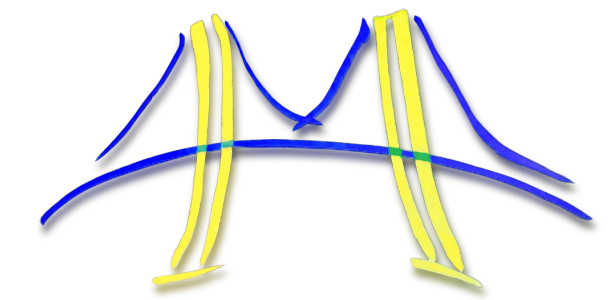# Parallel Gröbner Basis Computation using GASNet

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The Gröbner Basis Problem

- Given a set of polynomials $F = \{f_1, \dots\}$

- Consider the set of all polynomials I that may be constructed with polynomials in F by applying affine transformations (adding, subtracting, multiplying by a polynomial)

- Find a set of polynomials $G \subseteq I$ such that the leading term of any polynomial in I is divisible by the leading term of a polynomial in G. G is a Gröbner basis for F.

- Requires a defined *monomial ordering* for terms in polynomials (e.g. lexicographic)

[DAC07]

Buchberger's Algorithm

Algorithm 1: *Buchberger's Algorithm*

1: let $G = F$
2: **while** polynomials are added to g **do**
3:   pick $g_i, g_j \neq g_i \in G$
4:   minimally scale g_i and g_j so leading terms are equal
5:   let $S_{ij} = g_i - g_j$
    ▷ compute S-polynomial for g_i and g_j
6:   **for** $g \in G$ **do**
7:     reduce S_{ij} by g
8:   **end for**
9:   add S_{ij} to G if it doesn't become 0 during reduction
10: **end while**

- Worst case computation is intractable: algorithm runs in doubly-exponential time

- Normal case computation is frequently tractable, but depends heavily on choices of ordering and reduction

[Buc76]

Gröbner Basis Example

(Not following Buchberger's Algorithm)

1. Let

$$F = \{f_1 = x - y, f_2 y, f_3 = 2xy + z\}$$

2. Reduce $f_3 = xy + z$ by $f_1 = x - y$

  (a) Multiply $f_1 = x - y$ by $2y$ to get $2xy - y^2$
  (b) Subtract $2xy - y^2$ from $f_3 = 2xy + z$ to get $f_3' = y^2 + z$

3. Reduce $f_3' = y^2 - z$ by $f_2 = y$

  (a) Multiply $f_2 = y$ by y to get y^2 so leading terms are equal
  (b) Subtract y^2 from $f_3' = y^2 + z$ to get $f_3'' = z$

4. We can't reduce any more. Gröbner basis is

$$G = \{x - y, y, z\}$$

The GASNet Communication System

Global-Address Space Networking (GASNet)

- A low level network layer for parallel computing[GAS]

- A foundation for many parallel languages, including UPC [UPC] and Titanium[Tit]

- Uses active messages for communication

- Provides globally addressed arrays

Active Messages

- Messages are requests to run a function

1. Source processor sends function handle and input parameters

2. Destination processor calls function with specified parameters

3. (Optional) Destination processor asks source to call acknowledgment function

Multipol

Multipol System

- A distributed data structure library for parallel programs[Mul]
- Run time system manages infrastructure
- Data Structures include
  - `ObjectLayer` for distributed cashing and retrieving objects
  - `Bipartite` for storing a bipartite graph
  - Others...
- Used active messages for communication

Gröbner Basis for Multipol

- Chakrabati and Yelick [CY93] parallelized Buchberger's Algorithm
- Used Multipol's `ObjectLayer` data structure to store polynomials
- Ran on CM5 (now defunct)

CS267 Project

Port [CY93] to GASNet

- Detach `ObjectLayer` data structure from Multipol runtime system
- Translate parallelism to GASNet

Possible Extensions

- Port to modern parallel language, e.g. UPC.
- Port complete Multipol system.
- Improve Gröbner basis algorithm

References

[Buc76] B. Buchberger. A theoretical basis for the reduction of polynomials to canonical forms. *SIGSAM Bull.*, 10(3):19–29, 1976.

[CY93] Soumen Chakrabarti and Katherine Yelick. On the correctness of a distributed memory gröbner basis algorithm. Technical report, In Rewriting Techniques and Applications, 1993.

[DAC07] Donal O'Shea David A. Cox, John B. Little. *Ideals, Varieties, and Algorithms.* Springer, 2007.

[GAS] Gasnet communications system. http://gasnet.cs.berkeley.edu/.

[Mul] Multipol. http://www.eecs.berkeley.edu/ Research/Projects/CS/parallel/castle/multipol/.

[Tit] Titanium. http://titanium.cs.berkeley.edu/.

[UPC] Berkeley upc - unified parallel c. http://upc.lbl.gov/.