Problem Set 1

1. [ϵ-optimal strategies]
A strategy pair (x, y) for a zero sum game is ϵ-optimal if a player can gain at most ϵ by deviating from his strategy assuming that the other player does not deviate,

(a) Given a strategy pair (x, y) suggest an efficient procedure to verify that (x, y) is ϵ-optimal.

(b) Show that the value $x^t Ay$ of an ϵ optimal strategy is within ϵ of the value of the game.

2. [Irrelevant Attributes]
Given a function of n variables, which is, in fact, the OR of r of them, where $r << n$. Consider an algorithm to predict the function which maintains weights $w_1, \cdots, w_n$ and predicts 1 when $w_1x_1 + \ldots w_nx_n \geq n$ and 0 otherwise. Each time the prediction is incorrect halve or double the appropriate weights. Complete the description of the algorithm and show it makes $O(r \log n)$ mistakes.

3. [The matching game]
The matching game is played over the complete weighted bipartite graph $G(V, E)$. The edge player plays an edge $e \in E$ while the vertex player player plays a vertex $v \in V$ and if $v \in e$ the edge player pays $1/w_e$ to the vertex player.

(a) If the vertex player plays a uniformly random vertex what is the best response for the edge player?

(b) If the edge player plays a uniformly random edge from the maximum weight matching what is the best response for the vertex player?

(c) Are these two strategies optimal for this game? If not, what are a pair of strategies that are optimal for this game.

(d) Show how to use the multiplicative weights algorithm to find a fractional matching with cost within $(1 + \epsilon)$ factor of optimal. (A fractional matching is an assignment x_e to the edges such that for each vertex $\sum_{e \ni u} x_e \leq 1$.)