

Please write your name and SID number in the spaces below, and wait for the signal to start:

Name _____

SID _____

1. Let X be a random variable which is 1 iff the number on a toss of a fair die is even, 0 otherwise. Let Y be a random variable which is 1 iff the number on a *second* toss of the die is 4, 0 otherwise. Are X and Y independent? YES or NO.

2. What is the probability that 1, 2 and 3 occur in increasing order, but not consecutively, in a random permutation of length n ?

3. Give a big-O bound for randomized Treesort of n elements from class. To remind you, Treesort does a random permutation of the elements, then inserts them one at a time into a binary search tree, and finally does inorder traversal of the tree.

4. Let E_i for $i = 1, \dots, n$ be a set of events on some sample space. Put an appropriate inequality ($=, \neq, >, <, \geq, \leq$) between the two terms below:

$$\Pr[E_1 \vee E_2 \vee \dots \vee E_n] \qquad \Pr[E_1] + \Pr[E_2] + \dots + \Pr[E_n]$$

5. Which of the two tail bound techniques, Markov or Chebyshev, *generally* gives a tighter bound on the probability that $\Pr[X > v]$ where v is some value greater than $E[X]$? Assume the variance of X is known.

6. How many balls should be randomly placed into n bins to have a good probability that some bin gets two balls?

7. How many balls should be randomly placed into n bins to have a good probability that every bin is non-empty?