

This homework is due by 5pm on Thursday March 15th. Please hand it to the CS174 homework box on the second floor of Soda Hall.

1. Here is a different definition of a data-punctuated token tree. Let level 0 be the leaves of the tree, level 1 be their parents etc. Nodes are partitioned as follows: Assume every leaf is assigned a zero or one by a pseudo-random parity function. All tree nodes are defined non-recursively from the binary parity data at the leaves. i.e. there are no parity values at internal nodes.

At level  $k$ , draw boundaries at every transition from a sequence of exactly  $k$  zeros to a one. The nodes at level  $k$  represent the fingerprint of all leaves in between two boundaries at level  $k$ .

- (a) What is the minimum degree of an internal node in this tree?
- (b) Show that the expected number of transitions at level  $k$  in a sequence of  $n$  characters is  $n/2^{(k+1)}$ .
- (c) What is the expected number of nodes in an entire tree on  $n$  nodes?
- (d) Suppose one leaf node is changed or deleted. How many ancestor nodes at each level might change? Give an upper bound.
- (e) Now suppose that a contiguous sequence of  $m$  nodes is inserted in a tree with  $n$  nodes. How many nodes of the DPTT will change?