Angelic Hierarchical Planning: Optimal and Online Algorithms

Bhaskara Marthi

MIT/Willow Garage

bhaskara@csail.mit.edu

Stuart Russell

UC Berkeley russell@cs.berkeley.edu Jason Wolfe UC Berkeley jawolfe@cs.berkeley.edu

High-Level Actions (HLAs)

- Here, a high-level action (HLA) = a set of allowed immediate refinements:
 - each is a sequence of actions
 - may have associated preconditions
- Almost all actions we think about are high-level
 - Plan a trip
 - Vacuum the house
 - Go to work



- k-step lookahead >> 1-step lookahead
 - e.g., chess



- k-step lookahead >> 1-step lookahead
 - e.g., chess
- k-step lookahead no use if steps too small
 - e.g., first k turns in TSP of Australia



- k-step lookahead >> 1-step lookahead
 - e.g., chess
- *k*-step lookahead no use if steps too small
 - e.g., first k turns in TSP of Australia



- k-step lookahead >> 1-step lookahead
 - e.g., chess
- k-step lookahead no use if steps too small
 - e.g., first *k* turns in TSP of Australia
 - this is one small part of a human life,
 ≈ 20,000,000,000,000 primitive actions



- k-step lookahead >> 1-step lookahead
 - e.g., chess
- k-step lookahead no use if steps too small
 - e.g., first *k* turns in TSP of Australia
 - this is one small part of a human life,
 ≈ 20,000,000,000,000 primitive actions
- Abstract plans with HLAs are shorter



- k-step lookahead >> 1-step lookahead
 - e.g., chess
- k-step lookahead no use if steps too small
 - e.g., first k turns in TSP of Australia
 - this is one small part of a human life,
 ≈ 20,000,000,000,000 primitive actions
- Abstract plans with HLAs are shorter
 - Much shorter plans => exponential savings





is provably optimal

- k-step lookahead >> 1-step lookahead
 - e.g., chess
- k-step lookahead no use if steps too small
 - e.g., first k turns in TSP of Australia
 - this is one small part of a human life, $\approx 20,000,000,000,000$ primitive actions
- Abstract plans with HLAs are shorter
 - Much shorter plans => exponential savings
 - Can look ahead much further





looks like a good start

- k-step lookahead >> 1-step lookahead
 - e.g., chess
- k-step lookahead no use if steps too small
 - e.g., first k turns in TSP of Australia
 - this is one small part of a human life,
 ≈ 20,000,000,000,000 primitive actions
- Abstract plans with HLAs are shorter
 - Much shorter plans => exponential savings
 - Can look ahead much further
- Requires models for HLAs
 - i.e., transition and cost fns





looks like a good start

- k-step lookahead >> 1-step lookahead
 - e.g., chess
- k-step lookahead no use if steps too small
 - e.g., first k turns in TSP of Australia
 - this is one small part of a human life,
 ≈ 20,000,000,000,000 primitive actions
- Abstract plans with HLAs are shorter
 - Much shorter plans => exponential savings
 - Can look ahead much further
- Requires models for HLAs
 - i.e., transition and cost fns
 - No suitable models in literature





looks like a good start

- k-step lookahead >> 1-step lookahead
 - e.g., chess
- k-step lookahead no use if steps too small
 - e.g., first k turns in TSP of Australia
 - this is one small part of a human life,
 ≈ 20,000,000,000,000 primitive actions
- Abstract plans with HLAs are shorter
 - Much shorter plans => exponential savings
 - · Can look ahead much further
- Requires models for HLAs
 - i.e., transition and cost fns
 - No suitable models in literature
 - We extend our angelic semantics





looks like a good start

Models HLAs in deterministic domains



- Models HLAs in deterministic domains
- Central idea is reachable set of an HLA from some state



- Models HLAs in deterministic domains
- Central idea is reachable set of an HLA from some state
 - When extended to sequences of actions, ...



- Models HLAs in deterministic domains
- Central idea is reachable set of an HLA from some state
 - When extended to sequences of actions, ...



- Models HLAs in deterministic domains
- Central idea is reachable set of an HLA from some state
 - When extended to sequences of actions, ...



