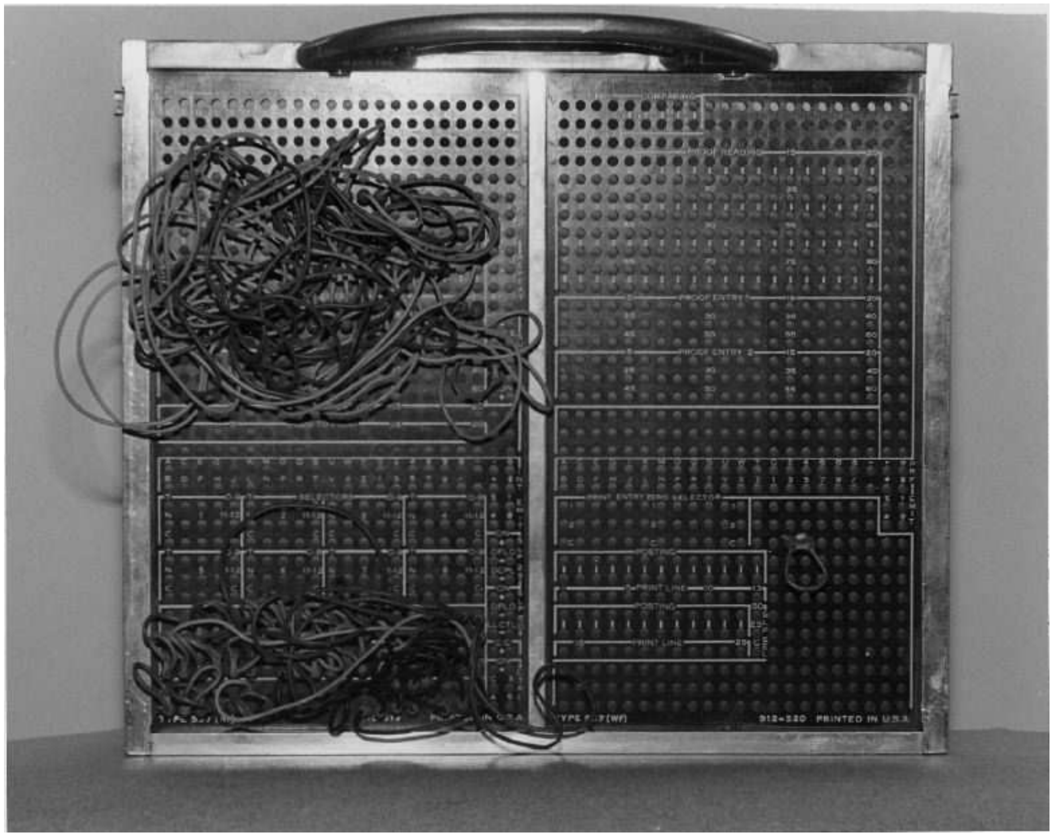


In the old days, they “defined procedures” like this.

4 Defining Your Own Procedures

Until now we've been using procedures that Scheme already knows when you begin working with it. In this chapter you'll find out how to create new procedures.

How to Define a Procedure

A Scheme program consists of one or more *procedures.* A procedure is a description of the process by which a computer can work out some result that we want. Here's how to define a procedure that returns the square of its argument:

```
(define (square x)
  (* x x))
```

(The value returned by `define` may differ depending on the version of Scheme you're using. Many versions return the name of the procedure you're defining, but others return something else. It doesn't matter, because when you use `define` you aren't interested in the returned value, but rather in the fact that Scheme remembers the new definition for later use.)

