

CS61B Lecture #19

Administrative:

- Please arrange alternative test times with us by e-mail.
- Labs this week devoted to test review. We suspect that Wednesday labs will get commandeered by project questions, so you might want to attend a Thursday lab as well if you want test review.
- A few review questions from the TAs are posted in the Lab #6 directory for this week (see the homework page).
- If you want timely response, use bug-submit.

Today:

- Trees

Readings for Today: *Data Structures, Chapter 5*

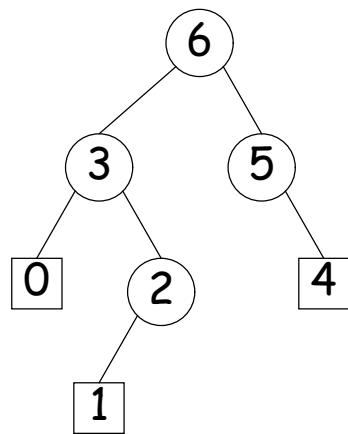
Readings for Next Topic: *Data Structures, Chapter 6*

A Recursive Structure

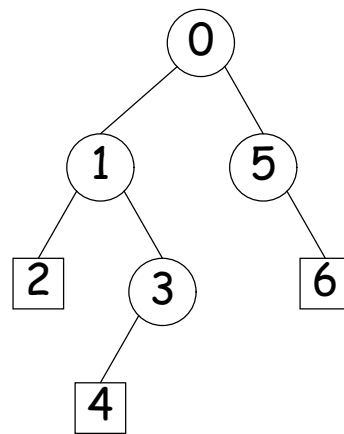
- Trees naturally represent recursively defined, hierarchical objects with more than one recursive subpart for each instance.
- Common examples: expressions, sentences.
 - Expressions have definitions such as "an expression consists of a literal or two expressions separated by an operator."
- Also describe structures in which we recursively divide a set into multiple subsets.

Fundamental Operation: Traversal

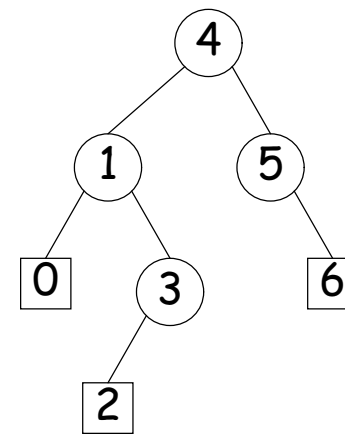
- *Traversing a tree* means enumerating (some subset of) its nodes.
- Typically done recursively, because that is natural description.
- As nodes are enumerated, we say they are *visited*.
- Three basic orders for enumeration (+ variations):
 - **Preorder**: visit node, traverse its children.
 - **Postorder**: traverse children, visit node.
 - **Inorder**: traverse first child, visit node, traverse second child (binary trees only).



Postorder



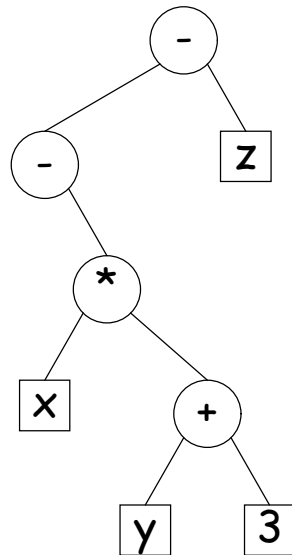
Preorder



inorder

Preorder Traversal and Prefix Expressions

Problem: Convert

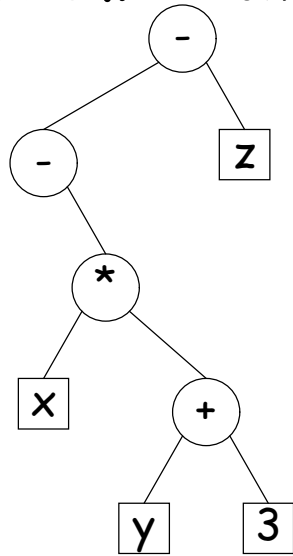


$\Rightarrow (- (- (* x (+ y 3))) z)$

```
static String toLisp (Tree<String> T) {
    if (T == null)
        return "";
    else if (T.degree () == 0)
        return T.label ();
    else {
        String R; R = "";
        for (int i = 0; i < T.numChildren (); i += 1)
            R += " " + toLisp (T.child (i));
        return String.format ("%s%s", T.label (), R);
    }
}
```

