Due Mar 18, 8:00am

You may work in groups of up to 3 people (no larger!). Please read the group collaboration policies on bSpace or `www.cs.berkeley.edu/~demmel/cs70_Spr11` before beginning group work. You *must* write up the solution set entirely on your own. You must never look at any other students' solutions (from any semester, not even a draft), nor share your own solutions (not even a draft).

Please begin your answer to each question on a new sheet of paper, and make sure that each sheet is labeled with your name, GSI name, the assignment number, the question number, and "CS70-Spring 2011".

Turn in each question in a different box in 283 Soda Hall: Question 1 in the box labeled "CS70 - 1", Question 2 in the box labeled "CS70 - 2", etc. Reason: Different problems will be graded in parallel by different readers.

Warning: You risk receiving no credit, or losing points, for any homework that does not conform to the above regulations! Please take the time to write clear and concise solutions; we will not grade messy or unreadable submissions. No late homeworks will be accepted. We will drop the lowest two homework scores.

1. (20 pts.)
Alice, Bob and Chuck are playing with a ball. If Alice has the ball, she throws it to Chuck. If Bob has the ball, he throws it to Alice or Chuck, with equal probabilities. If Chuck has the ball, he throws it to Alice or Bob, with equal probabilities.

At the beginning of the game the ball is given to one of Alice, Bob and Chuck, with equal probabilities. What is the probability that, after the ball is thrown once, Alice has it? That Bob has it? That Chuck has it?

2. (20 pts.)
From a shuffled deck of 52 cards one card is drawn. Let A be the event that a king is drawn, B the event that a club is drawn, and C the event that a black card (club or spade) is drawn. Which two of A, B and C are independent? What if a joker is added to the deck?

3. (20 pts.)
Let S be a sample space, and A and B two independent events in S. Let $\bar{A} = S - A$ and $\bar{B} = S - B$. Prove that $\bar{A}$ and $\bar{B}$ are also independent. Are A and $\bar{B}$ independent? Are A and $\bar{A}$ independent?

4. (20 pts.) Conditional Probability

1. I have a bag containing either a \$10 or \$20 bill (with probability $1/2$ for each of these two possibilities). I then add a \$10 bill to the bag, so it now contains two bills. The bag is shaken, and you randomly draw a bill from the bag (without looking). Suppose it turns out to be a \$10 bill. If a second student draws the remaining bill from the bag, what is the chance that it too is a \$10 bill? Show your calculations.

2. Your gambling buddy found a website online where he could buy trick coins that are heads or tails on both sides. He puts three coins into a bag: one coin that is heads on both sides, one coin that is tails on both sides, and one that is heads on one side and tails on the other side. You shake the bag, draw out a coin at random, put it on the table without looking at it, then look at the side that is showing. Suppose you notice that the side that is showing is heads. What is the probability that the other side is heads? Show your work.

5. (20 pts.) Discreet Math

A student organization wants to poll a sample of students to ask them whether they have ever cheated on their homework. This being an extremely sensitive subject, one obvious problem is that if the surveyers ask this question straight-out, respondents may lie to avoid embarrassing themselves.

The surveyers come up with the following clever scheme. They will ask the respondent to secretly roll a fair die. If the die comes up 1, 2, 3, or 4, the respondent is supposed to answer truthfully. If the die comes up 5 or 6, the respondent is supposed to answer the *opposite* of the truthful answer. The respondent is cautioned not to reveal what number came up on the die. Notice that if the respondent answers "Yes," this answer is not necessarily incriminating: for all the surveyer knows, this particular respondent might have rolled a 5 or 6 and might have never cheated in his/her life.

Let p be the probability that, if we select a student at random, then they will have cheated. (Of course, the surveyers do not know p; that is what they want to estimate.) Let q denote the probability that, if we select a student at random and have them follow the scheme above, then they will answer "Yes."

1. Calculate a simple formula for q, as a function of p.
2. Next, suppose the surveyers have estimated q. Now they want to solve for p. Find a simple formula for p, as a function of q.

6. (20 pts.) Bayes Rule

1. The probability that a randomly selected woman has breast cancer is 0.008. If she has breast cancer, the probability that a mammogram will show a positive result is 0.90. If a woman does not have breast cancer the probability of a positive result is 0.07. Take, for example, a woman who has a positive result. What is the probability that she actually has breast cancer? Show your work.
2. This question, which uses real data, was asked of real doctors. One-third of the doctors answered "0.90"; another one-third's estimates were in the range 0.50–0.80; one-sixth estimated something in the range 0.05–0.10; and another one-sixth estimated about 0.01. What fraction of the doctor's were in the right ballpark?