Motion Planning

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Many images from Lavalle, Planning Algorithms

Motion Planning

- **Problem**
  - Given start state x_S, goal state x_G
  - Asked for: a sequence of control inputs that leads from start to goal
- Why tricky?
  - Need to avoid obstacles
  - For systems with underactuated dynamics: can't simply move along any coordinate at will
    - E.g., car, helicopter, airplane, but also robot manipulator hitting joint limits

Solve by Nonlinear Optimization for Control?

- Could try by, for example, following formulation:

$$\begin{aligned} \min_{u,x} \quad & (x_T - x_G)^\top (x_T - x_G) \\ \text{s.t.} \quad & x_{t+1} = f(x_t, u_t) \ \ \forall t \\ & u_t \in \mathcal{U}_t \\ & x_t \in \mathcal{X}_t \\ & x_0 = x_S \end{aligned}$$

X_t can encode obstacles

- Or, with constraints, (which would require using an infeasible method):

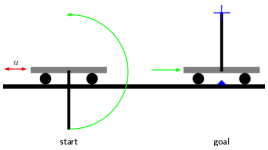
$$\begin{aligned} \min_{u,x} \quad & \|u\| \\ \text{s.t.} \quad & x_{t+1} = f(x_t, u_t) \ \ \forall t \\ & u_t \in \mathcal{U}_t \\ & x_t \in \mathcal{X}_t \\ & x_0 = x_S \\ & X_T = x_G \end{aligned}$$

- *Can work surprisingly well, but for more complicated problems with longer horizons, often get stuck in local maxima that don't reach the goal*