Inorder Traversal and Infix Expressions

Problem: Convert



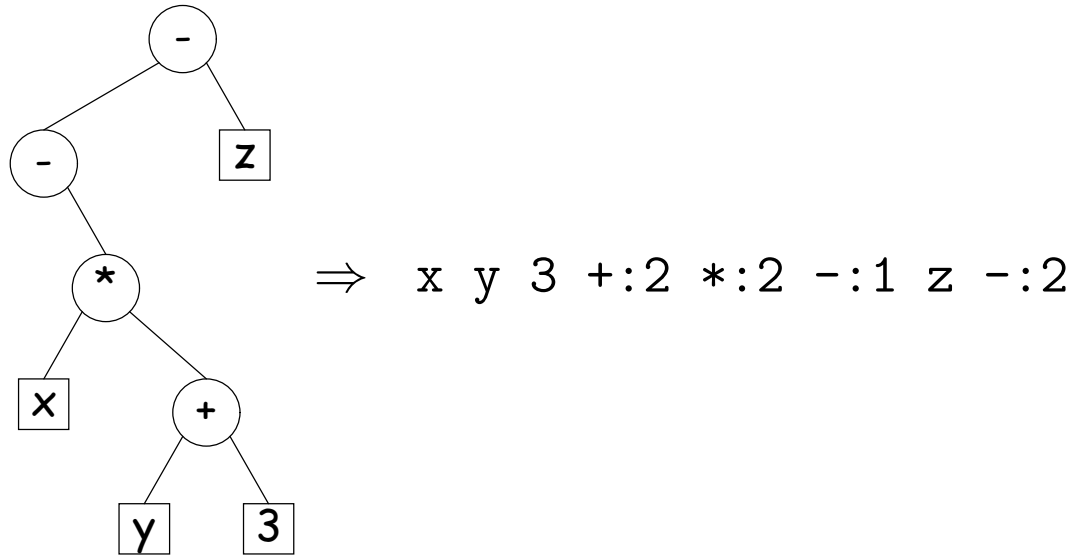
$\Rightarrow ((-(x*(y+3)))-z)$

To think about: how to get rid of all those parentheses.

```
static String toInfix (Tree<String> T) {
    if (T == null)
        return "";
    if (T.degree () == 0)
        return T.label ();
    else {
        return String.format ("%s%s%s",
                               toInfix (T.left ()), T.label (), toInfix (T.right ()))
    }
}
```

Postorder Traversal and Postfix Expressions

Problem: Convert



```
static String toPolish (Tree<String> T) {  
    if (T == null)  
        return "";  
    else {  
        String R; R = "";  
        for (int i = 0; i < T.numChildren (); i += 1)  
            R += toPolish (T.child (i)) + " ";  
        return String.format ("%s%s:%d", R, T.label (), T.degree ());  
    }  
}
```

A General Traversal: The Visitor Pattern

```
void preorderTraverse (Tree T<Label>, Action<Label> whatToDo)
{
    if (T != null) {
        whatToDo.action (T);
        for (int i = 0; i < T.numChildren (); i += 1)
            preorderTraverse (T.child (i), whatToDo);
    }
}
```

- What is Action?

```
interface Action<Label> {
    void action (Tree<Label> T);
}
```

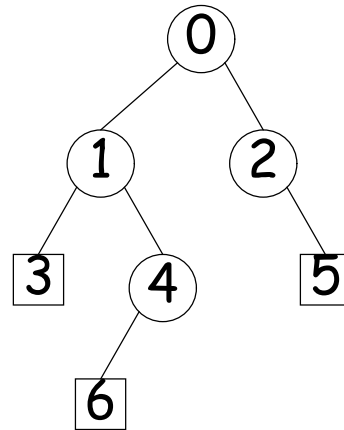
```
class Print implements Action<String> | preorderTraverse (myTree,
    void action (Tree<String> T) { | new Print ());
    System.out.print (T.label ()); |
    } |
}
```