- Models HLAs in deterministic domains
- Central idea is reachable set of an HLA from some state
 - When extended to sequences of actions, ...



- Models HLAs in deterministic domains
- Central idea is reachable set of an HLA from some state
 - When extended to sequences of actions, ...



- Models HLAs in deterministic domains
- Central idea is reachable set of an HLA from some state
 - When extended to sequences of actions, ...
 - ... allows proving that a plan can or cannot possibly reach the goal



- Models HLAs in deterministic domains
- Central idea is reachable set of an HLA from some state
 - When extended to sequences of actions, ...
 - ... allows proving that a plan can or cannot possibly reach the goal



- Models HLAs in deterministic domains
- Central idea is reachable set of an HLA from some state
 - When extended to sequences of actions, ...
 - ... allows proving that a plan can or cannot possibly reach the goal



- Models HLAs in deterministic domains
- Central idea is reachable set of an HLA from some state
 - When extended to sequences of actions, ...
 - ... allows proving that a plan can or cannot possibly reach the goal
- May seem related to nondeterminism ...



- Models HLAs in deterministic domains
- Central idea is reachable set of an HLA from some state
 - When extended to sequences of actions, ...
 - ... allows proving that a plan can or cannot possibly reach the goal
- May seem related to nondeterminism ...
 - but uncertainty is angelic: resolved by the agent, not an adversary



- Models HLAs in deterministic domains
- Central idea is reachable set of an HLA from some state
 - When extended to sequences of actions, ...
 - ... allows proving that a plan can or cannot possibly reach the goal
- May seem related to nondeterminism ...
 - but uncertainty is angelic: resolved by the agent, not an adversary



Angelic Semantics cont.

- Approximate descriptions provide lower & upper bounds on reachable sets
 - Descriptions are true: follow logically from hierarchy

Angelic Semantics cont.

- Approximate descriptions provide lower & upper bounds on reachable sets
 - Descriptions are true: follow logically from hierarchy
- Sound & complete planning algorithm uses descriptions to
 - Commit to provably successful abstract plans: Downward Refinement Property (DRP) automatically satisfied
 - potentially exponential speedup
 - Prune provably unsuccessful abstract plans (USP satisfied)

Contributions

- Extend angelic semantics with action costs
- Developed novel algorithms that do lookahead with HLAs
 - Angelic Hierarchical A* (AHA*)



• Angelic Hierarchical Learning Real-Time A* (AHLRTA*)

- Both require three inputs:
 - planning problem
 - action hierarchy (set of HLAs)
 - approximate models for HLAs



• Here, a planning problem =

- Here, a planning problem =
 - State space S



- Here, a planning problem =
 - State space S
 - Initial state so, terminal set G



- Here, a planning problem =
 - State space S
 - Initial state so, terminal set G
 - Primitive action set



- Here, a planning problem =
 - State space S
 - Initial state so, terminal set G
 - Primitive action set
 - Transition function: $S \times A \rightarrow S$
 - Cost function $: S \times A \rightarrow \mathbb{R} \cup \{\infty\}$



Transitions & costs for action a₁

Running Example: Warehouse World Domain



- Elaborated *Blocks World* with discrete spatial constraints
 - Gripper must stay in bounds
 - Can't pass through blocks
 - Can only turn at top row

Running Example: Warehouse World Domain



- Elaborated *Blocks World* with discrete spatial constraints
 - Gripper must stay in bounds
 - Can't pass through blocks
 - Can only turn at top row
- All actions have cost 1

Running Example: Warehouse World Domain



L, D, GetR, U, Turn, D, PutL, R, R, D, GetL, L, PutL, U, L, GetL, U, Turn, R, D, D, PutR

- Elaborated *Blocks World* with discrete spatial constraints
 - Gripper must stay in bounds
 - Can't pass through blocks
 - Can only turn at top row
- All actions have cost 1
- Goal: have C on T4
 - Can't just move directly
 - Final plan has 22 steps


L, D, GetR, U, Turn, D, PutL, R, R, D, GetL, L, PutL, U, L, GetL, U, Turn, R, D, D, PutR

- Elaborated *Blocks World* with discrete spatial constraints
 - Gripper must stay in bounds
 - Can't pass through blocks
 - Can only turn at top row
- All actions have cost 1
- Goal: have C on T4
 - Can't just move directly
 - Final plan has 22 steps