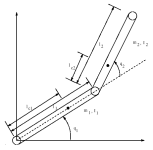
Examples

- Helicopter path planning

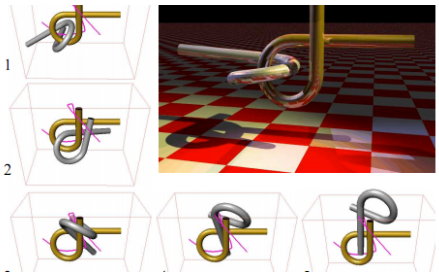
- Swinging up cart-pole

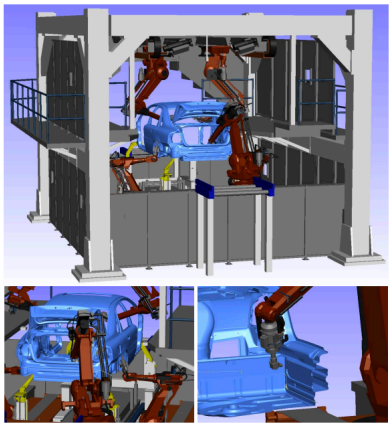


- Acrobot

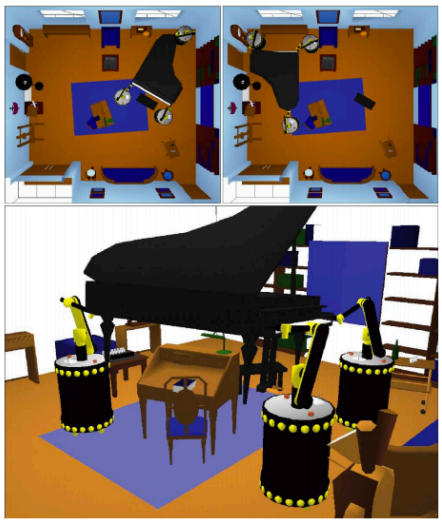


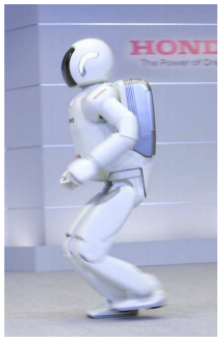
Examples

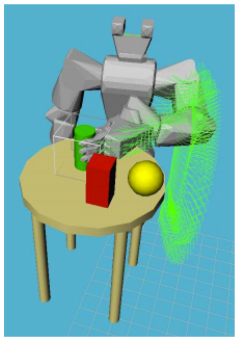




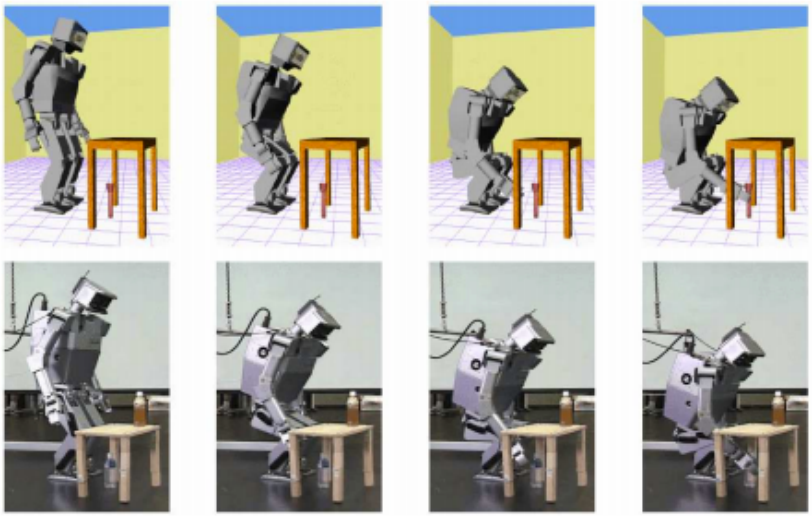
Examples







Examples



Motion Planning: Outline

- Configuration Space
- Probabilistic Roadmap
  - Boundary Value Problem
  - Sampling
  - Collision checking
- Rapidly-exploring Random Trees (RRTs)
- Smoothing

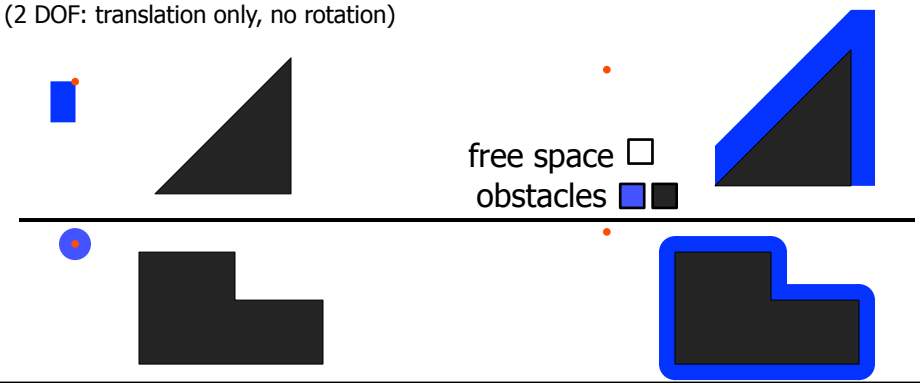
Configuration Space (C-Space)

= { x | x is a pose of the robot}

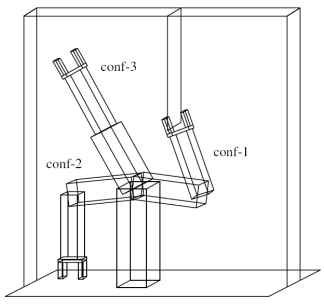
- obstacles → configuration space obstacles

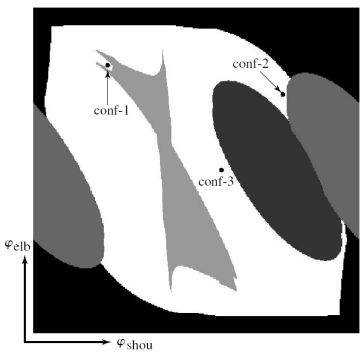
Workspace *Configuration Space*

(2 DOF: translation only, no rotation)

