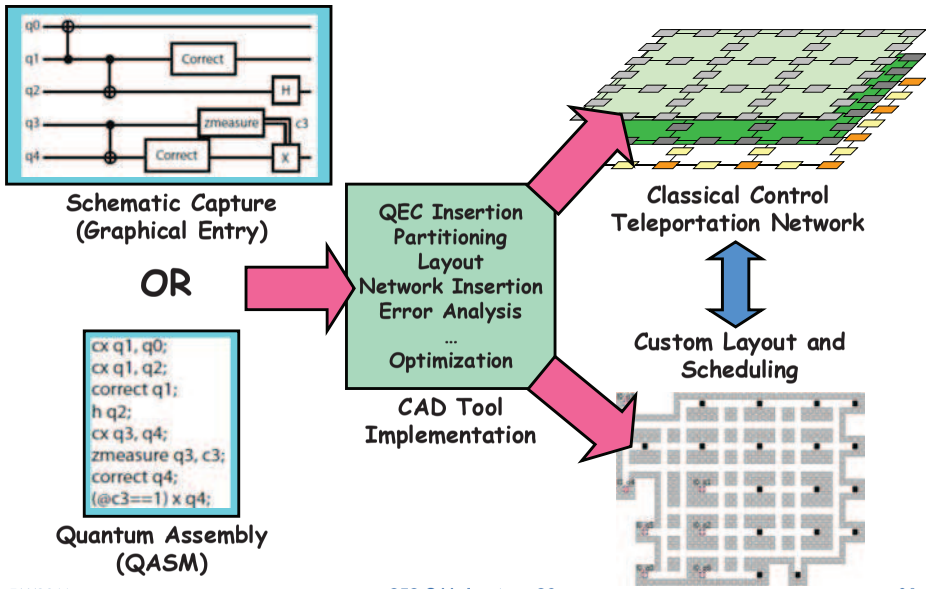
Vision of Quantum Circuit Design

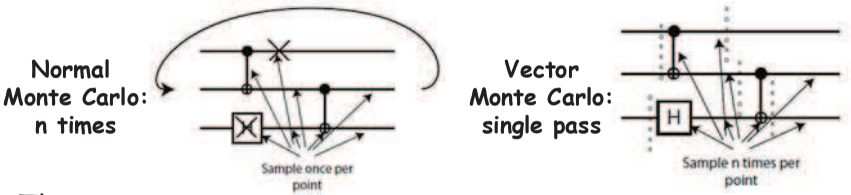


Important Measurement Metrics

- Traditional CAD Metrics:
  - Area
    - What is the total area of a circuit?
    - Measured in macroblocks (ultimately μm^2 or similar)
  - Latency ($Latency_{single}$)
    - What is the total latency to compute circuit *once*
    - Measured in seconds (or μs)
  - Probability of Success ($P_{success}$)
    - Not common metric for classical circuits
    - Account for occurrence of errors and error correction
- Quantum Circuit Metric: ADCR
  - Area-Delay to Correct Result: Probabilistic Area-Delay metric
  - $ADCR = Area \times E(Latency) = \frac{Area \times Latency_{single}}{P_{success}}$
  - $ADCR_{optimal}$: Best ADCR over all configurations
- Optimization potential: Equipotential designs
  - Trade Area for lower latency
  - Trade lower probability of success for lower latency

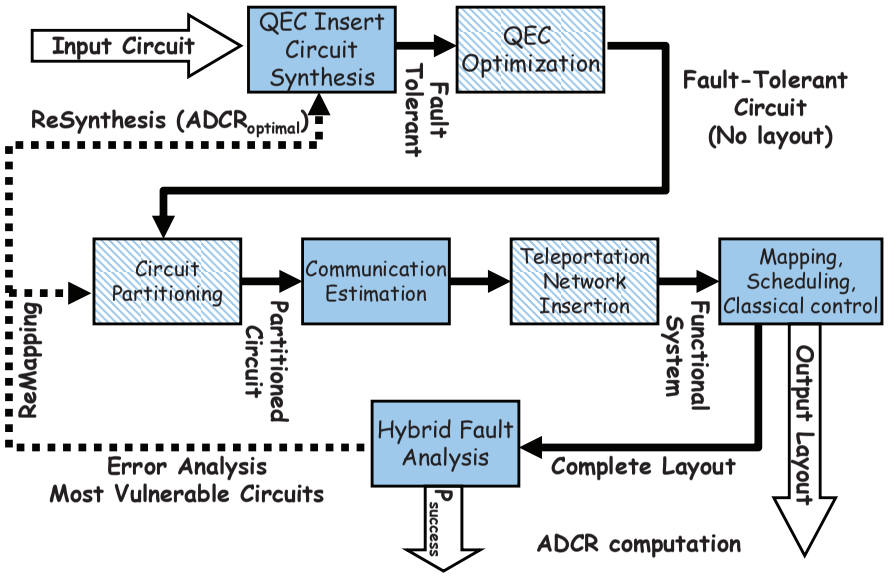
How to evaluate a circuit?

