Analysis of Integral Queries for TILU (Table of Integrals Look Up)

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Abstract

What can we learn from the range of over 7000 queries made to TILU, a symbolic integration problem-solving server, during the course of more than two years? We have saved all queries during this experiment, and based on our analysis, suggest improvements in the computer-human interaction components of our web server.

1 Introduction

TILU[1], a table lookup program for integrals [1] run at the University of California at Berkeley, received 7363 queries from the date from December 1995 through January 26, 1998. 6995 of these queries were successful from the web-interaction perspective, meaning TILU successfully returned some form of output, whether it was correct or incorrect as an integral. The base program doing the integral lookup (`sherlock`) has undergone some change during this period, and the web-server code has also been revised from time to time. The main issue we want to address here is how we can improve the user interaction, based on our evidence of the nature of user input errors and our perception of user expectations, some of which we hope we can easily correct.

When someone visits our web site and clicks on "activate integral" a block of data is sent to a port on our host machine which keeps track of all the information provided. This includes the integral being performed as well as a collection of other information: the hostname of the machine where the query originates, information about the browser (e.g. is it Netscape

[1] `http://http.cs.berkeley.edu/~fateman/htest.html`

version 3.01, Internet Explorer 4.01, or Lynx etc.) as well as available plug-ins. The hostname provided by the site where the query originated is not always adequate for our purposes; over half of the queries did not send their hostname, and since we did not keep track of their IP addresses, we cannot easily identify them. The remaining data, which we analyzed, consisted of queries from sites that did send their hostname. We may later analyze the other entries that did not send their hostname. (These are from hostnames specified as NIL in our record of interactions.)

2 Data

Over the time period studied, there were 3500 successful queries that sent their hostnames with the queries. Of these 3500 queries, 349 were from the Berkeley.EDU domain. These Berkeley queries were likely associated with our own tests and were thus ignored during analysis of the data. This brought the number of queries to analyze down to 3151, of which we reviewed 1014. Our most popular sites are listed in Table 1.

Table 1: Breakdown of TILU queries by organization type or locale.

Domain	Queries	Most popular sites with number of queries
edu (US schools)	1008	MIT (62), Stanford (61), Purdue (53), UCSB (43)
com (US companies)	653	Compuserve (49)
net (US network providers)	260	UUNet (53), AT&T WorldNet (32)
gov (US gov)	81	
.fr (France)	202	
.de (Germany)	189	
.se (Sweden)	118	
.fi (Finland)	77	
.ca (Canada)	71	
.uk (United Kingdom)	60	
.ch (Switzerland)	53	
.nl (Netherlands)	51	
.au (Australia)	43	

We divided the queries we analyzed into two parts:

1. Domains that had fewer than 5 queries total.
2. Domains that had 5 or more queries total.

We broke the queries up into these two groups under the suspicion that domains that made fewer than 5 queries would be more prone to gramatically incorrect queries. It turns out that this distinction between domains does make a difference, but not as large a difference as we first expected. Many of the domains that had more than 5 queries came from multiple sites under the same domain. To get more accurate results, we would need to break the domains into specific sites that made 5 or more queries and sites that made fewer than 5 queries.

Because of the huge number of queries, we did not analyze all of the results, but instead guessed some of the most relevant domains to analyze. Most of the queries to TILU came from domains that had 5 or more queries, whereas only a few came from domains that had fewer than 5 queries. Since there were so few domains that had fewer than 5 queries, we were able to analyze most of those queries. There were a large number of queries that came from domains with 5 or more queries, so we only analyzed only a portion of them. We did not analyze 2137 of the queries that sent a hostname with the query. Most of these unanalyzed queries came from domains that performed 5 or more queries. See our data in Table 2.

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Some percentages based on data in Table 2:

For those domains with 5 or more queries we have:

GIGO  261/689 = 37.9%
Good  240/689 = 34.8%
Well-formed 428/689 = 62.1%


For those domains with fewer than 5 queries we have:

GIGO 146/325 = 44.9%
Good 109/325 = 33.5%
Well-formed 179/325 = 55.1%


Totals for both:
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GIGO 407/1014 = 40.1%
Good 349/1014 = 34.4%
Well-formed 607/1014 = 59.9%
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As shown above, people submit well-formed integral queries about three fifths of the time, but only get a correct response from TILU about one third of the time. This means TILU returns an incorrect response to good queries about one fourth of the time. People perform badly-formed queries about two fifths of the time.

3 Analysis

Some interesting things we noticed were:

- people often integrate `(sin x)` and leave.
- people often integrate the same successful (or unsuccessful) integral over and over.
- people often try many different things with a badly formed integral, never correcting the bad form.
- people from domains that have 5 or more queries to TILU have a higher likelihood of having well-formed data, and thus have a higher likelihood of getting a correct response.