L, D, GetR, U, Turn, D, PutL, R, R, D, GetL, L, PutL, U, L, GetL, U, Turn, R, D, D, PutR

- Elaborated *Blocks World* with discrete spatial constraints
 - Gripper must stay in bounds
 - Can't pass through blocks
 - Can only turn at top row
- All actions have cost 1
- Goal: have C on T4
 - Can't just move directly
 - Final plan has 22 steps



L, D, GetR, U, Turn, D, PutL, R, R, D, GetL, L, PutL, U, L, GetL, U, Turn, R, D, D, PutR

- Elaborated *Blocks World* with discrete spatial constraints
 - Gripper must stay in bounds
 - Can't pass through blocks
 - Can only turn at top row
- All actions have cost 1
- Goal: have C on T4
 - Can't just move directly
 - Final plan has 22 steps



L, D, GetR, U, Turn, D, PutL, R, R, D, GetL, L, PutL, U, L, GetL, U, Turn, R, D, D, PutR

- Elaborated *Blocks World* with discrete spatial constraints
 - Gripper must stay in bounds
 - Can't pass through blocks
 - Can only turn at top row
- All actions have cost 1
- Goal: have C on T4
 - Can't just move directly
 - Final plan has 22 steps



L, D, GetR, U, Turn, D, PutL, R, R, D, GetL, L, PutL, U, L, GetL, U, Turn, R, D, D, PutR

- Elaborated *Blocks World* with discrete spatial constraints
 - Gripper must stay in bounds
 - Can't pass through blocks
 - Can only turn at top row
- All actions have cost 1
- Goal: have C on T4
 - Can't just move directly
 - Final plan has 22 steps



L, D, GetR, U, Turn, D, PutL, R, R, D, GetL, L, PutL, U, L, GetL, U, Turn, R, D, D, PutR

- Elaborated *Blocks World* with discrete spatial constraints
 - Gripper must stay in bounds
 - Can't pass through blocks
 - Can only turn at top row
- All actions have cost 1
- Goal: have C on T4
 - Can't just move directly
 - Final plan has 22 steps



L, D, GetR, U, Turn, D, PutL, R, R, D, GetL, L, PutL, U, L, GetL, U, Turn, R, D, D, PutR

- Elaborated *Blocks World* with discrete spatial constraints
 - Gripper must stay in bounds
 - Can't pass through blocks
 - Can only turn at top row
- All actions have cost 1
- Goal: have C on T4
 - Can't just move directly
 - Final plan has 22 steps



L, D, GetR, U, Turn, D, PutL, R, R, D, GetL, L, PutL, U, L, GetL, U, Turn, R, D, D, PutR

- Elaborated *Blocks World* with discrete spatial constraints
 - Gripper must stay in bounds
 - Can't pass through blocks
 - Can only turn at top row
- All actions have cost 1
- Goal: have C on T4
 - Can't just move directly
 - Final plan has 22 steps



L, D, GetR, U, Turn, D, PutL, R, R, D, GetL, L, PutL, U, L, GetL, U, Turn, R, D, D, PutR

- Elaborated *Blocks World* with discrete spatial constraints
 - Gripper must stay in bounds
 - Can't pass through blocks
 - Can only turn at top row
- All actions have cost 1
- Goal: have C on T4
 - Can't just move directly
 - Final plan has 22 steps



L, D, GetR, U, Turn, D, PutL, R, R, D, GetL, L, PutL, U, L, GetL, U, Turn, R, D, D, PutR

- Elaborated *Blocks World* with discrete spatial constraints
 - Gripper must stay in bounds
 - Can't pass through blocks
 - Can only turn at top row
- All actions have cost 1
- Goal: have C on T4
 - Can't just move directly
 - Final plan has 22 steps



L, D, GetR, U, Turn, D, PutL, R, R, D, GetL, L, PutL, U, L, GetL, U, Turn, R, D, D, PutR

- Elaborated *Blocks World* with discrete spatial constraints
 - Gripper must stay in bounds
 - Can't pass through blocks
 - Can only turn at top row
- All actions have cost 1
- Goal: have C on T4
 - Can't just move directly
 - Final plan has 22 steps