- First, generate a physical instance of circuit
  - Encode the circuit in one or more QEC codes
  - Partition and layout circuit: Highly dependant of layout heuristics!
    - Create a physical layout and scheduling of bits
    - Yields area and communication cost



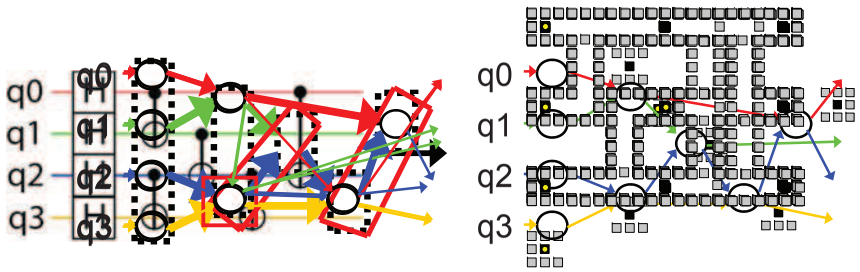
- Then, evaluate probability of success
  - Technique that works well for depolarizing errors: Monte Carlo
    - Possible error points: Operations, Idle Bits, Communications
  - Vectorized Monte Carlo: n experiments with one pass
  - Need to perform hybrid error analysis for larger circuits
    - Smaller modules evaluated via vector Monte Carlo
    - Teleportation infrastructure evaluated via fidelity of EPR bits
- Finally – Compute ADCR for particular result
  - Repeat as necessary by varying parameters to generate $ADCR_{optimal}$

Quantum CAD flow



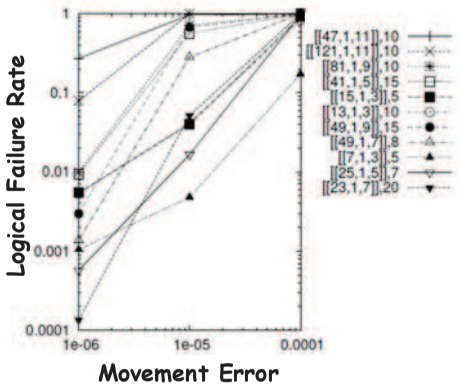
Example Place and Route Heuristic: Collapsed Dataflow

- Gate locations placed in dataflow order
  - Qubits flow left to right
  - Initial dataflow geometry folded and sorted
  - Channels routed to reflect dataflow edges
- Too many gate locations, collapse dataflow
  - Using scheduler feedback, identify latency critical edges
  - Merge critical node pairs
  - Reroute channels
- Dataflow mapping allows pipelining of computation!



Comparing Different QEC Codes

- Possible to perform a comparison between codes
  - Pick circuit/Run through CAD flow
  - Result depends on goodness of layout and scheduling heuristic
- Layout for CNOT gate (Compare with Cross, et. al)
  - Using Dataflow Heuristic
    - Validated with Donath's wire-length estimator (classical CAD)
  - Fully account of movement
  - Local gate model
- Failure Probability results
  - Best: [[23,1,7]] (Golay), [[25,1,5]] (Bacon-Shor), [[7,1,3]] (Steane)
  - Steane does particularly well with high movement errors
    - Simplicity particularly important in regime



- More info in Mark Whitney thesis
  - http://qarc.cs.berkeley.edu/publications

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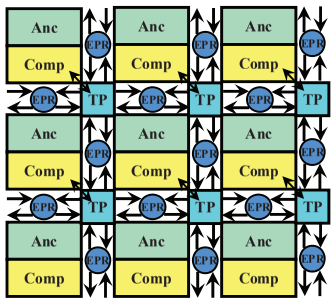
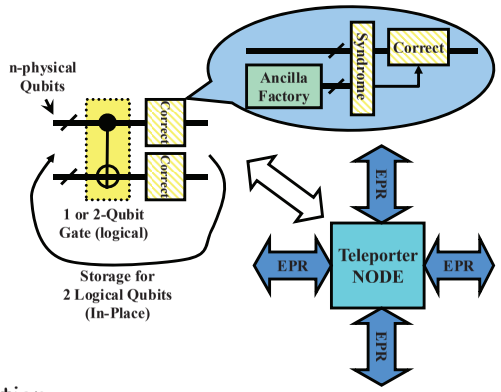
Quantum Logic Array (QLA)





