Rigging Tournament Brackets for Weaker Players

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Balanced Single-Elimination Tournaments

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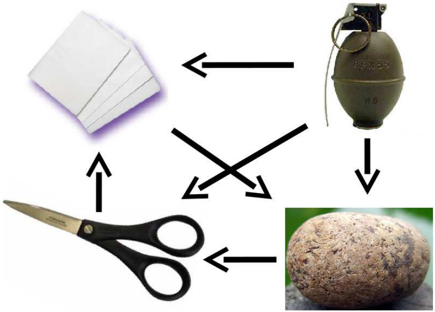
Tournament Fixing

If we had a tournament between *n* players, and knew all of the match outcomes in advance, could we pick a bracket for a single-elimination tournament so that our favourite player wins?



Why Study This Problem?

- SE tournaments are an incredibly popular form of competition
- From an election stand point, SE tournaments are equivalent to binary cup voting
- We inherently think that tournaments are 'fair'. It is important to confirm or deny this.

Tournament Fixing

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- Finding the most 'interesting' tournament when deterministic matches have an 'interestingness' rating is NP-hard
- Guaranteeing that certain players appear in prespecified rounds is NP-hard

We will focus on the deterministic balanced case and finding conditions that guarantee that we can manipulate easily.

Graph Theory Connection

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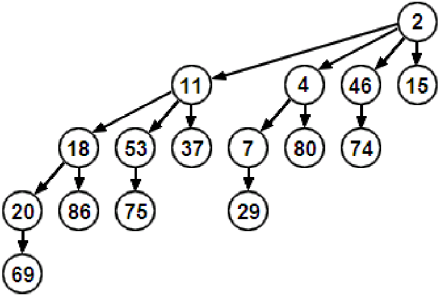
In deterministic tournament graphs, the problem of fixing a tournament is equivalent to finding a rooted binomial arboresence



This, in turn, is equivalent to a series of directed matchings.

The Strength of Players

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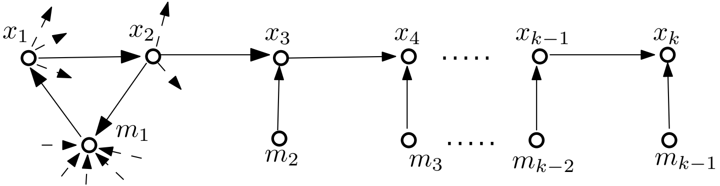
If we look at the ranking by number of wins, what can we say about lower ranked players?

Lemma

The second highest ranked player can win an SE tournament iff the highest ranked player is not a Condorcet winner.

Matchings

The third ranked player may be unable to win if there does not exist a matching onto the first and second ranked players.



We can extend this example to show that without a perfect matching onto the k higher ranked players, the $k+1$ ranked player may not be able to win.

Our Result

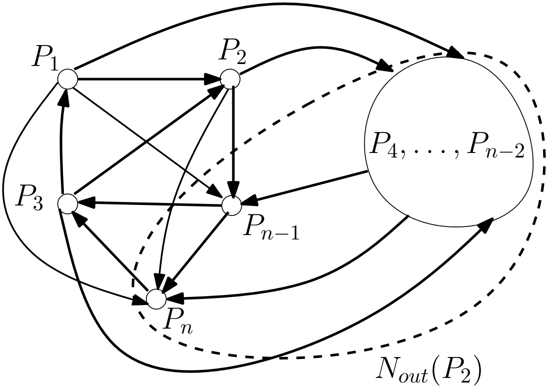
If there exists a perfect matching of size at most $\frac{n-6}{7}$ from the nodes ranked lower than $\mathcal{A}$ onto those ranked higher who $\mathcal{A}$ doesn't beat then $\mathcal{A}$ can be made to win an SE tournament

If $\mathcal{A}$ is ranked in the top $\frac{6n+7}{31} \approx 19\%$ of players and the matching exists, then $\mathcal{A}$ can always win an SE tournament!

For this constant fraction of players, you can greedily construct a bracket for each player - agenda control is easy!

How the Previous Approach Fails

If we assume that there is a matching from the lower ranked players onto the higher ranked players, the naive approach may fail.



Intuition

The construction is quite detailed, but the general steps are designed to reduce the situation over a series of rounds to a tournament where $\mathcal{A}$ is a king who beats at least half the players.

1. Use the matching given by the theorem statement
2. Find a maximal matching from the remaining players that $\mathcal{A}$ beats to those that beat $\mathcal{A}$
3. Use a greedy matching algorithm to make sure that the sources of the matching in step 1 that beat $\mathcal{A}$ are still covered
4. Match the remaining nodes arbitrarily amongst themselves.

The technical difficulty lies in applying Step 3.

Do Matchings Actually Exist?

Theorem (Erdős-Renyi)

Let c_n be any function of n, then consider $G = B(n, p)$ for $p = (\ln n + c_n)/n$. The probability that G contains a perfect matching is at least $1 - 2/e^{c_n}$.

In randomized tournament generating models, this means that with relatively small probabilities of upset – $O(\frac{\log n}{n})$ – the exact matching that we require exists with high probability.

See our WSCAI paper on tournaments in the Braverman-Mossel model!

Summary

- We have shown that under a fairly benign assumption, the top 19% of players may be able to win some single-elimination tournament.
- The results sidesteps the question of the computational complexity of fixing a tournament by showing that in many natural occurrences, fixing a tournament for important players is easy.
- One can view this as a ranking mechanism result, in the sense that the results from using a single-elimination tournament for ranking be very bad.

Come talk to us at the poster session on Thursday 4-5:20pm!

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