L, D, GetR, U, Turn, D, PutL, R, R, D, GetL, L, PutL, U, L, GetL, U, Turn, R, D, D, PutR

- Elaborated *Blocks World* with discrete spatial constraints
 - Gripper must stay in bounds
 - Can't pass through blocks
 - Can only turn at top row
- All actions have cost 1
- Goal: have C on T4
 - Can't just move directly
 - Final plan has 22 steps



L, D, GetR, U, Turn, D, PutL, R, R, D, GetL, L, PutL, U, L, GetL, U, Turn, R, D, D, PutR

- Elaborated *Blocks World* with discrete spatial constraints
 - Gripper must stay in bounds
 - Can't pass through blocks
 - Can only turn at top row
- All actions have cost 1
- Goal: have C on T4
 - Can't just move directly
 - Final plan has 22 steps



L, D, GetR, U, Turn, D, PutL, R, R, D, GetL, L, PutL, U, L, GetL, U, Turn, R, D, D, PutR

- Elaborated *Blocks World* with discrete spatial constraints
 - Gripper must stay in bounds
 - Can't pass through blocks
 - Can only turn at top row
- All actions have cost 1
- Goal: have C on T4
 - Can't just move directly
 - Final plan has 22 steps



L, D, GetR, U, Turn, D, PutL, R, R, D, GetL, L, PutL, U, L, GetL, U, Turn, R, D, D, PutR

- Elaborated *Blocks World* with discrete spatial constraints
 - Gripper must stay in bounds
 - Can't pass through blocks
 - Can only turn at top row
- All actions have cost 1
- Goal: have C on T4
 - Can't just move directly
 - Final plan has 22 steps



L, D, GetR, U, Turn, D, PutL, R, R, D, GetL, L, PutL, U, L, GetL, U, Turn, R, D, D, PutR

- Elaborated *Blocks World* with discrete spatial constraints
 - Gripper must stay in bounds
 - Can't pass through blocks
 - Can only turn at top row
- All actions have cost 1
- Goal: have C on T4
 - Can't just move directly
 - Final plan has 22 steps



L, D, GetR, U, Turn, D, PutL, R, R, D, GetL, L, PutL, U, L, GetL, U, Turn, R, D, D, PutR

- Elaborated *Blocks World* with discrete spatial constraints
 - Gripper must stay in bounds
 - Can't pass through blocks
 - Can only turn at top row
- All actions have cost 1
- Goal: have C on T4
 - Can't just move directly
 - Final plan has 22 steps



L, D, GetR, U, Turn, D, PutL, R, R, D, GetL, L, PutL, U, L, GetL, U, Turn, R, D, D, PutR

- Elaborated *Blocks World* with discrete spatial constraints
 - Gripper must stay in bounds
 - Can't pass through blocks
 - Can only turn at top row
- All actions have cost 1
- Goal: have C on T4
 - Can't just move directly
 - Final plan has 22 steps



L, D, GetR, U, Turn, D, PutL, R, R, D, GetL, L, PutL, U, L, GetL, U, Turn, R, D, D, PutR

- Elaborated *Blocks World* with discrete spatial constraints
 - Gripper must stay in bounds
 - Can't pass through blocks
 - Can only turn at top row
- All actions have cost 1
- Goal: have C on T4
 - Can't just move directly
 - Final plan has 22 steps



L, D, GetR, U, Turn, D, PutL, R, R, D, GetL, L, PutL, U, L, GetL, U, Turn, R, D, D, PutR

- Elaborated *Blocks World* with discrete spatial constraints
 - Gripper must stay in bounds
 - Can't pass through blocks
 - Can only turn at top row
- All actions have cost 1
- Goal: have C on T4
 - Can't just move directly
 - Final plan has 22 steps



L, D, GetR, U, Turn, D, PutL, R, R, D, GetL, L, PutL, U, L, GetL, U, Turn, R, D, D, PutR

- Elaborated *Blocks World* with discrete spatial constraints
 - Gripper must stay in bounds
 - Can't pass through blocks
 - Can only turn at top row
- All actions have cost 1
- Goal: have C on T4
 - Can't just move directly
 - Final plan has 22 steps