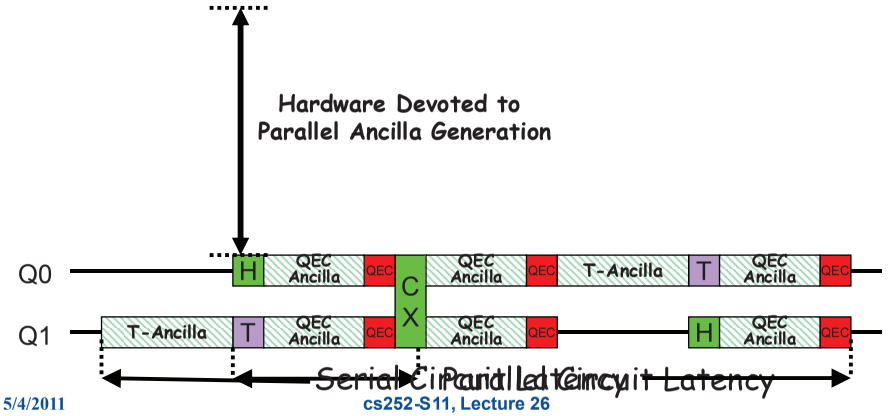
- Basic Unit:
  - Two-Qubit cell (logical)
  - Storage, Compute, Correction
- Connect Units with Teleporters
  - Probably in mesh topology, but details never entirely clear from original papers
- First Serious (Large-scale) Organization (2005)
  - Tzvetan S. Metodi, Darshan Thaker, Andrew W. Cross, Frederic T. Chong, and Isaac L. Chuang

Details

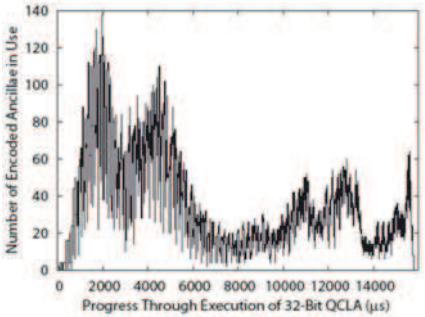
- Why Regular Array?
  - Distribute Ancilla generation where it is needed
  - Single 2-Qubit storage cell quite large
    - Concatenated [[7,1,3]] could have 343 or more physical Qubits/ logical Qubit
  - Size of single logical Qubit ⇒ makes sense to teleport between large logical blocks
  - Regularity easier to exploit for CAD tools!
    - Same reason we have ASICs with regular routing channels
- Assumptions:
  - Rate of ancilla consumption constant for every Qubit
  - Ratio of one Teleporter for every two Qubit gate is optimal
  - (Implicit) Error correction after every move or gate is optimal
  - Parallelism of quantum circuits can exploit computation on every Qubit in the system at same time
- Are these assumptions valid???

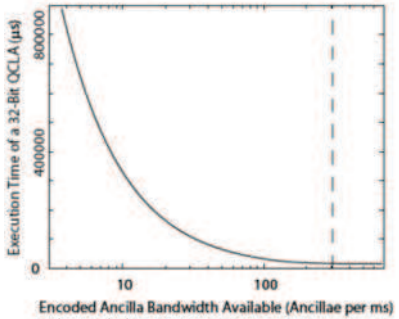
Running Circuit at "Speed of Data"

- Often, Ancilla qubits are independent of data
  - Preparation may be pulled offline
  - Very clear Area/Delay tradeoff:
    - Suggests Automatic Tradeoffs (CAD Tool)
- Ancilla qubits should be ready "just in time" to avoid ancilla decoherence from idleness



How much Ancilla Bandwidth Needed?

