

4. (20 pts.)

Let n_1, \dots, n_t be positive integers. Show that if $n_1 + n_2 + \dots + n_t - t + 1$ balls are placed into t bins, then for some $i \in \{1, 2, \dots, t\}$, the i -th bin contains at least n_i balls.

5. (20 pts.)

In the game of Set, you have a deck of cards. Each card has 1 or more objects drawn on it, which have the following properties:

- Each object has one of 3 shapes: diamond, oval or squiggle.
- Each object has one of 3 colors: purple, red or green.
- Each object has one of 3 shadings: solid, striped or empty.
- There can be 1, 2 or 3 objects drawn on each card (if there is more than 1 object on a card, all have the identical shape, color and shading).

To play the game, some of the cards are put face up on the table, and all players look for 3 cards that "match", where to match the three cards must

- all have the same shaped objects, or must have 3 different shapes
- all have the same color objects, or must have 3 different colors
- all have the same shaded objects, or must have 3 different shadings
- all have the same number of objects, or must have 3 different numbers

Whenever a player sees matching cards, they yell "match!", pick up the three matching cards, and replace them with 3 new cards. The game continues until no player can find any more matches, and the player who found the most matches wins.

1. If the deck of cards has exactly one of each kind of possible card, how many cards are there?
2. How many different subsets of 3 cards are there in the deck that match?

6. (20 pts.) Based on a 1st grade homework assignment from a local school.

1. Suppose you can have 9 pieces of fruit, which can be either apples or oranges. How many ways can you have 9 pieces of fruit (for example, you could have 4 apples and 5 oranges, or 0 apples and 9 oranges, etc.)?
2. Answer part 1 for n pieces of fruit.
3. Suppose you can have 9 pieces of fruit, which can be apples, oranges or pears. How many ways can you have 9 pieces of fruit (actual 1st grade assignment)?
4. Answer part 3 for n pieces of fruit.
5. Suppose you can have n pieces of fruit, and there are m different kinds of fruit. How many ways can you have n pieces of fruit?