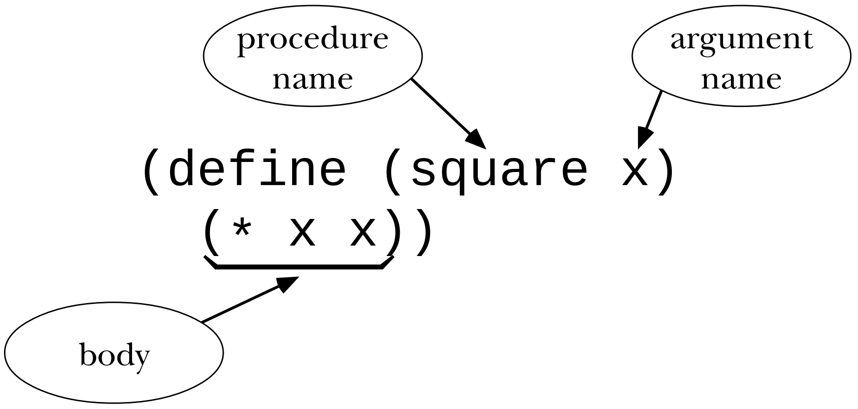
This is the definition of a procedure called `square`. `Square` takes one argument, a number, and it returns the square of that number. Once you have defined `square`, you can use it just the same way as you use primitive procedures:

```
> (square 7)
49

> (+ 10 (square 2))
14
```

```
> (square (square 3))
81
```

This procedure definition has four parts. The first is the word `define`, which indicates that you are defining something. The second and third come together inside parentheses: the name that you want to give the procedure and the name(s) you want to use for its argument(s). This arrangement was chosen by the designers of Scheme because it looks like the form in which the procedure will be invoked. That is, `(square x)` looks like `(square 7)`. The fourth part of the definition is the *body:* an expression whose value provides the function's return value.



Special Forms

`Define` is a *special form,* an exception to the evaluation rule we've been going on about.* Usually, an expression represents a procedure invocation, so the general rule is that Scheme first evaluates all the subexpressions, and then applies the resulting procedure to the resulting argument values. The specialness of special forms is that Scheme *doesn't* evaluate all the subexpressions. Instead, each special form has its own particular evaluation rule. For example, when we defined `square`, no part of the definition was evaluated: not `square`, not `x`, and not `(* x x)`. It wouldn't make sense to evaluate `(square x)` because you can't invoke the `square` procedure before you define it!

* Technically, the entire expression `(define (square x) ...)` is the special form; the word `define` itself is called a *keyword.* But in fact Lispians are almost always loose about this distinction and say "`define` is a special form," just as we've done here. The word "form" is an archaic synonym for "expression," so "special form" just means "special expression."

It would be possible to describe special forms using the following model: "Certain procedures want their arguments unevaluated, and Scheme recognizes them. After refraining from evaluating `define`'s arguments, for example, Scheme invokes the `define` procedure with those unevaluated arguments." But in fact the designers of Scheme chose to think about it differently. The entire special form that starts with `define` is just a completely different kind of thing from a procedure call. In Scheme there is no procedure named `define`. In fact, `define` is not the name of anything at all:

```
> +
#<PRIMITIVE PROCEDURE +>

> define
ERROR -- INVALID CONTEXT FOR KEYWORD DEFINE
```

Nevertheless, in this book, unless it's really important to make the distinction, we'll talk as if there were a procedure called `define`. For example, we'll talk about "`define`'s arguments" and "the value returned by `define`" and "invoking `define`."

Functions and Procedures

Throughout most of this book, our procedures will describe processes that compute *functions.* A function is a connection between some values you already know and a new value you want to find out. For example, the *square* function takes a number, such as 8, as its input value and returns another number, 64 in this case, as its output value. The *plural* function takes a noun, such as "computer," and returns another word, "computers" in this example. The technical term for the function's input value is its *argument.* A function may take more than one argument; for example, the `remainder` function takes two arguments, such as 12 and 5. It returns one value, the remainder on dividing the first argument by the second (in this case, 2).

We said earlier that a procedure is "a description of the process by which a computer can work out some result that we want." What do we mean by *process*? Consider these two definitions:

$$f(x) = 3x + 12$$
$$g(x) = 3(x + 4)$$

The two definitions call for different arithmetic operations. For example, to compute $f(8)$ we'd multiply 8 by 3, then add 12 to the result. To compute $g(8)$, we'd add 4 to

8, then multiply the result by 3. But we get the same answer, 36, either way. These two equations describe different *processes,* but they compute the same *function.* The function is just the association between the starting value(s) and the resulting value, no matter how that result is computed. In Scheme we could say

```
(define (f x)
  (+ (* 3 x) 12))

(define (g x)
  (* 3 (+ x 4)))
```

and we'd say that `f` and `g` are two procedures that represent the same function.