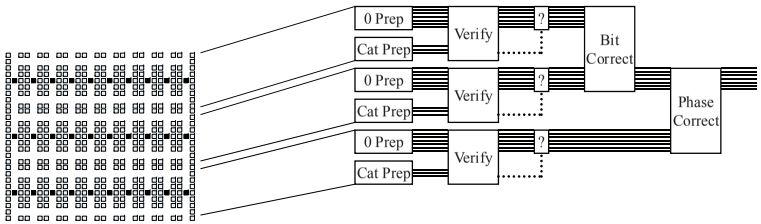




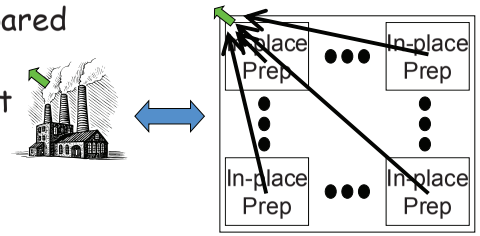
- 32-bit Quantum Carry-Lookahead Adder
  - Ancilla use very uneven (zero and T ancilla)
  - Performance is flat at high end of ancilla generation bandwidth
    - Can back off 10% in maximum performance an save orders of magnitude in ancilla generation area
- Many bits idle at any one time
  - Need only enough ancilla to maintain state for these bits
  - Many not need to frequently correct idle errors
- Conclusion: makes sense to compute ancilla requirements and share area devoted to ancilla generation

Ancilla Factory Design I

- "In-place" ancilla preparation

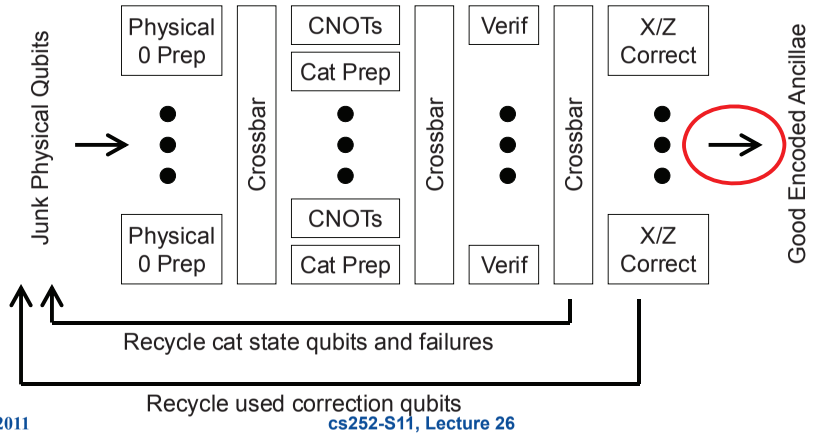


- Ancilla factory consists of many of these
  - Encoded ancilla prepared in many places, then moved to output port
  - Movement is costly!



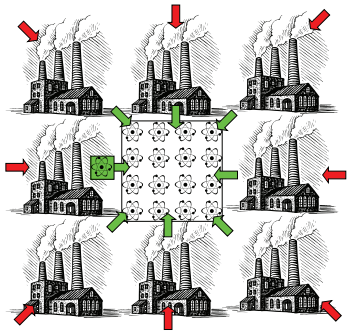
Ancilla Factory Design II

- Pipelined ancilla preparation: break into stages
  - Steady stream of encoded ancillae at *output port*
  - Fully laid out and scheduled to get area and bandwidth estimates

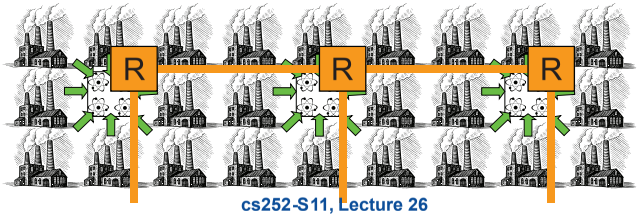


The Qalypso Datapath Architecture

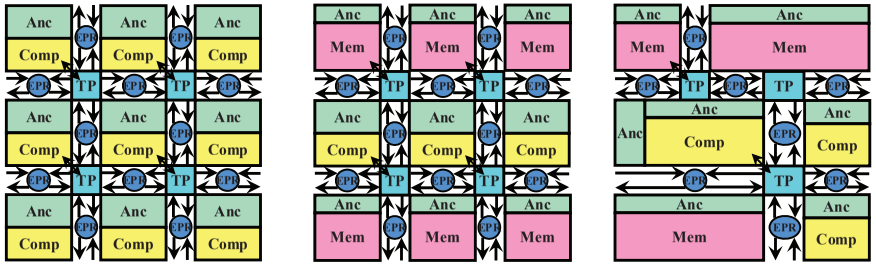
- Dense data region
  - Data qubits *only*
  - Local communication
- Shared Ancilla Factories
  - Distributed to data as needed
  - Fully multiplexed to all data
  - Output ports (➡): close to data
  - Input ports (➡): may be far from data (recycled state irrelevant)