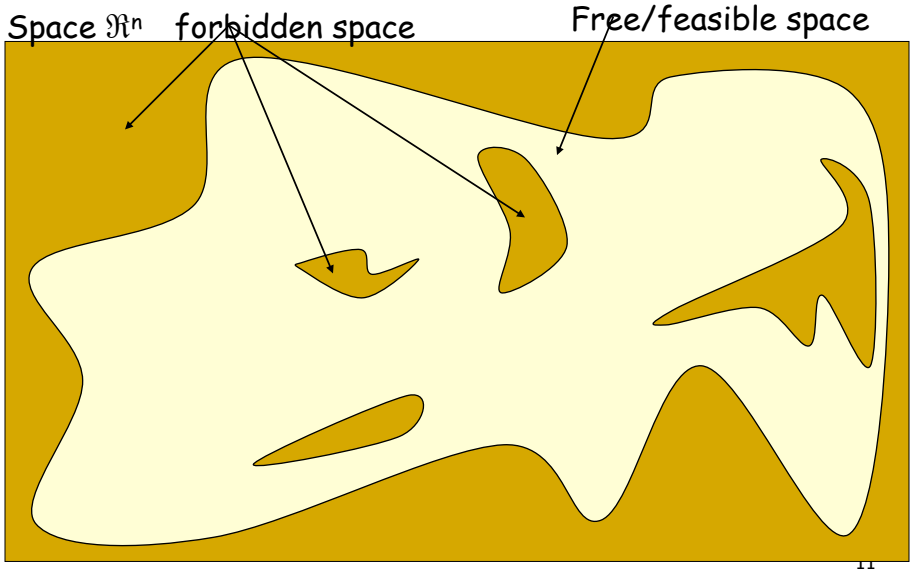


Motion planning





Probabilistic Roadmap (PRM)



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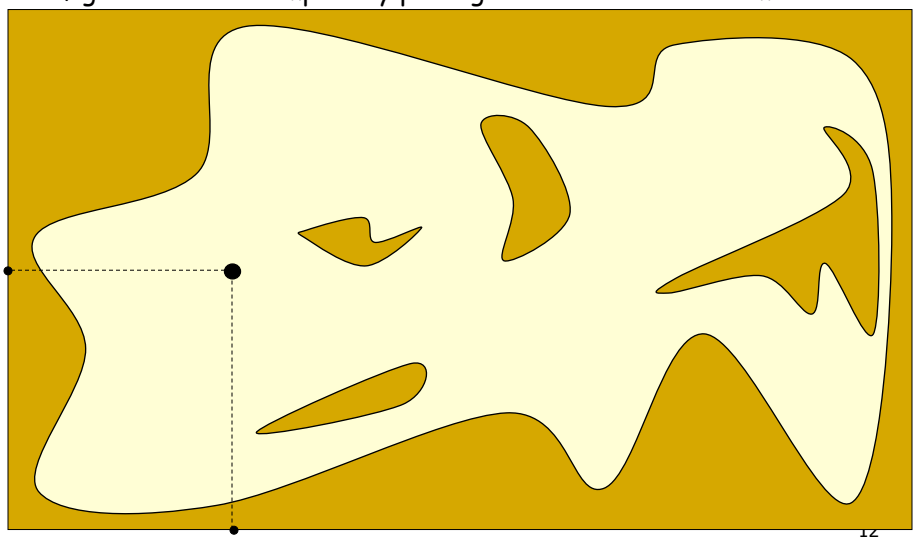
Probabilistic Roadmap (PRM)

Configurations are sampled by picking coordinates at random



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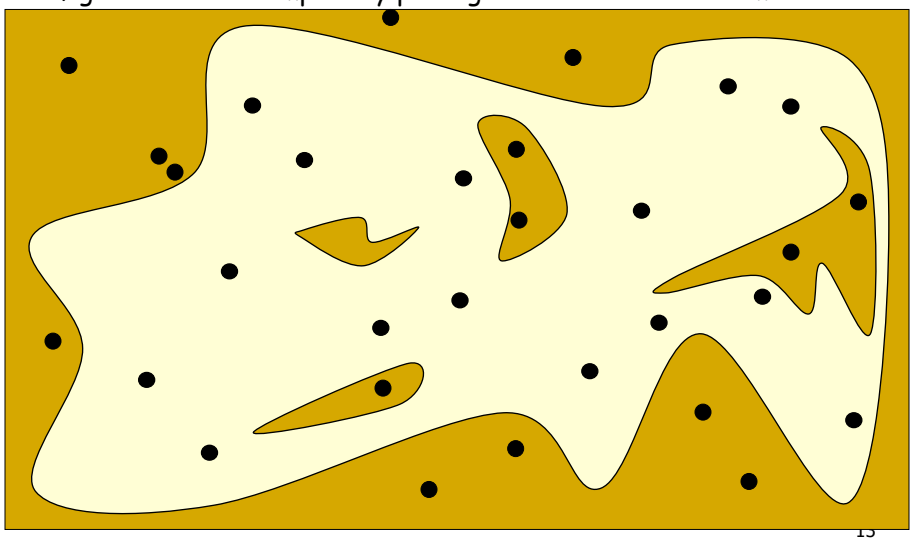

