1. Consider a CNOT gate whose second input (target qubit) is $1/\sqrt{2}|0\rangle - 1/\sqrt{2}|1\rangle$. Describe the action of the CNOT gate on the first (control) qubit.

   Now show that if the CNOT gate is applied in the Hadamard basis - i.e. apply the Hadamard gate to the inputs and outputs of the CNOT gate - then the result is a CNOT gate with the control and target qubit swapped.

2. You are given a qubit in the state $|0\rangle$ or $|psi\rangle = a|0\rangle + b|1\rangle$. To distinguish which you can measure in the standard basis and get an $|b|^2$ advantage in correctly guessing. What measurement would you perform to increase your advantage to $\Omega(|b|)$? Is it possible to do asymptotically better?

3. Alice and Bob share a Bell pair Φ^+. Alice wishes to send two classical bits to Bob. Show how she can accomplish this by sending him a single qubit. This shows that it is impossible to improve upon the two classical bits that Alice must send Bob in the teleportation protocol.

4. The controlled swap (C-SWAP) gate takes as input 3 qubits and swaps the second and third if and only if the first qubit is a 1.

   (a) Show that any classical circuit on n bits and with m gates can be simulated by a reversible circuit on $O(m+n)$ bits and using $O(m+n)$ NOT, CNOT and C-SWAP gates.

   (b) Show that given a circuit with n inputs and m gates, there is an equivalent reversible circuit with $O(n \log m)$ inputs and $O(m^{\log_2 3})$ gates.

   Can you generalize your construction to reduce the number of gates to $O(m^{1+\epsilon})$.