- Regions connected by teleportation networks



Tiled Quantum Datapaths

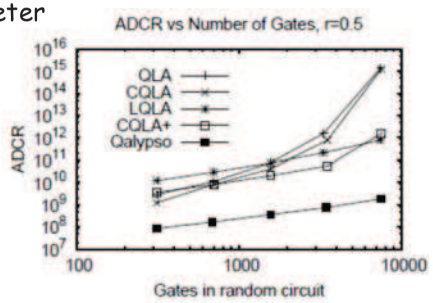


Previous: QLA, LQLA Previous: CQLA, CQLA+ Our Group: Qalypso

- Several Different Datapaths mappable by our CAD flow
  - Variations include hand-tuned Ancilla generators/factories
- Memory: storage for state that doesn't move much
  - Less/different requirements for Ancilla
  - Original CQLA paper used different QEC encoding
- Automatic mapping must:
  - Partition circuit among compute and memory regions
  - Allocate Ancilla resources to match demand (at knee of curve)
  - Configure and insert teleportation network

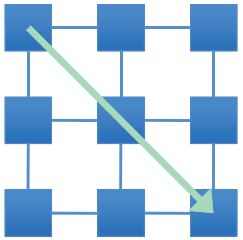
Which Datapath is Best?

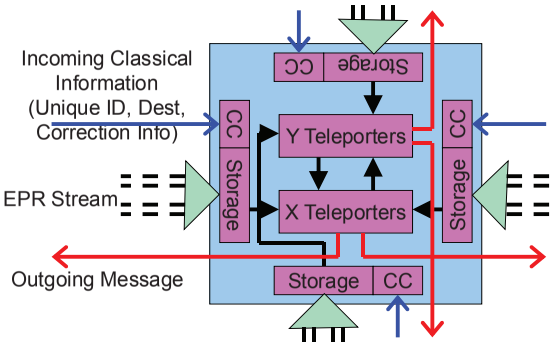
- Random Circuit Generation
  - f(Gate Count, Gate Types, Qubit Count, Splitting factor)
  - Splitting factor (r): measures connectivity of the circuit
    - Example: 0.5 splits Qubits in half, adds random gates between two halves, then recursively splits results
    - Closely related to Rent's parameter
- Qalypso clear winner (for all r)
  - 4x lower latency than LQLA
  - 2x smaller area than CQLA+
- Why Qalypso does well:
  - Shared, matched ancilla generation
  - Automatic network sizing (*not* one Teleporter for every two Qubits)
  - Automatic Identification of Idle Qubits (memory)



- LQLA and CQLA+ perform close second
  - Original datapaths supplemented with better ancilla generators, automatic network sizing, and Idle Qubit identification
  - Original QLA and CQLA do very poorly for large circuits

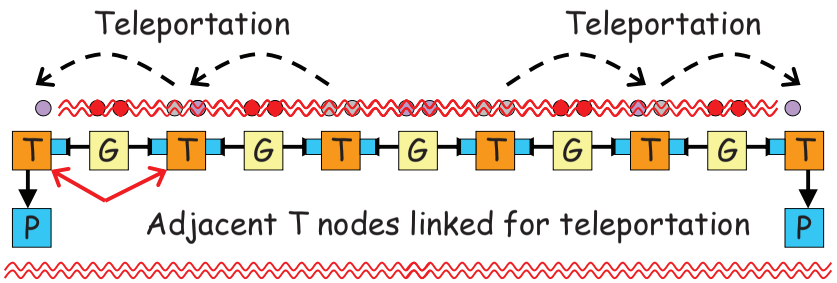
How to design Teleportation Network





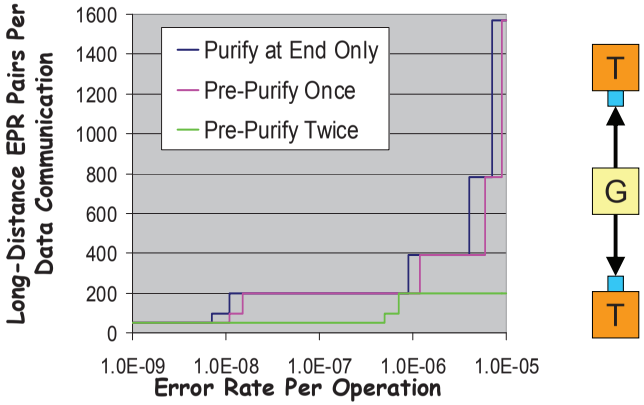
- What is the architecture of the network?
  - Including Topology, Router design, EPR Generators, etc..
- What are the details of EPR distribution?
- What are the practical aspects of routing?
  - When do we set up a channel?
  - What path does the channel take?

