Everything You Always Wanted To Know about Game Theory*
*but were afraid to ask

Dan Garcia, UC Berkeley
David Ginat, Tel-Aviv University
Peter Henderson, Butler University

What is "Game Theory"?
Combinatorial / Computational / Economic

Combinatorial	Computational	Economic
◊ Sprague and Grundy's 1939 *Mathematics and Games*	◊ R. Bell and M. Cornelius' 1988 *Board Games around the World*	◊ von Neumann and Morgenstern's 1944 *Theory of Games and Economic Behavior*
◊ Board (table) games	◊ Board (table) games	◊ Matrix games
◊ Nim, Domineering	◊ Tic-Tac-Toe, Chess	◊ Prisoner's dilemma
◊ Complete info, alternating moves	◊ Complete info, alternating moves	◊ Incomplete info, simultaneous moves
◊ Goal: Last move	◊ Goal: Varies	◊ Goal: Maximize payoff

Know Your Audience…

- How many have used games pedagogically?
- What is your own comfort level with GT? (hands down = none, one hand = ok; two hands = you could be teaching this session)
  - ◊ Combinatorial (Berlekamp-ish)
  - ◊ Computational (AI, Brute-force solving)
  - ◊ Economic (Prisoner's dilemma, matrix games)

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Here's our schedule:

("GT" = "Game Theory")

- Dan: Overview, Combinatorial GT basics
- David: Combinatorial GT examples
- Dan: Computational GT
- Peter: Economic GT & Two-person games
- Dan: Summary & Where to go from here

(All of GT in 75 min? Right!)

Why are games useful pedagogical tools?

- Vast resource of problems
  - ◊ Easy to state
  - ◊ Colorful, rich
  - ◊ Use in lecture or for projects
  - ◊ They can USE their projects when they're done
  - ◊ Project Reuse -- just change the games every year!
  - ◊ Algorithms, User Interfaces, Artificial Intelligence, Software Engineering

"Every game ever invented by mankind, is a way of making things hard for the fun of it!"
– John Ciardi

What is a combinatorial game?

- Two players (Left & Right) *alternating turns*
- No chance, such as dice or shuffled cards
- Both players have perfect information
  - ◊ No hidden information, as in Stratego & Magic
- The game is finite – it must eventually end
- There are no draws or ties
- Normal Play: Last to move wins!


Combinatorial Game Theory
The Big Picture
Whose turn is not part of the game
SUMS of games
You play games $G_1 + G_2 + G_3 + \ldots$
You decide which game is most important
You want the last move (in normal play)
Analogy: Eating with a friend, want the last bite
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Classification of Games
Impartial
Same moves available to each player
Example: Nim
Partisan
The two players have different options
Example: Domineering
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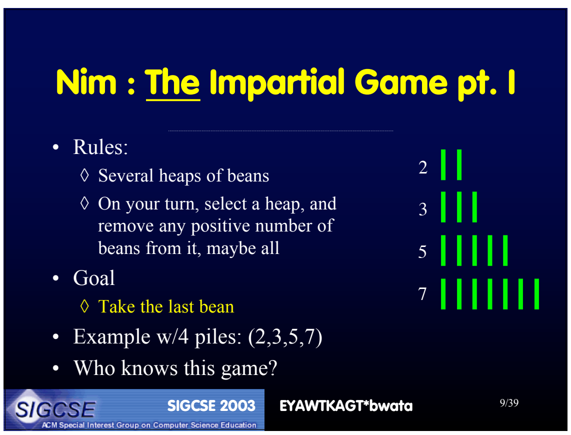
Nim : The Impartial Game pt. I
Rules:
Several heaps of beans
On your turn, select a heap, and remove any positive number of beans from it, maybe all
Goal
Take the last bean
Example w/4 piles: (2,3,5,7)
Who knows this game?
2
3
5
7
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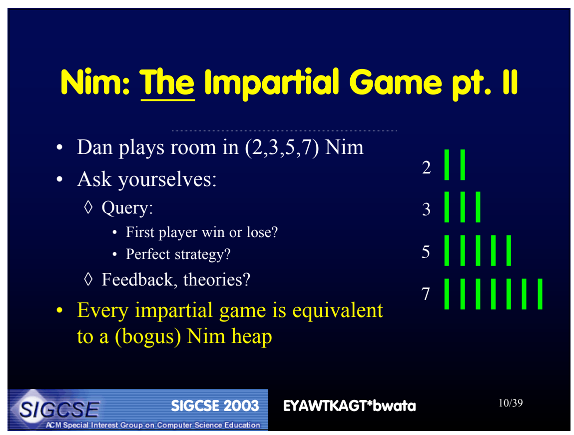
Nim: The Impartial Game pt. II
Dan plays room in (2,3,5,7) Nim
Ask yourselves:
Query:
First player win or lose?
Perfect strategy?
Feedback, theories?
Every impartial game is equivalent to a (bogus) Nim heap
2
3
5
7
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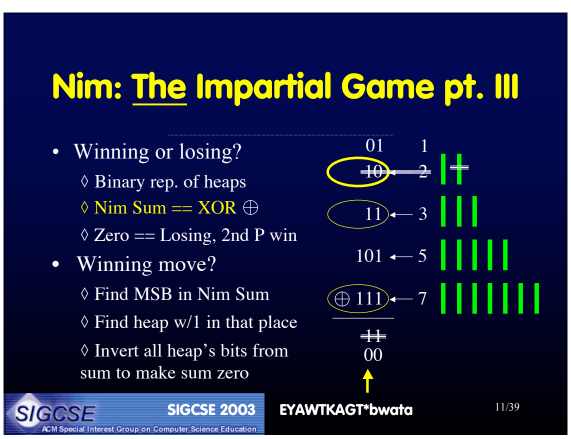
Nim: The Impartial Game pt. III
Winning or losing?
Binary rep. of heaps
Nim Sum == XOR ⊕
Zero == Losing, 2nd P win
Winning move?
Find MSB in Nim Sum
Find heap w/1 in that place
Invert all heap's bits from sum to make sum zero
01
1
10
2
11
3
101
5
⊕ 111
7
00
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Domineering: A partisan game
Rules (on your turn):
Place a domino on the board
Left places them North-South
Right places them East-West
Goal
Place the last domino
Example game
Query: Who wins here?
Left (bLue)
Right (Red)
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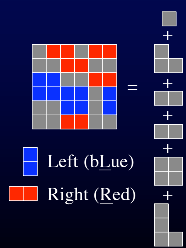
Domineering: A partisan game