L, D, GetR, U, Turn, D, PutL, R, R, D, GetL, L, PutL, U, L, GetL, U, Turn, R, D, D, PutR

- Elaborated *Blocks World* with discrete spatial constraints
 - Gripper must stay in bounds
 - Can't pass through blocks
 - Can only turn at top row
- All actions have cost 1
- Goal: have C on T4
 - Can't just move directly
 - Final plan has 22 steps

L D GetR U Turn D PutL









 Plans of interest are primitive refinements of special HLA Act

[Act]

- Plans of interest are primitive refinements of special HLA Act
- Each HLA has a set of immediate refinements into action sequences



- Plans of interest are primitive refinements of special HLA Act
- Each HLA has a set of immediate refinements into action sequences

[NavT(left of B), GetR, NavT(left of target), PutR]

Act

Move(B,C), Act

iff at G

- Plans of interest are primitive refinements of special HLA Act
- Each HLA has a set of immediate refinements into action sequences



Act

[Move(B,C), Act]

iff at G

• ALTs generalize lookahead trees for flat algs (e.g., A*)

- ALTs generalize lookahead trees for flat algs (e.g., A*)
 - Represent a set of potential plans



- ALTs generalize lookahead trees for flat algs (e.g., A*)
 - Represent a set of potential plans
 - Basic operation: refine a plan (replace with all refs. at some HLA)



- ALTs generalize lookahead trees for flat algs (e.g., A*)
 - Represent a set of potential plans
 - Basic operation: refine a plan (replace with all refs. at some HLA)



- ALTs generalize lookahead trees for flat algs (e.g., A*)
 - Represent a set of potential plans
 - Basic operation: refine a plan (replace with all refs. at some HLA)


- ALTs generalize lookahead trees for flat algs (e.g., A*)
 - Represent a set of potential plans
 - Basic operation: refine a plan (replace with all refs. at some HLA)



- ALTs generalize lookahead trees for flat algs (e.g., A*)
 - Represent a set of potential plans
 - Basic operation: refine a plan (replace with all refs. at some HLA)



- ALTs generalize lookahead trees for flat algs (e.g., A*)
 - Represent a set of potential plans
 - Basic operation: refine a plan (replace with all refs. at some HLA)



- ALTs generalize lookahead trees for flat algs (e.g., A*)
 - Represent a set of potential plans
 - Basic operation: refine a plan (replace with all refs. at some HLA)



- ALTs generalize lookahead trees for flat algs (e.g., A*)
 - Represent a set of potential plans
 - Basic operation: refine a plan (replace with all refs. at some HLA)



- ALTs generalize lookahead trees for flat algs (e.g., A*)
 - Represent a set of potential plans
 - Basic operation: refine a plan (replace with all refs. at some HLA)



- ALTs generalize lookahead trees for flat algs (e.g., A*)
 - Represent a set of potential plans
 - Basic operation: refine a plan (replace with all refs. at some HLA)



- ALTs generalize lookahead trees for flat algs (e.g., A*)
 - Represent a set of potential plans
 - Basic operation: refine a plan (replace with all refs. at some HLA)



- ALTs generalize lookahead trees for flat algs (e.g., A*)
 - Represent a set of potential plans
 - Basic operation: refine a plan (replace with all refs. at some HLA)
 - Nodes have optimistic & pessimistic valuations



• An HLA is fully characterized by planning problem + hierarchy



- An HLA is fully characterized by planning problem + hierarchy
 - But without abstraction, lose benefits of hierarchy



- An HLA is fully characterized by planning problem + hierarchy
 - But without abstraction, lose benefits of hierarchy
- Extension of idea from "Angelic Semantics for HLAs" [MRW '07]:
 - Valuation of HLA h from state s:
 - For each s', min cost of any primitive refinement of h that takes s to s'



- An HLA is fully characterized by planning problem + hierarchy
 - But without abstraction, lose benefits of hierarchy
- Extension of idea from "Angelic Semantics for HLAs" [MRW '07]:
 - Valuation of HLA h from state s:
 - For each s', min cost of any primitive refinement of h that takes s to s'
 - Exact description of h = valuation of h from each s



