Lecture 8

Grammars and Parsers
grammar and derivations, recursive descent parser vs. CYK parser, Prolog vs. Datalog

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Hack Your Language!
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Outline

Grammars: a concise way to define program syntax

Parsing: recognize syntactic structure of a program

Parser 1: recursive descent (backtracking)

Parser 2: CYK (dynamic programming algorithm)

Note: this file includes useful hidden slides which do not show in the PowerPoint Slide View.
Why parsing?

Parsers making sense of these sentences:

This lecture is dedicated to my parents, Mother Teresa and the pope.

The (missing) serial comma determines whether M.T.&p. associate with “my parents” or with “dedicated to”.

Seven-foot doctors filed a law suit.

does “seven” associate with “foot” or with “doctors”?

\[ \text{if } E_1 \text{ then } \begin{cases} \text{if } E_2 \text{ then } E_3 \text{ else } E_4 \end{cases} \]

typical semantics associates “else \( E_4 \)” with the closest if (ie, “if \( E_2 \)”).

In general, programs and data exist in text form

which needs to be understood by parsing (and converted to tree form)
The cs164 parsing story

From string generation to Earley parser

1. Write a random expression generator.

2. Invert this generator into a parser by inverting \texttt{print} into \texttt{scan} and \texttt{random()} into \texttt{ask0acle()}\texttt{.} The oracle constructs the parse tree.

3. Rewrite this parser in Prolog, which serves as your oracle. This gives you the ubiquitous recursive descent parser. Time = $O(2^n)$

4. Observe that this Prolog parser has no negation. It’s in a Datalog subset of Prolog (more or less).

5. Datalog programs are evaluated bottom-up (dynamic programming). Rewriting the Prolog parser into Datalog gives us the CYK parser. $O(n^3)$

6. Datalog evaluation can be optimized with the Magic Set Transformation, which gives us Earley Parser. (Covered in Lecture 9.) $O(n)$ with a suitable grammar. Earley is the basis for all efficient modern parsers.
Grammars
Grammar: a recursive definition of a language

Language: a set of (desired) strings

Example: the language of Regular Expressions (RE).

RE can be defined as a grammar:

*base case:* any input character $c$ is regular expression;

*inductive case:* if $e_1$, $e_2$ are regular expressions, then the following four are also regular expressions:

$e_1 | e_2 \quad e_1 e_2 \quad e_1^* \quad (e_1)$

Example:

a few strings in this language: $a \ a|a \ a|b^*$

a few strings not in this language: $\emptyset \ \emptyset$ empty string $(\epsilon) \ \emptyset | \emptyset$ (a)
Terminals, non-terminals, productions

The grammar notation:

\[
R ::= c \mid R\ R \mid R|R \mid R^* \mid (R)
\]

terminals (red): input characters
also called the alphabet of the language

non-terminals: substrings in the language
these symbols will be rewritten to terminals

start non-terminal: starts the derivation of a string
convention: always the first nonterminal mentioned

productions: rules governing string derivation
RE has five: \( R ::= c, \quad R ::= R\ R, \quad R ::= R|R, \quad R ::= R^*, \quad R ::= (R) \)
It’s grammar, not gramer.

“Not all writing is due to bad gramer.” (sic)

Saying “grammer” is a lexical error, not a syntactic (ie, grammatic) one.

In the compiler, this error is caught by the lexer.

lexer fails to recognize “grammer” as being in the lexicon.

In cs164, you learn which part of compiler finds errors.

lexer, parser, syntactic analysis, or runtime checks?
Deriving a string from a grammar

How is a string derived in a grammar:

1. write down the start non-terminal $S$
2. rewrite $S$ with the rhs of a production $S \rightarrow \text{rhs}$
3. pick a non-terminal $N$
4. rewrite $N$ with the rhs of a production $N \rightarrow \text{rhs}$
5. if no non-terminal remains, we have generated a string.
6. otherwise, go to 3.

Example:

grammar $G$: $E ::= T \mid T + E$  $T = F \mid F * T$  $F = a \mid (E)$

derivation of a string from $L(G)$: $E \rightarrow T + E \rightarrow F + E \rightarrow a + E$

$\rightarrow a + T \rightarrow a + F \rightarrow a + a$
Left- and right-recursive grammars
Grammars vs. languages

Write a grammar for the language all strings $ba^i$, $i > 0$.

**grammar 1:** $S ::= Sa \mid ba$

**grammar 2:** $S ::= baA \quad A ::= aA \mid \varepsilon$

A language can be described with multiple grammars

$L(G) =$ language (strings) described by grammar $G$

in our example, $L(\text{grammar 1}) = L(\text{grammar 2})$

Left recursive grammar:

Right-recursive grammar:

both $l$-rec and $r$-rec:
Why care about left-/right-recursion?

Some parser can’t handle left-recursive grammars.