- Key concepts
  - ◊ By moving correctly, you guarantee yourself future moves.
  - ◊ For many positions, you want to move, since you can steal moves. This is a "hot" game.
  - ◊ This game decomposes into non-interacting parts, which we separately analyze and bring results together.

What do we want to know about a particular game?

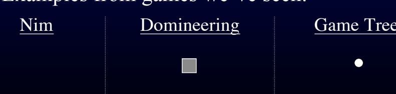
- What is the value of the game?
  - ◊ Who is ahead and by how much?
  - ◊ How big is the next move?
  - ◊ Does it matter who goes first?
- What is a winning / drawing strategy?
  - ◊ To know a game's value and winning strategy is to have solved the game
  - ◊ Can we easily summarize strategy?

Combinatorial Game Theory The Basics I - Game definition

- A game, G, between two players, Left and Right, is defined as a pair of sets of games:
  - ◊ $G = \{G^L \mid G^R\}$
  - ◊ G^L is the typical Left option (i.e., a position Left can move to), similarly for Right.
  - ◊ G^L need not have a unique value
  - ◊ Thus if $G = \{a, b, c, \ldots \mid d, e, f, \ldots\}$, G^L means *a* or *b* or *c* or … and G^R means *d* or *e* or *f* or ...

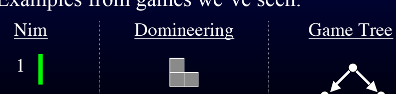
Combinatorial Game Theory The Basics II - Examples: 0

- The simplest game, the Endgame, born day 0
  - ◊ Neither player has a move, the game is over
  - ◊ { Ø | Ø } = { | }, we denote by 0 (a number!)
  - ◊ Example of *P*, previous/second-player win, losing
  - ◊ Examples from games we've seen:



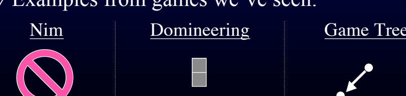
Combinatorial Game Theory The Basics II - Examples: *

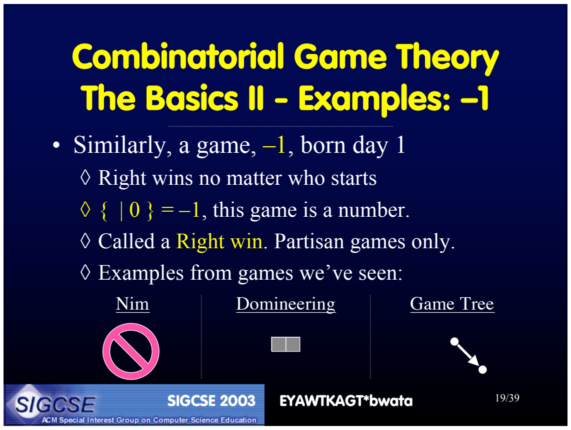
- The next simplest game, * ("Star"), born day 1
  - ◊ First player to move wins
  - ◊ { 0 | 0 } = *, this game is not a number, it's fuzzy!
  - ◊ Example of *N*, a next/first-player win, winning
  - ◊ Examples from games we've seen:



Combinatorial Game Theory The Basics II - Examples: 1

- Another simple game, 1, born day 1
  - ◊ Left wins no matter who starts
  - ◊ { 0 | } = 1, this game is a number
  - ◊ Called a Left win. Partisan games only.
  - ◊ Examples from games we've seen:




Combinatorial Game Theory
The Basics II - Examples: –1
Similarly, a game, –1, born day 1
Right wins no matter who starts
{ | 0 } = –1, this game is a number.
Called a Right win. Partisan games only.
Examples from games we've seen:
Nim
Domineering
Game Tree
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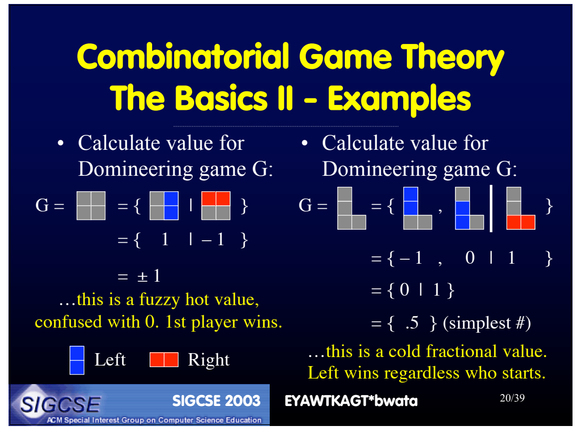
Combinatorial Game Theory
The Basics II - Examples
Calculate value for Domineering game G:
G = = { | }
= { 1 | – 1 }
= ± 1
…this is a fuzzy hot value, confused with 0. 1st player wins.
Left
Right
Calculate value for Domineering game G:
G = = { , | }
= { – 1 , 0 | 1 }
= { 0 | 1 }
= { .5 } (simplest #)
…this is a cold fractional value. Left wins regardless who starts.
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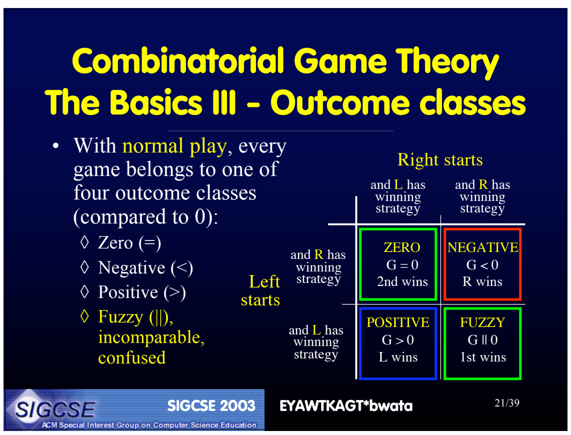
Combinatorial Game Theory
The Basics III - Outcome classes
With normal play, every game belongs to one of four outcome classes (compared to 0):
Zero (=)
Negative (<)
Positive (>)
Fuzzy (||), incomparable, confused
Right starts
and L has winning strategy
and R has winning strategy
Left starts
and R has winning strategy
and L has winning strategy
ZERO G = 0 2nd wins
NEGATIVE G < 0 R wins
POSITIVE G > 0 L wins
FUZZY G || 0 1st wins
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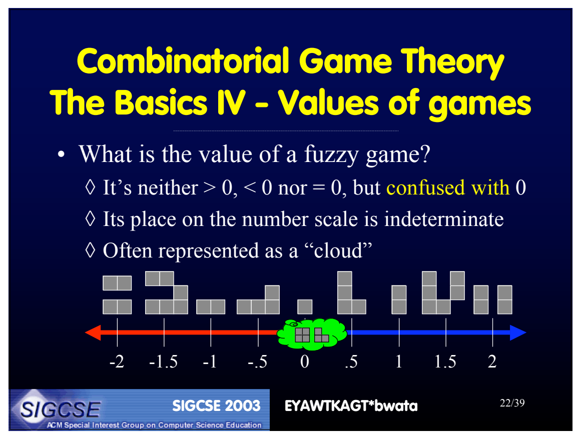
Combinatorial Game Theory
The Basics IV - Values of games
What is the value of a fuzzy game?
It's neither > 0, < 0 nor = 0, but confused with 0
Its place on the number scale is indeterminate
Often represented as a "cloud"
-2 -1.5 -1 -.5 0 .5 1 1.5 2
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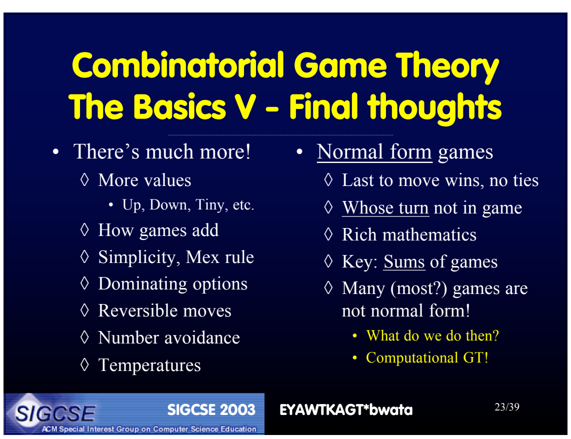
Combinatorial Game Theory
The Basics V - Final thoughts
There's much more!
More values
Up, Down, Tiny, etc.
How games add
Simplicity, Mex rule
Dominating options
Reversible moves
Number avoidance
Temperatures
Normal form games
Last to move wins, no ties
Whose turn not in game
Rich mathematics
Key: Sums of games
Many (most?) games are not normal form!
What do we do then?
Computational GT!
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And now over to David for more Combinatorial examples…
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Computational Game Theory (for non-normal play games)