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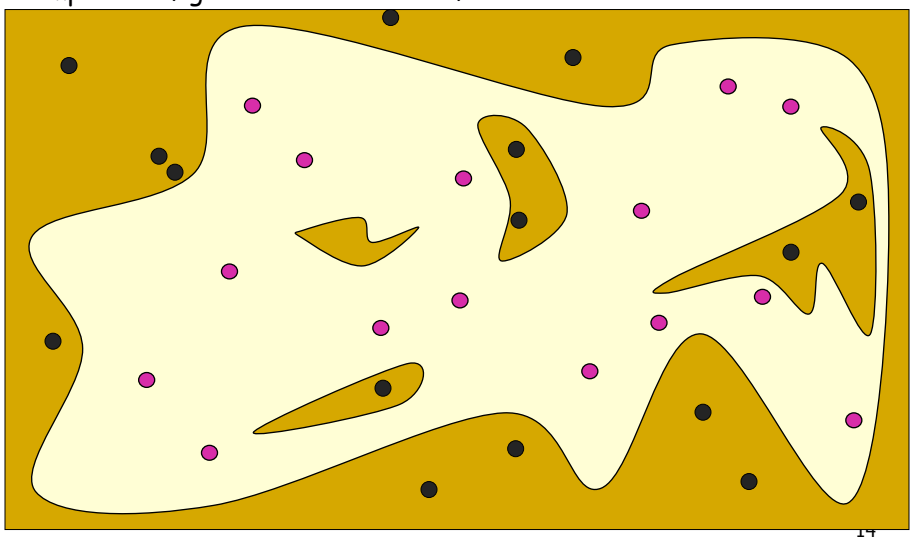
Probabilistic Roadmap (PRM)

Sampled configurations are tested for collision



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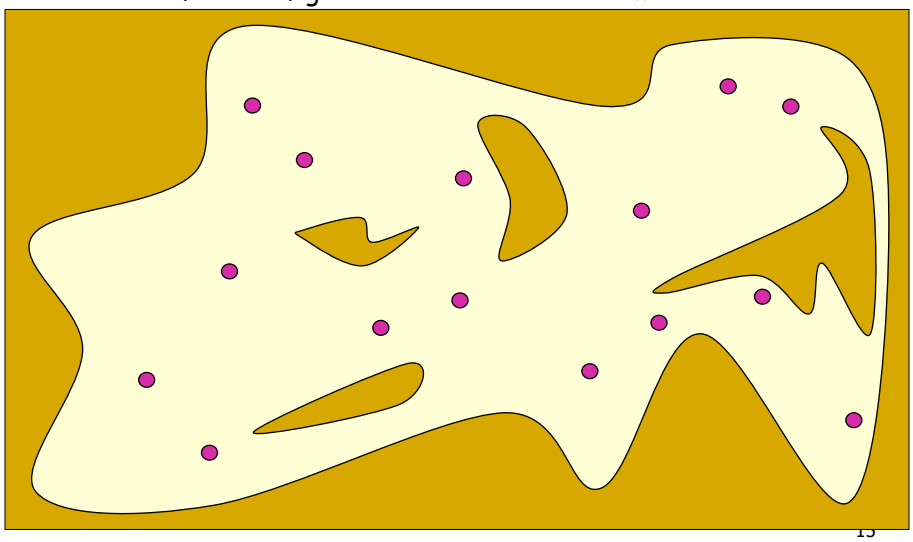
Probabilistic Roadmap (PRM)

The collision-free configurations are retained as milestones



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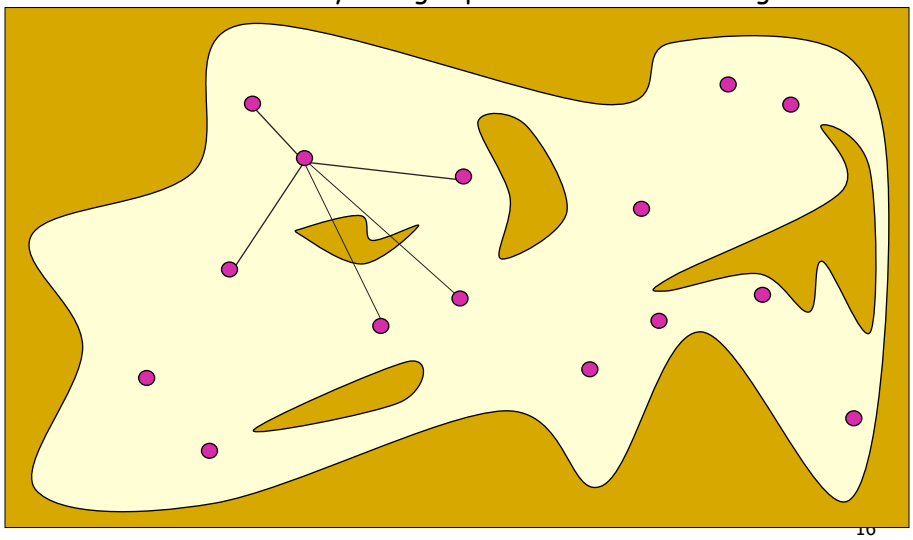
Probabilistic Roadmap (PRM)