It may get them into infinite recursion.

Same principle as in Prolog programs that do not terminate.

Luckily, we can rewrite a l-rec grammar into a r/r one.

while describing the same language

Example 1:

S ::= Sa | a can be rewritten to S ::= aS | a
The typical expression grammar

A grammar of expressions:

\[ G_1: \quad E ::= n \mid E + E \mid E * E \mid (E) \]

\[ G_1 \] is l-rec but can be rewritten to \( G_2 \) which is not

\[ G_2: \quad E ::= T \mid T + E \]
\[ T ::= F \mid F * T \]
\[ F ::= n \mid (E) \]

In addition to removing left recursion, nonterminals \( T \) (a term) and \( F \) (a factor) introduce desirable precedence and associativity. More in L9.

Is \( L(G_1) = L(G_2) \)?

That is, are these same sets of string? Yes.
The parsing problem
What the parser does

The *syntax-checking* parsing problem:

Given an input string $s$ and grammar $G$, check if $s \in L(G)$

The *parse-tree* parsing problem:

Given an input string $s \in L(G)$, return the parse tree of $s$
Really

A Poor Man’s Parser
Generate-and-test “parser”

We want to test if \( s \in L(G) \). Our “algorithm”:
- print a string \( p \in L(G) \), check if \( s = p \), repeat

The plan:
Write a function \( \text{gen}(G) \) that prints a string \( p \in L(G) \).
If \( L(G) \) is finite, \( \text{gen}(G) \) will eventually print all strings in \( L(G) \).

Does this algorithm work?
*Depends if you are willing to wait. 😊*
*Also, \( L(G) \) may be infinite.*

This parser is useful only for instructional purposes
in case it’s not clear already
gen(G)

Grammar G and its language L(G):

\[ G: \quad E ::= a \mid E + E \mid E \ast E \]

\[ L(G) = \{ a, a+a, a*a, a*a+a, \ldots \} \]

For simplicity, we hardcode G into gen()

```python
def gen() {  E(); print EOF }
def E() {  
    switch (choice()):  
    case 1: print "a"
    case 2: E(); print "+"; E()
    case 3: E(); print "\ast"; E()
}
Visualizing string generation with a parse tree

The tree that describe string derivation is parse tree.

Are we generating the string top-down or bottom-up?

Top-down. Can we do it other way around? Sure. See CYK.
Parsing

Parsing is the inverse of string generation:

given a string, we want to find the parse tree

If parsing is just the inverse of generation, let’s obtain the parser **mechanically** from the generator!

```python
def gen() {  E(); print EOF }
def E() {
    switch (choice()):
    case 1: print "a"
    case 2: E(); print "+"; E()
    case 3: E(); print "*"; E()
}
```
def gen() { E(); print EOF }
def E() {
    switch (choice()) {
        case 1: print "a"
        case 2: E(); print "+"; E()
        case 3: E(); print "*"; E()
    }
}
def parse() { E(); scan(EOF) }
def E() {
    switch (oracle()) {
        case 1: scan("a")
        case 2: E(); scan("+"); E()
        case 3: E(); scan("*"); E()
    }
    def scan(s) { if rest of input starts with s, consume s; else abort }
Reconstruct the Parse Tree
Parse tree

Parse tree: shows how the string is derived from $G$

leaves: input characters

internal nodes: non-terminals

children of an internal node: production used in derivation

Why do we need the parse tree?

We evaluate it to obtain the AST, or sometimes to directly compute the value of the program.

Test yourself: construct the AST from a parse tree.
Example: evaluate an expression on parse tree

Input: $2 \times (4 + 5)$

Grammar:

$E ::= T \mid T + E$
$T ::= F \mid F \times T$
$F ::= n \mid (E)$

Parse Tree (annotated with values):
Parse tree vs. abstract syntax tree

Parse tree = concrete syntax tree

– contains all syntactic symbols from the input
– including those that the parser needs "only" to discover
  • intended nesting: parentheses, curly braces
  • statement termination: semicolons

Abstract syntax tree (AST)

– abstracts away these artifacts of parsing,
– abstraction compresses the parse tree
  • flattens parse tree hierarchies
  • drops tokens
Add parse tree reconstruction to our parser