- Large games
  - ◊ Can theorize strategies, build AI systems to play
  - ◊ Can study endgames, smaller version of original
    - Examples: Quick Chess, 9x9 Go, 6x6 Checkers, etc.
- Small-to-medium games
  - ◊ Can have computer solve and teach us strategy
  - ◊ I wrote a system called GAMESMAN which I use in CS0 (a SIGCSE 2002 Nifty Assignment)

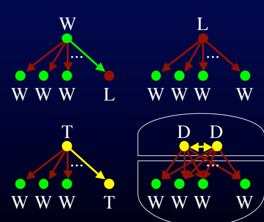
How do you build an AI opponent for large games?

- For each position, create Static Evaluator
- It returns a number: How much is a position better for Left?
  - ◊ (+ = good, – = bad)
- Run MINIMAX (or alpha-beta, or A*, or …) to find best move

White to move, wins in move 243 with Rd7xNe7

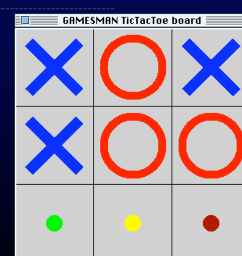
Computational Game Theory

- Simplify games / value
  - ◊ Store turn in position
  - ◊ Each position is (for player whose turn it is)
    - Winning (□ losing child)
    - Losing (All children winning)
    - Tieing (!□ losing child, but □ tieing child)
    - Drawing (can't force a win or be forced to lose)



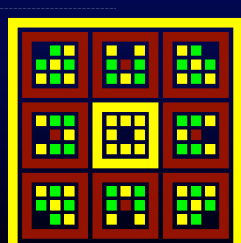
Computational Game Theory Tic-Tac-Toe

- Rules (on your turn):
  - ◊ Place your X or O in an empty slot
- Goal
  - ◊ Get 3-in-a-row first in any row/column/diag.
- Misére is tricky



Computational Game Theory Tic-Tac-Toe Visualization

- Visualization of values
- Example with Misére
  - ◊ Outer rim is position
  - ◊ Next levels are values of moves to that position
  - ◊ Recursive image
  - ◊ Legend: Lose, Tie, Win



Use of games in projects (CS0) Language: Scheme & C

- Every semester…
  - ◊ New games chosen
  - ◊ Students choose their own graphics & rules (I.e., open-ended)
  - ◊ Final Presentation, best project chosen, prizes
- Demonstrated at SIGCSE 2002 Nifty Assignments

And now over to Peter…

- Two player games
- More motivation
- Prisoner's Dilemma

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Summary

- Games are wonderful pedagogic tools
  - ◊ Rich, colorful, easy to state problems
  - ◊ Useful in lecture *or* for homework / projects
  - ◊ Can demonstrate so many CS concepts
- We've tried to give broad theoretical foundations & provided some nuggets…

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Resources

- www.cs.berkeley.edu/~ddgarcia/eyawtkagtbwata/
- www.cut-the-knot.org
- E. Berlekamp, J. Conway & R. Guy: *Winning Ways I & II* [1982]
- R. Bell and M. Cornelius: *Board Games around the World* [1988]
- K. Binmore: *A Text on Game Theory* [1992]

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