Each milestone is linked by straight paths to its nearest neighbors



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Probabilistic Roadmap (PRM)

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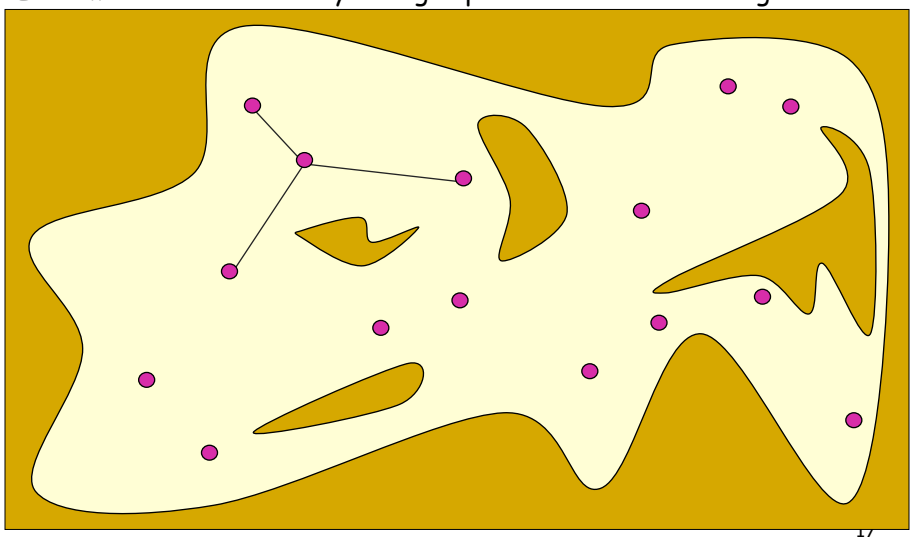

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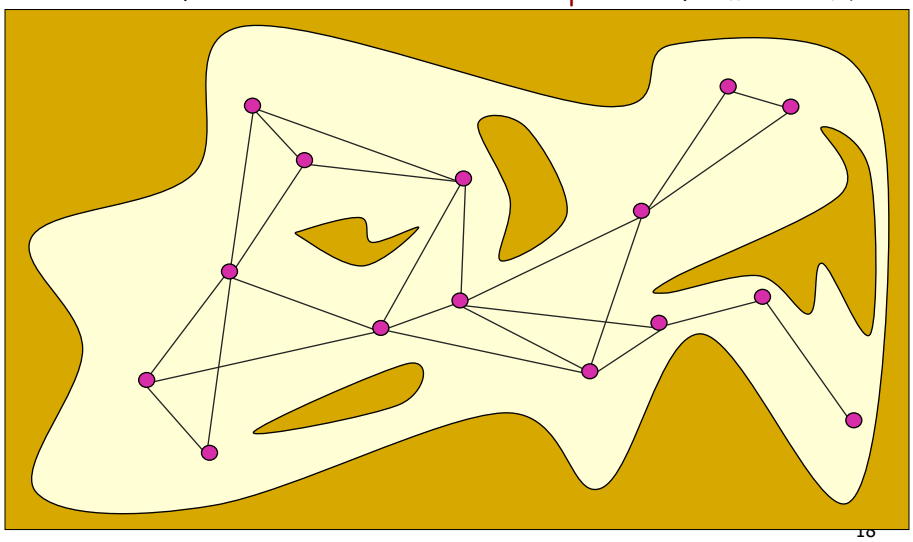
Probabilistic Roadmap (PRM)

