On Motion Planning for Dexterous Manipulation, Part I: The Problem Formulation

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Abstract

In the study of multifingered robot hands, the process of manipulating an object from one grasp configuration to another is called dextrous manipulation. Motion planning for dextrous manipulation amounts to generating a sequence of trajectories of the fingers so that a final grasp configuration, through the effect of contact constraints, can be reached from an initial grasp configuration. In this paper, we formulate the motion planning problem for dextrous manipulation and in a forthcoming paper we will construct solutions based on this formulation. First, dextrous manipulation is decomposed into (i) coordinated manipulation; (ii) rolling motion; (iii) sliding motion and (iv) finger relocation. Then, we develop motion constraints for each of the manipulation modes, and show that for finger motions that satisfy these constraints these exists a well defined lift to the total space which links two contact configurations. Special to this paper is the incorporation of nonholonomic as well as holonomic, unilateral as well as bilateral constraints into motion planning.

1 Introduction

In the study of multifingered robot hands, the procedure of adjusting grasp configurations without the risk of dropping the object is called dextrous manipulation. In order to perform such a manipulation, the robot hand has to rely on rolling contact, sliding contact as well as on finger relocation. Since *rolling constraint is in general non-holonomic*, this makes a hand manipulation system an non-integrable system. Note that the notion of *non-integrability* has not been seen in the study of a single manipulator system. In other words, a single manipulator system is integrable, while a robot hand system is not, if we were to exploit the advantages associated with dextrous manipulation.

The study of dextrous manipulation is complicated not simply because a number of finger manipulators are involved ([Ker85], [LHS88]) but rather because of the non-integrability nature of the system. As a matter fact, approaches developed for studying integrable systems do not generalize easily to a non-integrable system. For example, motion planning for a rigid body under rolling constraints (i.e., non-holonomic) is very different from and much more difficult than motion planning under sliding constraints (i.e., holonomic) (see [LC89]).

The aim of this paper is to understand the dextrous manipulation problem. An outline of the paper is as follows: First, we formulate precisely what do we mean by dextrous manipulation. Then, we will decompose dextrous manipulation into the following manipulation modes: (a) *Coordinated manipulation*; (b) *Rolling motion*; (c) *Sliding motion* and (d) *Finger relocation.* For each of the manipulation modes appropriate constraints will be imposed. We then show that, for finger trajectories which satisfy these constraints the trajectory of the object is well defined. Our hope is to construct a set of trajectories of the fingers that links two grasp configurations. This will be the goal of the part II of the paper.

2. The Problem Statement

Consider the hand manipulation system shown in Figure 1. We assume that each finger contacts the object over its most distal link

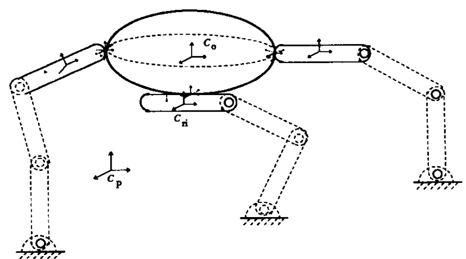

Figure 1: A hand manipulation system

only. Thus, by attaching a set of coordinate frames to the respective bodies and a reference frame to the hand palm, the configuration space of the object relative to the palm can be identified with a copy of the Euclidean group, denoted by $SE_o(3)$, and similarly the configuration space of finger $i, i = 1, ..k$, by $SE_i(3)$, where k denotes the number of fingers. We further assume that (A1) *the boundaries of the object and of the fingers are smooth, and (A2) the relative curvature form between the object and finger $i, i = 1, ..k$, is invertable.*

Definition 2.1 *The configuration space, P, of the hand manipulation system is*

$$P = SE_o(3) \times SE_1(3) \times ... \times SE_k(3)$$

where a configuration $z \in P$ is of the form $z = (g_0, g_1 ... g_k)$, with

$g_0 \in SE_o(3), g_i \in SE_i(3)$. *A point z in P is also called a* grasp configuration.

As a consequence of this definition, the following possibilities exist for a given grasp configuration $z \in P$: (1) none of the fingers is in contact with the object (*i.e., a null grasp*), (2) a subgroup of the fingers are in contact with the object (*i.e., this may correspond to finger relocation*) and (3) every finger is in contact with the object. We will restrict our attentions to a subset of P where overlaps between any two bodies do not occur ([Can88]). Note that nothing has been said about the nature of the grasp (e.g., force closure condition, etc.).

