CS 70 Discrete Mathematics and Probability Theory
Spring 2011 Demmel HW 10

Due April 8, 6:00pm

You may work in groups of up to 3 people (no larger!). Please read the group collaboration policies on bSpace or `www.cs.berkeley.edu/~demmel/cs70_Spr11` before beginning group work. You *must* write up the solution set entirely on your own. You must never look at any other students' solutions (not even a draft), nor share your own solutions (not even a draft).

Please begin your answer to each question on a new sheet of paper, and make sure that each sheet is labeled with your name, section number, GSI name, the assignment number, the question number, and "CS70–Spring 2011".

Turn in each question in a different box in 283 Soda Hall: Question 1 in the box labeled "CS70 - 1", Question 2 in the box labeled "CS70 - 2", etc. Reason: Different problems will be graded in parallel by different readers.

Warning: You risk receiving no credit, or losing points, for any homework that does not conform to the above regulations! Please take the time to write clear and concise solutions; we will not grade messy or unreadable submissions. No late homeworks will be accepted. We will drop the lowest two homework scores.

1. (10 pts.) Consider the following game: flip a fair coin, and bet \$1 on H. (i.e. if it is a H, you win and get \$1; if it is a T, you lose and pay \$1). If you win, the game stops. Otherwise, you flip the coin again, this time doubling your bet. If you win, stop; else, repeat this game of flipping and doubling your bet each time, until you win and stop. For example, if you get 3 Tails in a row and then a Head, you lose $1+2+4=7$ dollars on the first three throws, and win 8 dollars on the last throw, for a total win of $8-7=1$ dollar.

(a). Suppose f is the sum of all wins and losses at the end of the game. What is the distribution of f? Calculate its expectation and variance.

(b). Suppose g is the maximum value you could lose before you finally win, i.e. the sum of all the losses right before a H is thrown. What is the distribution of g? Explain whether the expectation of g is finite or infinite. If finite, calculate it.

2. (10 pts.) Suppose you are playing the same game, but now with an unfair coin with probability of H being p. What will be the distributions and expectations of f and g? Explain whether for some values of p, expectation becomes infinite.

3. (10 pts.) Suppose you are playing the same game with a fair coin (as in Question 1), but this time you have only a finite pile of money and the game stops either when you lose all your money, or you throw a H and win. What will be the distributions and expectations of f and g? You may assume that there are 2^n-1 dollars in your pile of money, for some fixed value of n. Assuming you only have a finite amount of money, is this game worth playing?