Times

- The traversal algorithms have roughly the form of the boom example in §1.3.3 of *Data Structures*—an exponential algorithm.
- However, the role of M in that algorithm is played by the *height* of the tree, not the number of nodes.
- In fact, easy to see that tree traversal is *linear*: $\Theta(N)$, where N is the # of nodes: Form of the algorithm implies that there is one visit at the root, and then one visit for every *edge* in the tree. Since every node but the root has exactly one parent, and the root has none, must be $N - 1$ edges in any non-empty tree.
- In positional tree, is also one recursive call for each empty tree, but # of empty trees can be no greater than kN , where k is arity.
- For k -ary tree (max # children is k), $h + 1 \leq N \leq \frac{k^{h+1}-1}{k-1}$, where h is height.
- So $h \in \Omega(\log_k N) = \Omega(\lg N)$ and $h \in O(N)$.
- Many tree algorithms look at one child only. For them, time is proportional to the *height* of the tree, and this is $\Theta(\lg N)$, assuming that tree is *bushy*—each level has about as many nodes as possible.

Level-Order (Breadth-First) Traversal

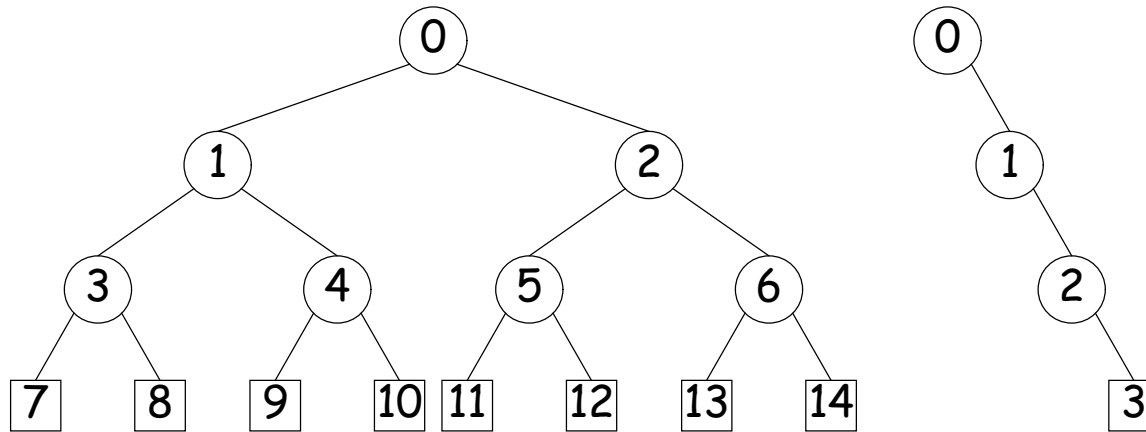
Problem: Traverse all nodes at depth 0, then depth 1, etc:



- One technique: *Iterative Deepening*. For each level, k , from 0 to h , call `doLevel(T,k)`

```
void doLevel (Tree T, int lev) {  
    if (lev == 0)  
        visit T  
    else  
        for each non-null child, C, of T {  
            doLevel (C, lev-1);  
        }  
}
```

Iterative Deepening Time?



- Let h be height, N be # of nodes.
- Count # edges traversed (i.e, # of calls, not counting null nodes).
- First (full) tree: 1 for level 0, 3 for level 1, 7 for level 2, 15 for level 3.
- Or in general $(2^1 - 1) + (2^2 - 1) + \dots + (2^{h+1} - 1) = 2^{h+2} - h \in \Theta(N)$, since $N = 2^{h+1} - 1$ for this tree.
- Second (*right leaning*) tree: 1 for level 0, 2 for level 2, 3 for level 3.
- Or in general $(h + 1)(h + 2)/2 = N(N + 1)/2 \in \Theta(N^2)$, since $N = h + 1$ for this kind of tree.