Let $G = SE(3)$ denote the group of rigid motion of $\mathbf{R}^3$. G acts on P by right translation, i.e, there exists a map

$$\Phi : P \times G \longrightarrow P : (z, h) \longmapsto (g_o h, g_1 h, ... g_k h).$$

Physically, this corresponds to a rigid motion h on the entire system by the base (or the macro-) manipulator where the hand system is attached to. Clearly, for each $h \neq e$(e is the identity element of G), the map $\Phi_h : P \longrightarrow P : z \longmapsto \Phi(z, h)$ is one-to-one(i.e., the action is free). Thus, we can define an equivalence relation $\sim$ on P as follows: $z_1 \sim z_2$ *if there exists a* $h \in G$ *such that* $z_2 = z_1 h$. In other words, two grasp configurations are equivalent *if one is related to the other by a rigid motion.* We let P/G denote the space obtaind from P under this equivalence relation. A point $[z]$ in P/G is of the form $[z] = zG$. *P/G is called the space of shapes* and there exists a natural projection from P to P/G, given by

$$\pi : P \longrightarrow P/G : z \longmapsto zG.$$

π is seen to be onto. The triplet $(P, G, P/G)$ is called a principal $G-$bundle. P is sometimes called the *total space*, P/G the *base space* and G the *structure group.*

Proposition 2.1 *Let $M = SE_1(3) \times .. \times SE_k(3)$ denote the configuration space of the fingers. Then, the space of shapes, P/G, is homeomorphic to M.*

Proof. Let $[z] = zG = (g_o, g_1, ... g_k)G$ be an element in P/G. Then, setting $h = g_o^{-1}$ we see that $zG \sim (e, g_1 g_o^{-1}, .. g_k g_o^{-1})G$. We call $[\hat{z}] = (e, g_1, .. g_k)G$ the unique representative of $[z]$. For if $[z]$ and $[z']$ are two different points in P/G. Then, $[z] \neq [z']$ if and only if $[\hat{z}] \neq [\hat{z}']$ if and only if $(g_1, ... g_k) \neq (g_1', ... g_k')$. We define the homeomorphism f by

$$f : P/G \longrightarrow M : zG \longmapsto (g_1, ... g_k), \text{ where } zG \sim (e, g_1, .. g_k)G.$$

This construction completes the proof. $\square$

This identification of P/G with M corresponds to setting the palm frame initially at the location of the object frame.

Since finger motion can be controlled by actuators located at the finger joints, M is also called the *control space.* A trajectory

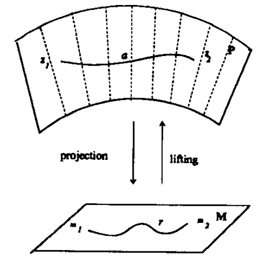

Figure 2: A global picture of dextrous manipulation

in M can be realized by specifying appropriate torque inputs. On the other hand, if a grasp is both stable (or force closure) and manipulable ([LHS88]) then the object motion can be effected by the finger motion. The setup of the problem is as follows (see Figure 2).

Problem Statement: *Given two grasp configurations z_1, z_2 in P, construct an admissible piecewise continuous curve $\gamma(t), t \in [0, t_f]$, in M such that (1) γ can be lifted to a curve α in P, i.e., $\pi(\alpha) = \gamma$ and α connects z_1 to z_2, i.e., $\alpha(0) = z_1$ and $\alpha(t_f) = z_2$.*

In the rest of the paper, we will define the precise meaning of a curve γ in M being admissible, and then we show that if γ is admissible, then, starting from an initial grasp configuration, there exists an unique lift α of γ to P. The construction of such an curve is under current investigation. A somewhat simplified answer is given in [LC89].

We now define some terminologies that will be needed and refer the readers to ([LCS89], [LHS88], [Mon86]) for further references.

Notation 2.1 Let (v_x^i, v_y^i, v_z^i) and (w_x^i, w_y^i, w_z^i) denote the components of the instantaneous contact velocity of finger i relative to the object, and ψ_i be the angle of contact.

