CS 70 Discrete Mathematics for CS
Fall 2003 Wagner

HW 8

Due Thursday November 6

Show your work on all questions on this problem set.

1. **(4 pts.) Any questions?**
   Is there anything you'd like to see explained better in lecture or discussion sections?

2. **(10 pts.) Binary strings**

   (a) Count the number of 2003-bit strings that don't include 01 as a substring. Simplify your expression as much as possible.

   (b) Count the number of n-bit strings that don't contain 110 as a substring. It's enough to find a recurrence relation; you don't need to solve the recurrence relation.

3. **(10 pts.) De Méré's problem**
   The birth of the probability theory has been partially attributed (by some) to two questions that the Chevalier de Méré posed to Pascal and Fermat in the 17th century, thereby triggering Pascal and Fermat to study this field systematically. Here is the first of de Méré's two questions.

   (a) What is the probability of getting at least one double six in 24 throws of a pair of dice?

   (b) What is the probability of getting at least one six in 4 throws of a die?

   Historical side note, for the curious: When Pascal told M. de Méré that the answer to part (a) is different from part (b), de Méré reportedly was scandalized. The good chevalier initially refused to believe it, insisting that $24/36 = 4/6$ and therefore the two cases must be equivalent. One of Pascal's letters to Fermat begged Fermat to please explain this to de Méré, because Pascal was striking out. Hey, nobody's perfect!

4. **(10 pts.) Homeworks**
   Suppose n of you do this homework. I collect your solutions, and grade them, shuffle the stack of graded solutions (so that all permutations of the solutions are equally likely), and then randomly hand back one graded solution to each of you who turned in a solution. What is the probability that you get your own homework back?

5. **(10 pts.) Correlation**
   It was suggested in class that, when $\Pr[A|B] > \Pr[A]$, then A and B may be viewed intuitively as being positively correlated. One might wonder whether "being positively correlated" is a symmetric relation. Prove or disprove: If $\Pr[A|B] > \Pr[A]$ holds, then $\Pr[B|A] > \Pr[B]$ must necessarily hold, too. (You may assume that both $\Pr[A|B]$ and $\Pr[B|A]$ are well-defined, i.e., neither $\Pr[A]$ nor $\Pr[B]$ are zero.)

6. (20 pts.) Independence (due to H.W. Lenstra)

Suppose we pick a random card from a standard deck of 52 playing cards. Let A represent the event that the card is an queen, B the event that the card is a spade, and C the event that a red card (a heart or a diamond) is drawn.

(a) Which two of A, B, and C are independent? Justify your answer carefully.

[In other words: For each pair of events (AB, AC, and BC), state and prove whether they are independent or not.]

(b) What if a joker is added to the deck? Justify your answer carefully.

7. (16 pts.) Independence (due to H.W. Lenstra)

Let Ω be a sample space, and let $A, B \subseteq \Omega$ be two *independent* events. Let $\overline{A} = \Omega - A$ and $\overline{B} = \Omega - B$ (sometimes written $\neg A$ and $\neg B$) denote the complementary events.

For the purposes of this question, you may use the following definition of independence: Two events A, B are *independent* if $\Pr[A \cap B] = \Pr[A] \Pr[B]$.

(a) Prove or disprove: $\overline{A}$ and $\overline{B}$ are necessarily independent.

(b) Prove or disprove: A and $\overline{B}$ are necessarily independent.

(c) Prove or disprove: A and $\overline{A}$ are necessarily independent.

(d) Prove or disprove: It is possible that $A = B$.

8. (10 pts.) Burnt pancakes

I have a bag containing three pancakes: One golden on both sides, one burnt on both sides, and one golden on one side and burnt on the other. You shake the bag, draw a pancake at random, look at one side, and notice that it is burnt. What is the probability that the other side is burnt?

9. (10 pts.) Money bags

I have a bag containing either a \$1 or \$5 bill (with equal probability assigned to both possibilities). I then add a \$1 bill to the bag, so it now contains two bills. The bag is shaken, and you draw out a \$1 bill. If a second student draws the remaining bill from the bag, what is the chance that it is a \$1 bill?