In real life, functions are not always represented by procedures. We could represent a function by a *table* showing all its possible values, like this:

Alabama	Montgomery
Alaska	Juneau
Arizona	Phoenix
Arkansas	Little Rock
California	Sacramento
...	...

This table represents the State Capital function; we haven't shown all the lines of the complete table, but we could. There are only a finite number of U.S. states. Numeric functions can also be represented by *graphs,* as you probably learned in high school algebra. In this book our focus is on the representation of functions by procedures. The only reason for showing you this table example is to clarify what we mean when we say that a function *is represented by* a procedure, rather than that a function *is* the procedure.

We'll say "the procedure `f`" when we want to discuss the operations we're telling Scheme to carry out. We'll say "the function represented by `f`" when our attention is focused on the value returned, rather than on the mechanism. (But we'll often abbreviate that lengthy second phrase with "the function `f`" unless the context is especially confusing.)*

* Also, we'll sometimes use the terms "domain" and "range" when we're talking about procedures, although technically, only functions have domains and ranges.

Argument Names versus Argument Values

"It's long," said the Knight, "but it's very, *very* beautiful. Everybody that hears me sing it—either it brings the *tears* into their eyes, or else—"

"Or else what?" said Alice, for the Knight had made a sudden pause.

"Or else it doesn't, you know. The name of the song is called '*Haddock's Eyes.*' "

"Oh, that's the name of the song, is it?" Alice said, trying to feel interested.

"No, you don't understand," the Knight said, looking a little vexed. "That's what the name is *called.* The name really is '*The Aged Aged Man.*' "

"Then I ought to have said 'That's what the *song* is called'?" Alice corrected herself.

"No, you oughtn't; that's quite another thing! The *song* is called '*Ways And Means*': but that's only what it's *called,* you know!"

"Well, what *is* the song, then?" said Alice, who was by this time completely bewildered.

"I was coming to that," the Knight said. "The song really is '*A-sitting On A Gate*': and the tune's my own invention."

—Lewis Carroll, *Through the Looking-Glass, and What Alice Found There*

Notice that when we *defined* the `square` procedure we gave a *name,* `x`, for its argument. By contrast, when we *invoked* `square` we provided a *value* for the argument (e.g., `7`). The word `x` is a "place holder" in the definition that stands for whatever value you use when you call the procedure. So you can read the definition of `square` as saying, "In order to `square` a number, multiply *that number* by *that number.*" The name `x` holds the place of the particular number that you mean.

Be sure you understand this distinction between defining a procedure and calling it. A procedure represents a general technique that can be applied to many specific cases. We don't want to build any particular case into the procedure definition; we want the definition to express the general nature of the technique. You wouldn't want a procedure that only knew how to take the square of 7. But when you actually get around to using `square`, you have to be specific about which number you're squaring.

The name for the name of an argument (whew!) is *formal parameter.* In our `square` example, `x` is the formal parameter. (You may hear people say either "formal" alone or "parameter" alone when they're feeling lazy.) The technical term for the actual value of the argument is the *actual argument.* In a case like

```
(square (+ 5 9))
```

you may want to distinguish the *actual argument expression* `(+ 5 9)` from the *actual argument value* 14. Most of the time it's perfectly clear what you mean, and you just say "argument" for all of these things, but right now when you're learning these ideas it's important to be able to talk more precisely.

The `square` procedure takes one argument. If a procedure requires more than one argument, then the question arises, which actual argument goes with which formal parameter? The answer is that they go in the order in which you write them, like this:

```
(define (f a b)
  (+ (* 3 a) b))

> (f 5 8)
23

> (f 8 5)
29
```

Procedure as Generalization

What's the average of 17 and 25? To answer this question you could add the two numbers, getting 42, and divide that by two, getting 21. You could ask Scheme to do this for you:

```
> (/ (+ 17 25) 2)
21
```

What's the average of 14 and 68?

```
> (/ (+ 14 68) 2)
41
```

Once you understand the technique, you could answer any such question by typing an expression of the form

```
(/ (+ __________ __________) 2)
```

to Scheme.