- An HLA is fully characterized by planning problem + hierarchy
 - But without abstraction, lose benefits of hierarchy
- Extension of idea from "Angelic Semantics for HLAs" [MRW '07]:
 - Valuation of HLA h from state s:
 - For each s', min cost of any primitive refinement of h that takes s to s'
 - Exact description of h = valuation of h from each s
 - But this description has no compact, efficient representation in general



• Instead, use approximate valuations



- Instead, use approximate valuations
- We choose a simple form: reachable set + cost bound on set



- Instead, use approximate valuations
- We choose a simple form: reachable set + cost bound on set
- Optimistic valuations never overestimate best achievable cost



- Instead, use approximate valuations
- We choose a simple form: reachable set + cost bound on set
- Optimistic valuations never overestimate best achievable cost
- Pessimistic valuations never underestimate best achievable cost



Descriptions specify propositions (possibly) added/deleted by HLA

Descriptions specify propositions (possibly) added/deleted by HLA



- Descriptions specify propositions (possibly) added/deleted by HLA
 - Also include a cost bound



- Descriptions specify propositions (possibly) added/deleted by HLA
 - Also include a cost bound
 - · Can condition on features of initial state

$NavT(x_t, y_t)$	(Pre: At(<i>x</i> _s , <i>y</i> _s))
Opt: $-At(x_s, y_s)$, $+At(x_t, y_t)$, $\tilde{-}FaceR$, $\tilde{+}FaceR$ $cost \ge x_s - x_t + y_s - y_t $	S t
Pess: IF Free $(x_t, y_t) \land \forall x$ Free (x, y_{max}) : -At (x_s, y_s) , +At (x_t, y_t) , ~FaceR, ~FaceR $cost \leq x_s - x_t + 2 y_{max} - y_t - y_s + 1$	

- Descriptions specify propositions (possibly) added/deleted by HLA
 - Also include a cost bound
 - Can condition on features of initial state

NavT(x_t, y_t	.)	(Pre: At(<i>x</i> _s , <i>y</i> _s))
Opt: -	$At(x_s, y_s), +At(x_t, y_t), \text{``FaceR}, \text{``FaceR}, \\ cost \geq x_s - x_t + y_s - y_t $	S t
Pess: IF -	Free $(x_t, y_t) \land \forall x$ Free (x, y_{max}) : At (x_s, y_s) , +At (x_t, y_t) , ~FaceR, ~FaceR $cost \leq x_s - x_t + 2 y_{max} - y_t - y_s + 1$ SE:	
l	nil	t

- Descriptions specify propositions (possibly) added/deleted by HLA
 - Also include a cost bound
 - Can condition on features of initial state
- An simple algorithm progresses a valuation (DNF + #) through an NCSTRIPS description to produce next valuation



Angelic Hierarchical A* (AHA*)

- Construct an ALT with the single plan [Act]
- Loop
 - Select a plan with minimal optimistic cost to G
 - If primitive, return it
 - Otherwise, refine one of its HLAs
 - Prune dominated refinements





























Analysis of AHA*

- AHA* is hierarchically optimal (HO)
 - Optimistic valuation → admissible heuristic
 - Pruning never rules out all HO plans
- Better descriptions lead to lower runtime
 - optimistic \rightarrow directed search
 - pessimistic → pruning (refine HO plans w/o backtracking)
- Reduces to A* given "flat" hierarchy: Act \rightarrow [Prim, Act]

Solution Length	A*	AHA*
7	0.9	0.6
16	10	4.7
25	40	11
37	550	30
44	> 10000	68

runtimes in seconds on five warehouse world instances of increasing solution length
Online Search

- Situated agents must cope with passage of time
 - offline planning rarely feasible
 - common alternative: real-time search
- Korf's Learning Real-Time A* (LRTA*):
 - Combines limited lookahead + learning
 - Always reaches goal, converges to optimal
- Angelic Hierarchical LRTA* (AHLRTA*)
 - Performs hierarchical lookahead
 - Shares LRTA*'s guarantees
 - Reduces to LRTA* given "flat" hierarchy



Online Results



1 AHLRTA* refinement \approx 5 LRTA* refinements



Model-based hierarchical planning is theoretically interesting, shows promising empirical performance