The collision-free links are retained as local paths to form the PRM



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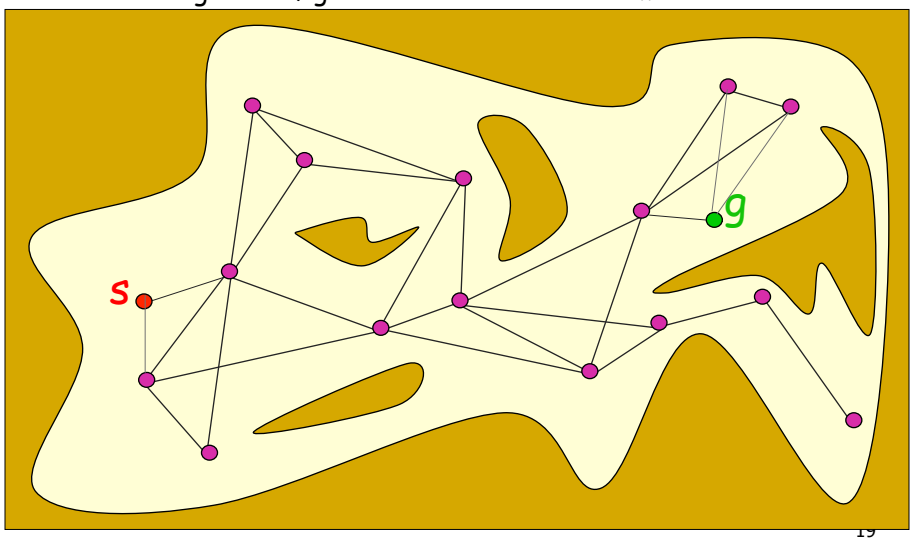
Probabilistic Roadmap (PRM)

The start and goal configurations are included as milestones



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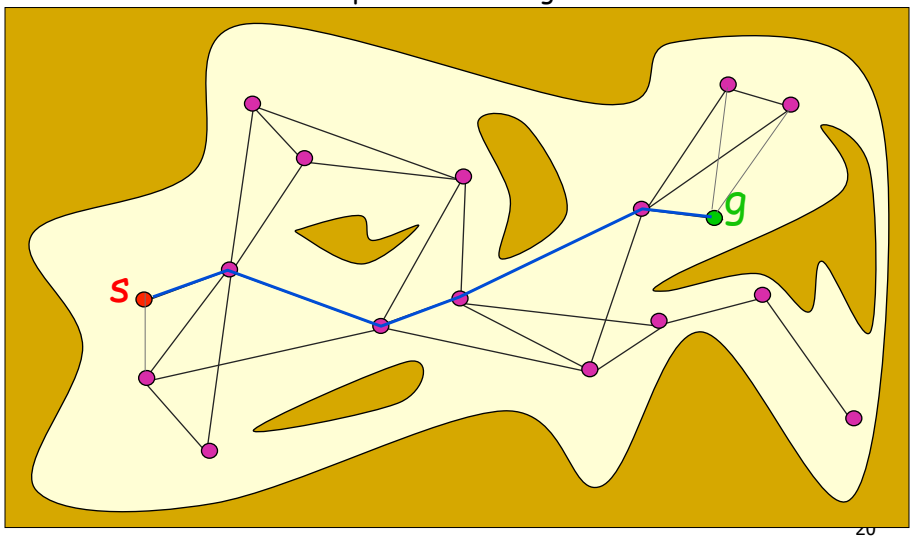
Probabilistic Roadmap (PRM)

The PRM is searched for a path from s to g



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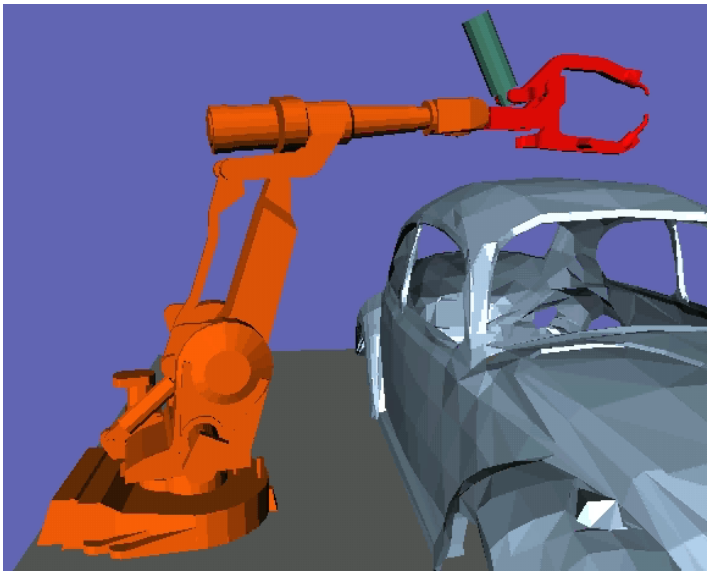
Probabilistic Roadmap