But if you're going to be faced with more such problems, an obvious next step is to *generalize* the technique by defining a procedure:

```
(define (average a b)
  (/ (+ a b) 2))
```

With this definition, you can think about the next problem that comes along in terms of the problem itself, rather than in terms of the steps required for its solution:

```
> (average 27 4)
15.5
```

This is an example of what we meant when we defined "abstraction" as noticing a pattern and giving it a name. It's not so different from the naming of such patterns in English; when someone invented the name "average" it was, probably, after noticing that it was often useful to find the value halfway between two other values.

This naming process is more important than it sounds, because once we have a name for some idea, we can use that idea without thinking about its pieces. For example, suppose that you want to know not only the average of some numbers but also a measure of whether the numbers are clumped together close to the average, or widely spread out. Statisticians have developed the "standard deviation" as a measure of this second property. You'd rather not have to think about this mysterious formula:

$$\sigma_x = \sqrt{\frac{\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2}{n}}$$

but you'd be happy to use a procedure `standard-deviation` that you found in a collection of statistical programs.

After all, there's no law of nature that says computers automatically know how to add or subtract. You could imagine having to instruct Scheme to compute the sum of two large numbers digit by digit, the way you did in elementary school. But instead someone has "taught" your computer how to add before you get to it, giving this technique the name `+` so that you can ask for the sum of two numbers without thinking about the steps required. By inventing `average` or `standard-deviation` we are extending the repertoire of computations that you can ask for without concerning yourself with the details.

Composability

We've suggested that a procedure you define, such as `average`, is essentially similar to one that's built into Scheme, such as `+`. In particular, the rules for building expressions are the same whether the building blocks are primitive procedures or defined procedures.

```
> (average (+ 10 8) (* 3 5))
16.5

> (average (average 2 3) (average 4 5))
3.5

> (sqrt (average 143 145))
12
```

Any return value can be used as an end in itself, as the return value from `sqrt` was used in the last of these examples, or it can provide an argument to another procedure, as the return value from `*` was used in the first of these examples.

These small examples may seem arbitrary, but the same idea, composition of functions, is the basis for all Scheme programming. For example, the complicated formula we gave for standard deviation requires computing the squares of several numbers. So if we were to write a `standard-deviation` procedure, it would invoke `square`.

The Substitution Model

We've paid a lot of attention to the details of formal parameters and actual arguments, but we've been a little handwavy* about how a procedure actually computes a value when you invoke it.

We're going to explain what happens when you invoke a user-defined procedure. Every explanation is a story. No story tells the entire truth, because there are always some details left out. A *model* is a story that has just enough detail to help you understand whatever it's trying to explain but not so much detail that you can't see the forest for the trees.

Today's story is about the *substitution* model. When a procedure is invoked, the goal is to carry out the computation described in its body. The problem is that the body is written in terms of the formal parameters, while the computation has to use the actual argument values. So what Scheme needs is a way to associate actual argument values with formal parameters. It does this by making a new copy of the body of the procedure, in

* You know, that's when you wave your hands around in the air instead of explaining what you mean.

which it substitutes the argument values for every appearance of the formal parameters, and then evaluating the resulting expression. So, if you've defined `square` with

```
(define (square x)
  (* x x))
```

then the body of `square` is `(* x x)`. When you want to know the square of a particular number, as in `(square 5)`, Scheme substitutes the 5 for `x` everywhere in the body of square and evaluates the expression. In other words, Scheme takes

```
(* x x)
```

then does the substitution, getting

```
(* 5 5)
```

and then evaluates that expression, getting 25.

If you just type `(* x x)` into Scheme, you will get an error message complaining that `x` doesn't mean anything. Only after the substitution does this become a meaningful expression.

By the way, when we talk about "substituting into the body," we don't mean that the procedure's definition is changed in any permanent way. The body of the procedure doesn't change; what happens, as we said before, is that Scheme constructs a new expression that looks like the body, except for the substitutions.*

There are little people who specialize in `square`, just as there are little people who specialize in `+` and `*`. The difference is that the little people who do primitive procedures can do the work "in their head," all at once. The little people who carry out user-defined procedures have to go through this substitution business we're talking about here. Then they hire other little people to help evaluate the resulting expression, just as Alonzo hires people to help him evaluate the expressions you type directly to Scheme.