Basic Idea: Chained Teleportation



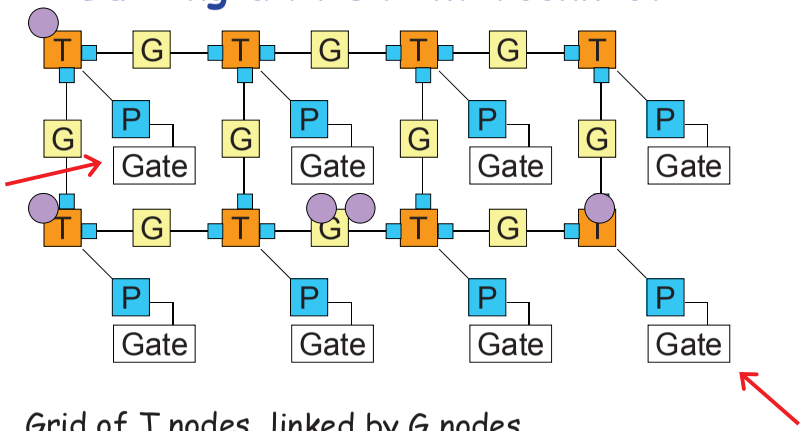
- Positive Features
  - Regularity (can build classical network topologies)
  - T node linking not on critical path
  - Pre-purification part of link setup
    - Fidelity amplification of the line
  - Allows continuous stream of EPR correlations to be established for use when necessary

Pre-Purification



- Experiment: Transmit enough EPR pairs over network to meet required fidelity of channel
  - Measure total global traffic
  - Higher Fidelity local EPR pairs ⇒ less global EPR traffic
- Benefit: decreased congestion at T Nodes

Building a Mesh Interconnect

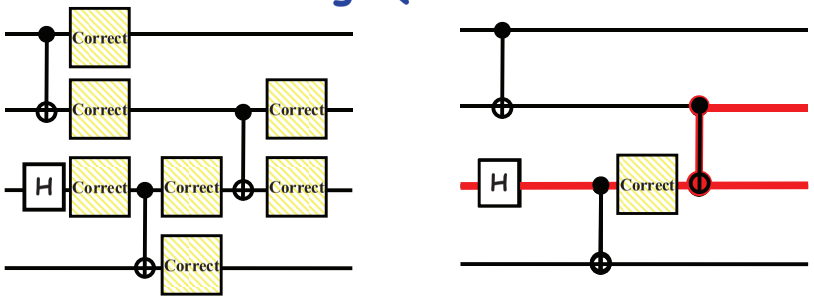


- Grid of T nodes, linked by G nodes
- Packet-switched network
  - Options: Dimension-Order or Adaptive Routing
  - Precomputed or on-demand start time for setup
- Each EPR qubit has associated classical message

Outline

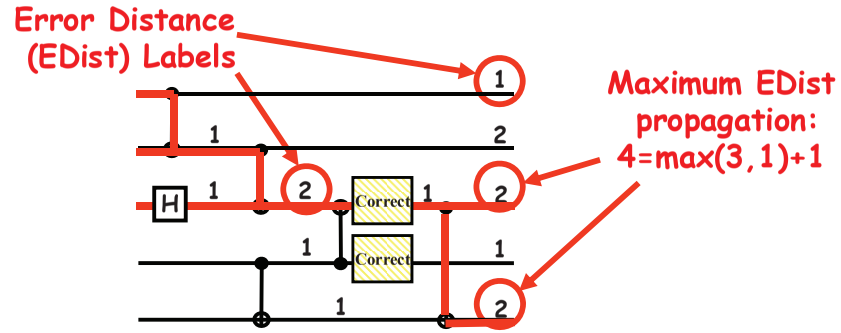
- Quantum Computing
- Ion Trap Quantum Computing
- Quantum Computer Aided Design
  - Area-Delay to Correct Result (ADCR) metric
  - Comparison of error correction codes
- Quantum Data Paths
  - QLA, CQLA, Qalypso
  - Ancilla factory and Teleportation Network Design
- Error Correction Optimization ("Recorrection")
- Shor's Factoring Circuit Layout and Design

