Solutions for CS174 Homework 7

1. Solution;

   (a) The minimum degree of an internal node is one. Consider the sequence 001001001. At the first level there are three nodes, all with degree 3. The level 2 boundaries are all in the same place, so each level 1 node has a unique parent. Those parents have only one child.

   (b) At each position at level k, the probability of being a transition point is $1/2^{(k+1)}$. So in a sequence of n characters, the expected number of transitions is $n/2^{(k+1)}$.

   (c) Let N_k denote the expected number of nodes at level k. So $N_k = n/2^{(k+1)} + 1$. So the expected number of nodes in the tree is

   $$n + \sum_{1 \le k \le \log n - 1} \left(\frac{n}{2^{(k+1)}} + 1\right) = 3n/2 + o(n).$$

   (d) We can see that any change to one node at leaf level at most requires changes to two consecutive nodes at level k. The level of the tree is about $\log n$. So the number of changes required is bounded by $2 \log_2 n$.

   (e) Inserting m nodes requires inserting about $3m/2$ nodes into the tree. Since the m nodes are inserted continuously, the changes required to old nodes in the tree is bounded by $2 \log n$. So the total changes is about $3m/2 + 2 \log n$.