Iterative Traversals

- Tree recursion conceals data: a *stack* of nodes (all the T arguments) and a little extra information. Can make the data explicit, e.g.:

```
void preorderTraverse2 (Tree T<T>, Action whatToDo) {
    Stack s = new Stack ();
    s.push (T);
    while (! s.isEmpty ()) {
        Tree node = (Tree) s.pop ();
        if (node == null)
            continue;
        whatToDo.action (node);
        for (int i = node.numChildren ()-1; i >= 0; i -= 1)
            s.push (node.child (i));
    }
}
```

- To do a breadth-first traversal, use a queue instead of a stack, replace push with add, and pop with removeFirst.
- Makes breadth-first traversal worst-case linear time in all cases, but also linear *space* for "bushy" trees.

Iterators for Trees

- Frankly, iterators are not terribly convenient on trees.
- But can use ideas from iterative methods.

```
class PreorderTreeIterator<T> implements Iterator<T> {
    private Stack<Tree<T>> s = new Stack<Tree<T>> ();

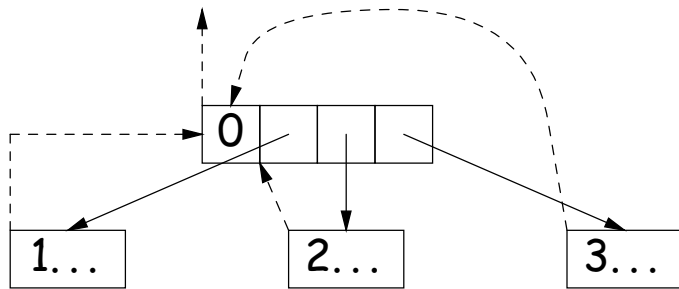
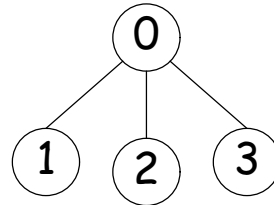
    public PreorderTreeIterator (Tree<T> T) { s.push (T); }

    public boolean hasNext () { return ! s.isEmpty (); }
    public T next () {
        Tree<T> result = s.pop ();
        for (int i = result.numChildren ()-1; i >= 0; i -= 1)
            s.push (result.child (i));
        return result.label ();
    }
    void remove () { throw new UnsupportedOperationException (); }
}
```

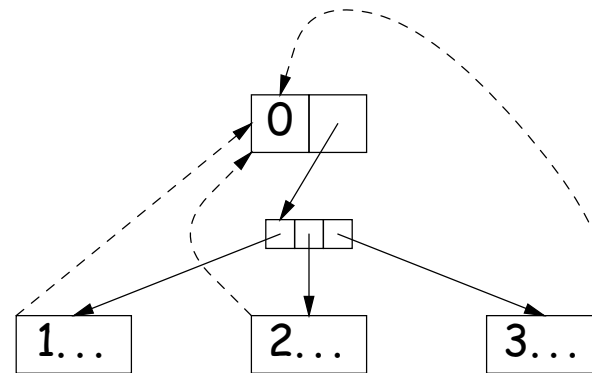
Example: (what do I have to add to class Tree first?)

```
for (String label : aTree) System.out.print (label + " ");
```

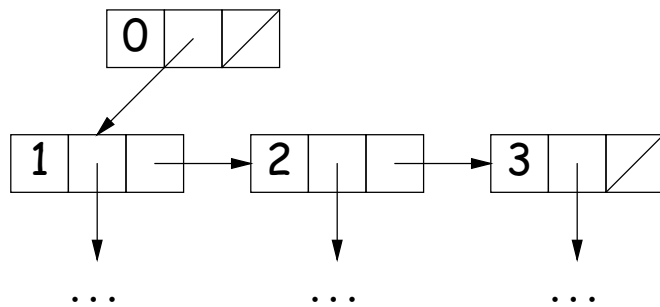
Representation Choices



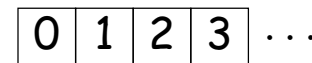
(a) Embedded child pointers
(+ optional parent pointers)



(b) Array of child pointers
(+ optional parent pointers)



(c) child/sibling pointers



(d) pre-order array
(complete trees)