Solutions for CS174 Homework 6

1. From the lecture notes, $\mathrm{E}[\sum H_{ij}] \leq n/2$. If the probability of a given packet is delayed more than $T(n)$ steps is bounded by 2^{-2n}, then we can guarantee that all packets reach their destination in time $T(n)$ with probability $1 - 2^{-n}$.

   If we assume $\delta > 2e - 1, \Pr[\sum H_{ij} > (1+\delta)\mu] \leq 2^{-\mu\delta}$. If we set $\mu\delta = 2n$, we get $\delta = 4 < 2e - 1$. So we should use the other formula, where when $\delta \leq 2e - 1, \Pr[\sum H_{ij} > (1+\delta)\mu] \leq exp(-\mu\delta^2/4)$. We set $exp(-\mu\delta^2/4) = 2^{-2n}$, so $\delta = \sqrt{16 \ln 2} \doteq 3.3$. So $T(n)$ should be $(3.3+1)n/2 = 2.15n$.

2. This is equivalent to check whether $AB = I$. Hence we can simply use the matrix multiplication checker program in the lecture notes to check whether $AB = I$. The running time of the checker program is $O(n^2)$.

3. $p(a_{11}, \ldots, a_{ij}, \ldots, a_{nn})$ is a multivariate polynomial total degree n. So the program checker simply picks $a_{11}, \ldots, a_{ij}, \ldots, a_{nn}$ uniformly at random from $\{0, \ldots, M-1\}$. We can compute such the determinant of the corresponding matrix A, denoted as $x = |A|$. We then compare the x with the output of the polynomial p. Due to Schwartz-Zippel theorem, the error bound is n/M. The running time to compute a determinant is $O(n^3)$. So the running time of the program checker is $O(n^3)$.

4. Let b_i denote the substring $a_{i+1} \cdots a_{i+k}$. Let $f(a, x) = \sum_i a_i x^i \mod p$ denote the finger print function. So $f(b_i, x) = (f(b_{i-1}, x) - a_i)x^{-1} + a_{i+k}x^{k-1} \mod p$. Thus we can compute $f(b_i, x)$ recursively in $O(n)$ time independent of k.