Reducing QEC Overhead



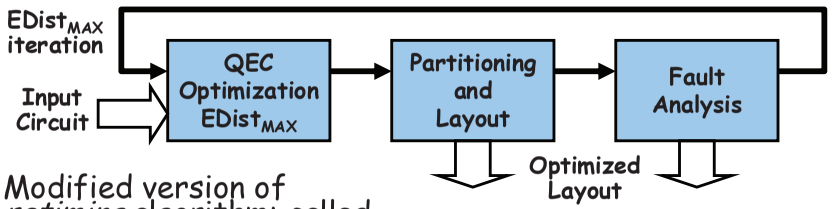
- Standard idea: correct after every gate, and long communication, and long idle time
  - This is the easiest for people to analyze
  - Urban Legend? Must do in order to keep circuit fault tolerant!
- This technique is suboptimal (at least in some domains)
  - Not every bit has same noise level!
- Different idea: identify critical Qubits
  - Try to identify paths that feed into noisiest output bits
  - Place correction along these paths to reduce maximum noise

Simple Error Propagation Model



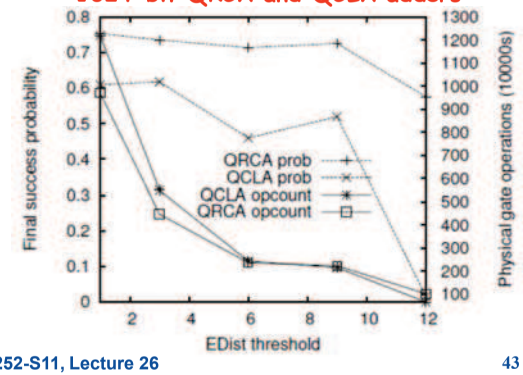
- EDist model of error propagation:
  - Inputs start with EDist = 0
  - Each gate propagates max input EDist to outputs
  - Gates add 1 unit of EDist, Correction resets EDist to 1
- Maximum EDist corresponds to Critical Path
  - Back track critical paths that add to Maximum EDist
- Add correction to keep EDist below critical threshold
  - Example: Added correction to keep $EDist_{MAX} \leq 2$

QEC Optimization

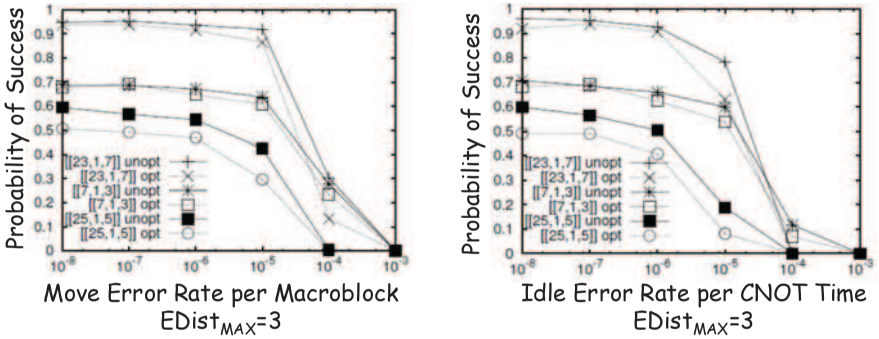


- Modified version of *retiming* algorithm: called "recorrection:"
  - Find minimal placement of correction operations that meets specified $MAX(EDist) \leq EDist_{MAX}$
- Probably of success *not* always reduced for $EDist_{MAX} > 1$
  - But, operation count and area drastically reduced
- Use Actual Layouts and Fault Analysis
  - Optimization *pre-layout*, evaluated *post-layout*

1024-bit QRCA and QCLA adders



Recorrection in presence of different QEC codes



- 500 Gate Random Circuit (r=0.5)
- Not all codes do equally well with Recorrection
  - Both [[23,1,7]] and [[7,1,3]] reasonable candidates
  - [[25,1,5]] doesn't seem to do as well
- Cost of communication and Idle errors is clear here!
- However – real optimization situation would vary EDist to find optimal point

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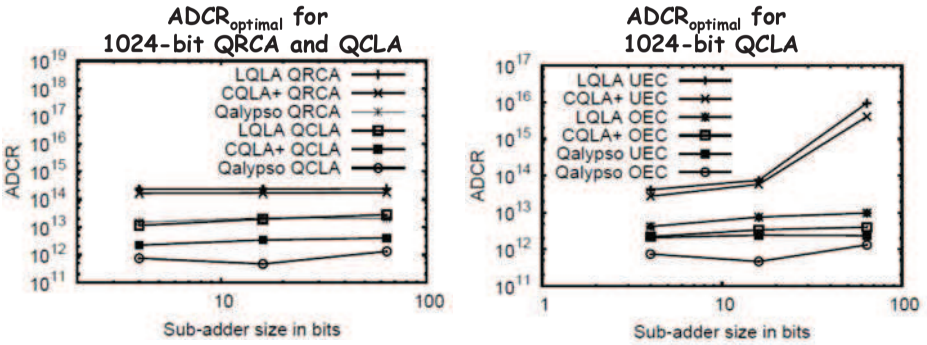
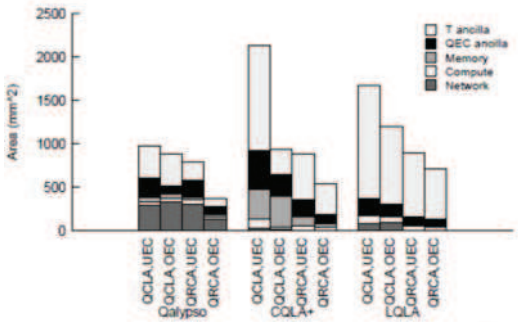
Comparison of 1024-bit adders