- Initialize set of points with x_S and x_G
- Randomly sample points in configuration space
- Connect nearby points if they can be reached from each other
- Find path from x_S to x_G in the graph
  - alternatively: keep track of connected components incrementally, and declare success when x_S and x_G are in same connected component

PRM example



PRM example 2

PRM: Challenges

1. Connecting neighboring points: Only easy for holonomic systems (i.e., for which you can move each degree of freedom at will at any time). Generally requires solving a Boundary Value Problem

$$\begin{aligned} \min_{u,x} \quad & \|u\| \\ \text{s.t.} \quad & x_{t+1} = f(x_t, u_t) \quad \forall t \\ & u_t \in \mathcal{U}_t \\ & x_t \in \mathcal{X}_t \\ & x_0 = x_S \\ & X_T = x_G \end{aligned}$$

Typically solved without collision checking; later verified if valid by collision checking

2. Collision checking:

Often takes majority of time in applications (see Lavalle)

3. Sampling: how to sample uniformly (or biased according to prior information) over configuration space?

Sampling

- How to sample uniformly at random from $[0,1]^n$?
  - Sample uniformly at random from [0,1] for each coordinate
- How to sample uniformly at random from the surface of the n-D unit sphere?
  - Sample from n-D Gaussian → isotropic; then just normalize
- How to sample uniformly at random for orientations in 3-D?

PRM's Pros and Cons

- Pro:
  - Probabilistically complete: i.e., with probability one, if run long the graph will contain a solution path if one exists.
- Cons:
  - Required to solve 2 point boundary value problem
  - Build graph over state space but no particular focus on generating a path

Rapidly exploring Random Trees

- Basic idea:
  - Build up a tree through generating "next states" in the tree by executing random controls
  - However: not exactly above to ensure good coverage

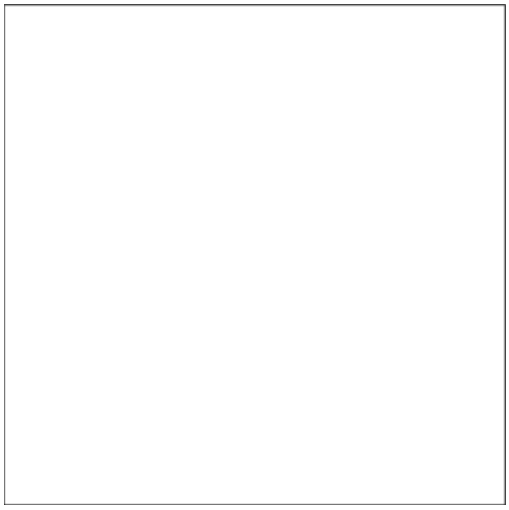
Rapidly-exploring Random Trees (RRT)

GENERATE_RRT($x_{init}, K, \Delta t$)
1 $\mathcal{T}$.init(x_{init});
2 **for** $k = 1$ **to** K **do**
3 $x_{rand} \leftarrow$ RANDOM_STATE();
4 $x_{near} \leftarrow$ NEAREST_NEIGHBOR($x_{rand}, \mathcal{T}$);
5 $u \leftarrow$ SELECT_INPUT(x_{rand}, x_{near});
6 $x_{new} \leftarrow$ NEW_STATE($x_{near}, u, \Delta t$);
7 $\mathcal{T}$.add_vertex(x_{new});
8 $\mathcal{T}$.add_edge(x_{near}, x_{new}, u);
9 Return $\mathcal{T}$

RRT Practicalities

- Finding (approximate) nearest neighbor efficiently
  - KD Trees data structure (upto 20-D) [e.g., FLANN]
  - Locality Sensitive Hashing

Growing RRT



Bi-directional RRT

Multi-directional RRT

Resolution-Complete RRT

Smoothing

- Shortcutting
- Nonlinear optimization for optimal control

Example: Swing up Pendulum

Example: Swing up Acrobot