Let's say Sam, a little person who specializes in `square`, has been asked to compute `(square 6)`. Sam carries out the substitution, and is left with the expression `(* 6 6)` to

* You may be thinking that this is rather an inefficient way to do things—all this copying and replacement before you can actually compute anything. Perhaps you're afraid that your Scheme programs will run very slowly as a result. Don't worry. It really happens in a different way, but the effect is the same except for the speed.

evaluate. Sam then hires Tessa, a multiplication specialist, to evaluate this new expression. Tessa tells Sam that her answer is 36, and, because the multiplication is the entire problem to be solved, this is Sam's answer also.

Here's another example:

```
(define (hypotenuse a b)
  (sqrt (+ (square a) (square b))))

> (hypotenuse 5 12)
```

Suppose Alonzo hires Harry to compute this expression. Harry must first substitute the actual argument values (5 and 12) into the body of `hypotenuse`:

```
(sqrt (+ (square 5) (square 12)))
```

Now he evaluates that expression, just as Alonzo would evaluate it if you typed it at a Scheme prompt. That is, Harry hires four little people: one `sqrt` expert, one `+` expert, and two `square` experts.* In particular, some little person has to evaluate `(square 5)`, by substituting 5 for `x` in the body of `square`, as in the earlier example. Similarly, we substitute 12 for `x` in order to evaluate `(square 12)`:

```
(hypotenuse 5 12)                  ; substitute into HYPOTENUSE body
(sqrt (+ (square 5) (square 12)))  ; substitute for (SQUARE 5)
         (* 5 5)
         25
(sqrt (+ 25         (square 12)))  ; substitute for (SQUARE 12)
                    (* 12 12)
                    144
(sqrt (+ 25         144))
      (+ 25         144)           ; combine the results as before
      169
(sqrt 169)
13
```

* Until we started defining our own procedures in this chapter, all of the little people were hired by Alonzo, because all expressions were typed directly to a Scheme prompt. Now expressions can come from the bodies of procedures, and so the little people needed to compute those expressions are hired by the little person who's computing that procedure. Notice also that each little person *reports to* another little person, not necessarily the one who *hired* her. In this case, if Harry hires Shari for `sqrt`, Paul for `+`, and Slim and Sydney for the two `square`s, then Slim reports to Paul, not to Harry. Only Shari reports directly to Harry.

Don't forget, in the heady rush of learning about the substitution model, what you already knew from before: Each piece of this computation is done by a little person, and some other little person is waiting for the result. In other words, the substitution model tells us how *each compound procedure* is carried out, but doesn't change our picture of the way in which procedure invocations are *composed* into larger expressions.

Pitfalls

⇒ Don't forget that a function can have only *one* return value. For example, here's a program that's supposed to return the sum of the squares of its two arguments:

```
(define (sum-of-squares x y)                ;; wrong!
  (square x)
  (square y))
```

The problem is that the body of this procedure has two expressions, instead of just one. As it turns out, Scheme just ignores the value of the first expression in cases like this, and returns the value of the last one. What the programmer wants is the *sum* of these two values, so the procedure should say

```
(define (sum-of-squares x y)
  (+ (square x)
     (square y)))
```

⇒ Another pitfall comes from thinking that a procedure call changes the value of a parameter. Here's a faulty program that's supposed to compute the function described by $f(x) = 3x + 10$:

```
(define (f x)                               ;; wrong!
  (* x 3)
  (+ x 10))
```

Again, the first expression has no effect and Scheme will just return the value $x + 10$.*

⇒ A very common pitfall in Scheme comes from choosing the name of a procedure as a parameter. It doesn't come up very often with procedures like the ones in this chapter

* This is especially problematic for people who used to program in a language like Pascal or BASIC, where you say things like "`X = X * 3`" all the time.

whose domains and ranges are both numbers, but it will be more likely later. If you have a program like this:

```
(define (square x)
  (* x x))

(define (area square)                          ;; wrong!
  (square square))
```

then you'll get in trouble when you invoke the procedure, for example, by saying `(area 8)`. The `area` little person will substitute `8` for `square` everywhere in the procedure definition, leaving you with the expression `(8 8)` to evaluate. That expression would mean to apply the procedure `8` to the argument `8`, but `8` isn't a procedure, so an error message results.