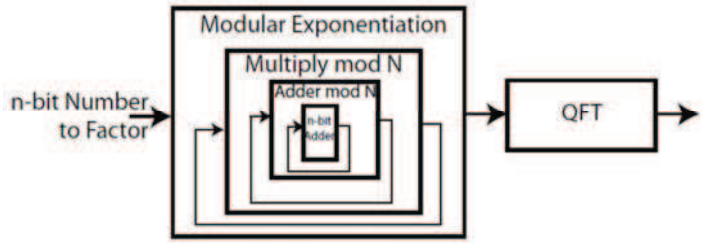
- 1024-bit Quantum Adder Architectures
  - Ripple-Carry (QRCA)
  - Carry-Lookahead (QCLA)
- Carry-Lookahead is better in all architectures
- QEC Optimization improves ADCR by order of magnitude in some circuit configurations

Area Breakdown for Adders



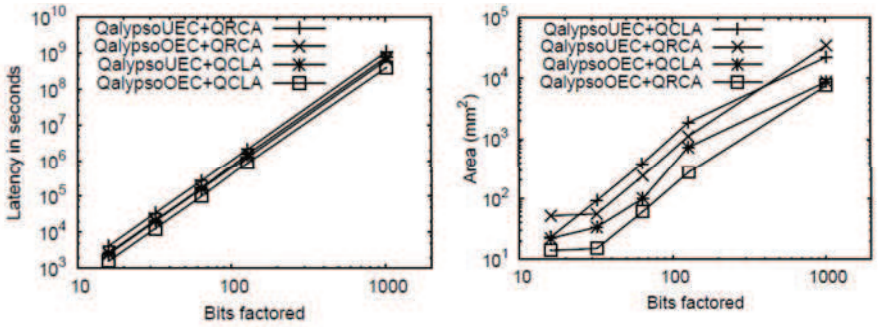
- Error Correction is *not* predominant use of area
  - Only 20-40% of area devoted to QEC ancilla
  - For Optimized Qalypso QCLA, 70% of operations for QEC ancilla generation, but only about 20% of area
- T-Ancilla generation is major component
  - Often overlooked
- Networking is significant portion of area when allowed to optimize for ADCR (30%)
  - CQLA and QLA variants didn't really allow for much flexibility

Investigating 1024-bit Shor's



- Full Layout of all Elements
  - Use of 1024-bit Quantum Adders
  - Optimized error correction
  - Ancilla optimization and Custom Network Layout
- Statistics:
  - Unoptimized version: 1.35×10^{15} operations
  - Optimized Version 1000X smaller
  - QFT is only 1% of total execution time

1024-bit Shor's Continued



- Circuits too big to compute $P_{success}$
  - Working on this problem
- Fastest Circuit: 6×10^8 seconds ~ 19 years
  - Speedup by classically computing recursive squares?
- Smallest Circuit: 7659 mm^2
  - Compare to previous *estimate* of 0.9 m^2 = 9×10^5 mm^2

Conclusion

- Quantum Computer Architecture:
  - Considering details of Quantum Computer systems at larger scale (1000s or millions of components)
  - See http://qarc.cs.berkeley.edu
- Argued that CAD tools may have a place in Quantum Computing Research
  - Presented Some details of a Full CAD flow (Partitioning, Layout, Simulation, Error Analysis)
  - New Evaluation Metric: ADCR = Area × E(Latency)
  - Full mapping and layout accounts for communication cost
- "Recorrection" Optimization for QEC
  - Simplistic model (EDist) to place correction blocks
  - Validation with full layout
  - Can improve ADCR by factors of 10 or more
    - Improves latency and area significantly, can improve probability under some circumstances as well
- Full analysis of Adder architectures and 1024-bit Shor's
  - Still too long (and too big), but smaller than previous *estimates*
  - Total circuit size still too big for our error analysis – but have hope that we can improve this