Among the "good ideas" that we incorporated along the way were:

- removing the option that allowed the user to state the variable of integration. Sometimes this was erroneously set or even erased. Instead, we tell the user that we will integrate with respect to x.
- insisting that a choice for input language be provided, making the user aware of the requirement that some well-defined language is needed, and "math-ese" is not it!
- keeping a panel of links on the left side of the web page for those users with browsers that support frames. These links give users easy access to pertinent information, such as why TILU's output was not what they were looking for, without requiring them to read through a lot of text or pressing the "back" button to get to the information.

4 Work for the Future

Some comments about other problems, and also some suggestions as to what should be done in the future:

- One way to solve the problem of users attempting "trivial" definite integrals is by adding a huge number of definite integrals to the table. We could also, somehow, make it more obvious that users should read the document about why definite integrals generally will give an "integral not found" response. We could easily add to the capabilities of TILU and look up the indefinite integrals for such problems, in case they are well defined. Unfortunately, we do not include a mechanism in TILU for assuring correctness. (For example, assuring the integral is continuous and differentiable in the given range, and thus the fundamental theorem of calculus would be valid.)

- The problem of people editing the variable of integration should have been solved by eliminating the option of changing it. We now assume that you are integrating with respect to x.

- If someone enters an empty string, we should possibly return an answer saying, "You must enter something to integrate."

- The numerous failures of TILU to find the response to a query could be solved by adding additional integrals to the table. We could also have TILU look up the failed integral request at a server such as the one at http://www.integrals.com, and return the results from that server.

- To prevent users from writing in "math-ese," we could possibly place a warning in large, bold text near the integrand box, stating that users MUST write their answers in Mathematica or LISP or their answers will be bizarre.

- The Mathematica parser should be adjusted to ignore a carriage return at the beginning of the line so it doesn't give incorrect responses.

- Users should be able to figure out for themselves that they can only integrate one thing at a time because TILU currently outputs the result of the first integral of the many that the users input. If necessary, we could use a text box, instead of a text area, in which to type the integrand. This would also solve the problem of hitting a carriage return before typing the integral.

- In general, "Garbage In" could be reduced by improving the parser to catch errors and respond with a message that lets the user know TILU could not parse the query, instead of responding with a vague "integral not found." This improvement could eliminate many "wrong language," "bizarre input," and "mathese" errors and encourage users to try again with improved input.

- One exciting, and entirely feasible possibility of future work is to extend TILU to include most information in Gradshteyn and Ryzhik's Table of Integrals, Series, and Products[2]. Gradshteyn and Ryzhik have an online version of their book which contains nearly 20,000 integrals, series, and products in TeX format. To accomplish this, we would need to create a parser that takes the TeX integrals and converts them to our LISP format. We will not be able to convert everything because many of the items in Gradshteyn and Ryzhik's book contain mathematical constructs which are difficult to translate into a computer, but we should be able to convert a large number of the integrals. Hopefully we could then integrate TILU with Aztec Corporation's proposed integral look up interface[2].

- When returning the results of the query, we could retrieve all matching integrals in the hash table, instead of looking for the most precise one. This way, people will be able to pick the best match for their purposes, instead of leaving it up to TILU to decide on the closest one.

References

[1] Richard J. Fateman, "Network Servers for Symbolic Mathematics," *Proc. of Int'l Symp. on Symbolic and Algebraic Computation*, ISSAC-97 (ACM Press) Maui, Hawaii, July, 1997, 249—256.

[2] I.S. Gradshteyn and I.M. Ryzhik, *Table of Integrals, Series, and Products*, 5th Ed., A. Jeffrey, ed. Academic Press: San Diego, 1997.

[2] `http://www.az-tec.com/epub/gr/sample.htm`

Table 2: Breakdown, by number of queries per domain, of TILU responses to user queries.

	5 or more	Fewer than 5
Good. (*)	240	109
TILU couldn't find integral (*)	162	56
GIGO - Bizarre input.	106	45
GIGO - Used the wrong language (LISP instead of Mathematica or vice-versa).	78	28
GIGO - Used "math-ese."	61	52
User attempted a definite integral. (*)	16	10
GIGO - User put integrand in variable of integration box instead of in integrand box.	9	6
User hit return before Mathematica integrand. (*)	9	4
GIGO - User left integrand blank.	7	14
"INTEGRAL_REQUEST_DENIED" error. (*)	1	0
GIGO - User attempted to integrate multiple items.	0	1
Totals	689	325

Notes:
"*" means that the request was well-formed, but TILU gave something other than the correct answer to the integral.
"Good" means correct input and correct output.
"GIGO" means "garbage in, garbage out."
"math-ese" refers to writing the integrand as if converting directly from an integrand written on paper in a math class.
"TILU couldn't find integral" errors are specified as "FAILURE_OF," "INTEGRAL-NOT-FOUND," or "CANT-FIND-INTEGRAL" in our log file queries.