

UNIVERSITY OF CALIFORNIA  
Department of Electrical Engineering  
and Computer Sciences  
Computer Science Division

CS 282  
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Assignment 1

**Due:** Monday, 30 Jan 2006

(Since we're not require any texts, we encourage you to use the notes by Yap, F. Wright (both on-line), or do some Google searching if you get stuck, or ask questions (I have spare copies of some of the texts mentioned.))

Now to the business at hand in this assignment. There are numerous ways of representing standard kinds of "algebraic" objects. These problems are puzzles to make you think about alternative abstractions for polynomials. I'd like hard-copy of your responses, though if you need hints or clarifications send me e-mail ([fateman@cs](mailto:fateman@cs)).

1. What is wrong with this proof that  $1=-1$ ?

$$1 = \sqrt{1} = \sqrt{(-1) \cdot (-1)} = (\sqrt{-1}) \cdot (\sqrt{-1}) = i \cdot i = -1$$

2. Find the first 4 coefficients in the  $p$ -adic expansion (if it exists), for  $p = 5$  and  $p = 7$  for  $2/7$ ,  $\sqrt{2}$ ,  $\sqrt{6}$ . Show your hand computations (or your computer program).
3. What's the (repeating) decimal expansion for  $3/17$ ? (Use a computer!) Prove that every rational number can be represented by a binary floating-point number with a repeating tail. Prove that every binary floating-point number with a repeating tail is a rational number. In decimal,  $1.0=0.999\dots$ . By analogy, some rational numbers have two different binary expansions. Characterize them.
4. A certain polynomial  $q(x)$  with integer coefficients each in magnitude less than 100 has the values  $q(13) = 11338558$ ,  $q(19) = 78818476$ , and (although you shouldn't need it)  $q(23) = 208194088$ .  
What is  $q(-0.01)$ ? Explain your reasoning. You may use a computer for calculation if you wish. You may ask for hints.

5. Odd as it may seem, it is generally accepted among certain mathematicians, that  $\cos(n \arccos(x))$  is a polynomial in  $x$  of degree  $n$ . What polynomial do you get (for  $n$  0, 1, 2, 3)? What can you say about non-integer or negative  $n$ ? Do you believe that this expression is a polynomial for all  $x$ ?

6. If you have the official prerequisite programming courses or or at least modest experience in some non-numeric programming, it is likely that you have encountered the representation of an algebraic expression using a tree-like data structure. We can print out a such a tree as a parenthesized prefix form, for human consumption. For example,  $x^3 + (x + 2)^2$  might be printed out, in Lisp, as

```
(plus (expt x 3)(expt (plus x 2) 2)).
```

Assume you have two such expressions built up from the variable  $x$ , the integers (positive and negative) the operators `plus`, `times`, `expt` (only positive integer-valued powers allowed).

*Sketch* algorithms for the following (in order from easiest to hardest):

- A program returning the product of the two expressions.
- A program returning the highest degree in  $x$  that occurs with a non-zero coefficient in one of the expressions.
- A predicate returning true if the two expressions are equivalent (otherwise false). You may wish to clarify what you mean by equivalent.
- A program returning two expressions, the quotient and remainder of one expression divided by the other. (Note: you have to define what is meant by a quotient and a remainder.)

Consider: How much complication is added by allowing exponents to be *negative or positive* integers?