```python
def parse() {
    root = E();
    scan(EOF);
    return root
}

def E() {
    switch (oracle()) {
        case 1: scan("a")
            return ("a",)
        case 2:
            left = E()
            scan("+")
            right = E()
            return ("+", left, right)
        case 3: // analogous
    }
}
```

Python tuple
Recursive Descent Parser
(by implementing the oracle with Prolog)
Recall \textit{amb}: the nondeterministic evaluator from cs61A

\[(\text{amb 1 2 3 4 5})\] evaluates to 1 or .. or 5

Which option does \textit{amb} choose? One leading to success.

in our case, success means parsing successfully

How was \textit{amb} implemented?

backtracking

Our parser with \textit{amb}:

\[
\text{def } E() \{ \text{ switch (amb(1,2,3)) } \{ \\
\text{ case 1: scan("a")} \\
\text{ case 2: E(); scan("+"); E()} \\
\text{ case 3: E(); scan("*"); E()} \} \}
\]

Note: \textit{amb} may not work with any left-recursive grammar
How do we implement the oracle

We could implement it with coroutines.

We’ll use use **logic programming** instead.

After all, we already have oracle functionality in our Prolog

We will define a parser as a logic program

backtracking will give it exponential time complexity
Backtracking parser in Prolog

Example grammar:

\[ E ::= a \]
\[ E ::= a + E \]

We want to parse a string a+a, using a query:

\[ ?- \text{parse}([a,+,a]). \]

true \[ \iff \text{a+a is in } L(G) \]

Backtracking Prolog parser for this grammar

\[ e([a|\text{Out}], \text{Out}). \]
\[ e([a,+R], \text{Out}) :- e(R,\text{Out}). \]
\[ \text{parse}(S) :- e(S,[]). \]
How does this parser work? (1)

Let’s start with simple Prolog queries:

?- [H | T] = [a,+,a].
H = a,
T = [+ , a].

?- [a,+,b,+,c]=[a, + | Rest].
Rest = [b, +, c].
How does this parser work? (2)

Let’s start with this (incomplete) grammar:

\[ e([a|T], T). \]

Sample queries:

\[ e([a,+a], Rest). \]

--> Rest = [+a]

\[ e([a], Rest). \]

--> Rest = []

\[ e([a], []). \]

--> true  // parsed successfully
Parser for the full expression grammar

\[
E = T | T + E \\
T = F | F * T \\
F = a
\]

e(In,Out) :- t(In, Out).
e(In,Out) :- t(In, [+|R]), e(R,Out).

t(In,Out) :- f(In, Out).
t(In,Out) :- f(In, [!*|R]), t(R,Out).

f([a|Out],Out).

parse(S) :- e(S,[]).

?- parse([a,+,a,*,a],T). --> true
Construct also the parse tree

\[ E = T \mid T + E \quad T = F \mid F \ast T \quad F = a \]

e(In,Out,e(T1)) :- t(In, Out, T1).
e(In,Out,e(T1,+ ,T2)) :- t(In, [+|R], T1), e(R,Out,T2).
t(In,Out,e(T1)) :- f(In, Out, T1).
t(In,Out,e(T1,*,T2)) :- f(In, [*|R], T1), t(R,Out,T2).
f([a|Out],Out,a).

parse(S,T) :- e(S, [], T).

?- parse([a,+ ,a,*,a],T).
T = e(e(a), +, e(e(a, *, e(a))))
Construct also the AST

\[
E = T | T + E \quad T = F | F * T \quad F = a
\]

\[
e(\text{In,Out}, T1) \quad \text{:- t(\text{In, Out, T1}).}
\]

\[
e(\text{In,Out}, \text{plus}(T1, T2)) \quad \text{:- t(\text{In, [+|R], T1}), e(R,Out,T2).}
\]

\[
t(\text{In,Out},T1) \quad \text{:- f(\text{In, Out, T1}).}
\]

\[
t(\text{In,Out},\text{times}(T1,T2)) \quad \text{:- f(\text{In, [*|R], T1}), t(R,Out,T2).}
\]

\[
f([a|Out],\text{Out, a}).
\]

\[
\text{parse}(S,T) \quad \text{:- e(S,[]),T).}
\]

\[
? - \text{parse}([a,+,a,*,a],T).
\]

\[
T = \text{plus}(a, \text{times}(a, a))
\]
Running time of the backtracking parser

We can analyze either version. They are the same.

amb:

```python
def E() { switch (oracle(1,2,3)) {
    case 1: scan("a")
    case 2: E(); scan("+"); E()
    case 3: E(); scan("*"); E() }}
```

Prolog:

```prolog
e(In,Out) :- In==[a|Out].
e(In,Out) :- e(In,T1), T1==[+|T2], e(T2,Out)
e(In,Out) :- e(In,T1), T1==[*|T2], e(T2,Out)
```
Recursive descent parser

This parser is known as recursive descent parser (rdp)

The parser for the calculator (Lec 2) is an rdp. Study its code. rdp is the way to go when you need a small parser.

Crafting its grammar carefully removes exponential time complexity.

Because you can avoid backtracking by facilitating making choice between rules based on immediate next input. See the calculator parser.
Summary
Summary

Languages vs grammars

- a language can be described by many grammars

Grammars

- string generation vs. recognizing if string is in grammar
- random generator and its dual, oracular recognizer

Parse tree:

- result of parsing is parse tree

Recursive descent parser

- runs in exponential time.