CS174 Sp2001 **Homework 1** **out: Jan 18, 2001**
J. Canny **due: Jan 25, 2001**

This homework is due by 5pm on Thursday Jan 25th. Please hand it to the CS174 homework box on the second floor of Soda Hall.

1. A fair 6-sided die is tossed, and let:

$$X = \begin{cases} 1 & \text{if the number is even} \\ 0 & \text{otherwise} \end{cases} \qquad Y = \begin{cases} 1 & \text{if the number is in } \{4,5,6\} \\ 0 & \text{otherwise} \end{cases}$$

   Are these random variables independent? What is $\mathrm{E}[XY]$?

2. Let X be a random number with the uniform distribution on $\{1,\ldots,n\}$ so $\Pr[X=j]=1/n$ for $j=1,\ldots,n$. Let

$$Y = \begin{cases} 1 & \text{if } X \text{ is even} \\ 0 & \text{otherwise} \end{cases} \qquad Z = \begin{cases} 1 & \text{if } X \text{ is divisble by 3} \\ 0 & \text{otherwise} \end{cases}$$

   For what values of n are Y and Z independent?

3. Assume the cards in a 52-card deck are numbered from 1 to 52. Suppose a card is drawn at random, and let X be the number of the card. Now draw a second card at random without replacing the first. Let Y be the number of this card. Are X and Y independent? Explain.

4. Suppose you have a biased coin with $\Pr[\text{Head}] \neq 0.5$. How could you use this coin to simulate a fair coin? Hint: the solution is a Las Vegas algorithm. Think about pairs of coin tosses.

5. The random permutation generator from lecture 2 can be thought of as an "unsorting" version of selection sort. Can you derive a random permutation generator from Bubblesort? What is the probability of a particular swap? Does your algorithm give a uniform distribution on permutations (try to do this)? What is the expected running time?