It isn't a problem if the formal parameter is the name of a procedure that you don't use inside the body. The problem arises when you try to use the same name, e.g., `square`, with two meanings within a single procedure. But special forms are an exception; you can never use the name of a special form as a parameter.

⇒ A similar problem about name conflicts comes up if you try to use a keyword (the name of a special form, such as `define`) as some other kind of name—either a formal parameter or the name of a procedure you're defining. We're listing this separately because the result is likely to be different. Instead of getting the wrong value substituted, as above, you'll probably see a special error message along the lines of "improper use of keyword."

⇒ Formal parameters *must* be words. Some people try to write procedures that have compound expressions as the formal parameters, like this:

```
(define (f (+ 3 x) y)                          ;; wrong!
  (* x y))
```

Remember that the job of the procedure definition is only to provide a *name* for the argument. The *actual* argument isn't pinned down until you invoke the procedure. People who write programs like the one above are trying to make the procedure definition do some of the job of the procedure invocation.

Boring Exercises

4.1 Consider this procedure:

```
(define (ho-hum x y)
  (+ x (* 2 y)))
```

Show the substitution that occurs when you evaluate

```
(ho-hum 8 12)
```

4.2 Given the following procedure:

```
(define (yawn x)
  (+ 3 (* x 2)))
```

list all the little people that are involved in evaluating

```
(yawn (/ 8 2))
```

(Give their names, their specialties, their arguments, who hires them, and what they do with their answers.)

4.3 Here are some procedure definitions. For each one, describe the function in English, show a sample invocation, and show the result of that invocation.

```
(define (f x y) (- y x))

(define (identity x) x)

(define (three x) 3)

(define (seven) 7)

(define (magic n)
  (- (/ (+ (+ (* 3 n)
              13)
           (- n 1))
        4)
     3))
```

Real Exercises

4.4 Each of the following procedure definitions has an error of some kind. Say what's wrong and why, and fix it:

```
(define (sphere-volume r)
  (* (/ 4 3) 3.141592654)
  (* r r r))

(define (next x)
  (x + 1))

(define (square)
  (* x x))

(define (triangle-area triangle)
  (* 0.5 base height))

(define (sum-of-squares (square x) (square y))
  (+ (square x) (square y)))
```

4.5 Write a procedure to convert a temperature from Fahrenheit to Celsius, and another to convert in the other direction. The two formulas are $F = \frac{9}{5}C + 32$ and $C = \frac{5}{9}(F - 32)$.

4.6 Define a procedure `fourth` that computes the fourth power of its argument. Do this two ways, first using the multiplication function, and then using `square` and not (directly) using multiplication.

4.7 Write a procedure that computes the absolute value of its argument by finding the square root of the square of the argument.

4.8 "Scientific notation" is a way to represent very small or very large numbers by combining a medium-sized number with a power of 10. For example, 5×10^7 represents the number 50000000, while 3.26×10^{-9} represents 0.00000000326 in scientific notation. Write a procedure `scientific` that takes two arguments, a number and an exponent of 10, and returns the corresponding value:

```
> (scientific 7 3)
7000
```

```
> (scientific 42 -5)
0.00042
```

Some versions of Scheme represent fractions in *a*/*b* form, and some use scientific notation, so you might see `21/50000` or `4.2E-4` as the result of the last example instead of `0.00042`, but these are the same value.

(A harder problem for hotshots: Can you write procedures that go in the other direction? So you'd have

```
> (sci-coefficient 7000)
7

> (sci-exponent 7000)
3
```

You might find the primitive procedures `log` and `floor` helpful.)

4.9 Define a procedure `discount` that takes two arguments: an item's initial price and a percentage discount. It should return the new price:

```
> (discount 10 5)
9.50

> (discount 29.90 50)
14.95
```

4.10 Write a procedure to compute the tip you should leave at a restaurant. It should take the total bill as its argument and return the amount of the tip. It should tip by 15%, but it should know to round up so that the total amount of money you leave (tip plus original bill) is a whole number of dollars. (Use the `ceiling` procedure to round up.)

```
> (tip 19.98)
3.02

> (tip 29.23)
4.77

> (tip 7.54)
1.46
```