Definition 2.2 *We say that finger i contacts the object by (a) fixed point of contact if $v_x^i = v_y^i = v_z^i = 0$ and $w_x^i = w_y^i = 0$; (b) rolling contact if $v_x^i = v_y^i = v_z^i = 0$ and $w_z^i = 0$ and (c) sliding contact if $v_z^i = 0$ and $w_x^i = w_y^i = w_z^i = 0$.*

Notation 2.2 $(v_{o,p}^t, w_{o,p}^t)^t \in \mathbf{R}^6$ denotes the velocity vector of the object, and $\xi_{fi,p} = (v_{fi,p}^t, w_{fi,p}^t)^t \in \mathbf{R}^6$ the velocity vector of finger i, and $\xi_{f,p} = (\xi_{f1,p}^t, ... \xi_{fk,p}^t)^t \in \mathbf{R}^{6k}$. $K_i \in \mathbf{R}^3$ denotes the friction cone of *ith* contact, and $K = K_1 \oplus ... \oplus K_k$ the force cone of the hand. $c_{oi} \in \mathbf{R}^3$ (or $c_{fi} \in \mathbf{R}^3$) denotes the position vector of the *ith* contact point relative to the object (or finger i) and $p_{oi} \in \mathbf{R}^2$ (or $p_{fi} \in \mathbf{R}^2$) the coordinates of c_{oi} (or of c_{fi}). $\underline{c}_o = (c_{o1}^t, ... c_{ok}^t)^t \in \mathbf{R}^{3k}$ and $\underline{p}_o = (p_{o1}^t, ... p_{ok}^t)^t$, and etc. $\underline{k} = \{1, ... k\}$. Let $m \subset \underline{k}$ be a subset of $\underline{k}$, then $\underline{k} \backslash m$ denotes its complement.

The following relation exists for contact type (a) and (b)

$$G^t \begin{bmatrix} v_{o,p} \\ w_{o,p} \end{bmatrix} = \hat{J}\xi_{f,p}$$

where G is the grip Jacobian ([LHS88]) and $\hat{J}$ is the pseudo hand-Jacobian. G depends $\underset{\sim}{c}_o$ and we write $G = G(\underset{\sim}{c}_o) = G(\underset{\sim}{p}_o)$ to emphasize this dependence, and similarly, $\hat{J}$ depends on $\underset{\sim}{c}_f$.

Definition 2.3 *A set of contact points $\underset{\sim}{c}_o$ is said to satisfy the condition of a grasp if $G(\underset{\sim}{c}_o)(K) = \mathbf{R}^6$.*

Notation 2.3 $M_{oi} \in \mathbf{R}^{2\times 2}$ denotes the metric form of the object at the *ith* point of contact, $K_{oi} \in \mathbf{R}^{2\times 2}$ the curvature form, and $T_{oi} \in \mathbf{R}^{1\times 2}$ the "torsion" form. Similarly, for finger i, M_{fi} denotes the metric form, and etc. (See [Mon86] for more detail). When the grasp condition is satisfied we let $F = (GG^t)^{-1}G\hat{J} = F(\underset{\sim}{p}_o, \underset{\sim}{p}_f, \psi)$.

The following set of equations that describe the kinematics of contact will be called the Montana's Equations ([Mon86]).

$$\begin{aligned}
\dot{p}_{oi} &= M_{oi}^{-1}(K_{oi} + \hat{K}_{fi})^{-1}\left(\begin{bmatrix} -w_y^i \\ w_x^i \end{bmatrix} - \hat{K}_{fi}\begin{bmatrix} v_x^i \\ v_y^i \end{bmatrix}\right), && (1)\\
\dot{p}_{fi} &= M_{fi}^{-1}A_{\psi_i}(K_{oi} + \hat{K}_{fi})^{-1}\left(\begin{bmatrix} -w_y^i \\ w_x^i \end{bmatrix} + K_{oi}\begin{bmatrix} v_x^i \\ v_y^i \end{bmatrix}\right), && (2)\\
\dot{\psi}_i &= w_z^i + T_{oi}M_{oi}\dot{p}_{oi} + T_{fi}M_{fi}\dot{p}_{fi}, && (3)\\
v_z^i &= 0, && (4)
\end{aligned}$$

where $\hat{K}_{fi} = A_{\psi_i}K_{fi}A_{\psi_i}$ is the curvature form of finger i seen by the object.

3 Classification of Motion Constraints

In this section, we classify the set of basic constraints on finger motion in M. These include *(i) constraints for collision avoidance and (ii) constraints by the kinematic structures of the fingers.* These constraints are unilateral and holonomic and thus can be easily dealt with.

We assume that geometries of the fingers and the object are known, and they satisfy assumptions (A1) and (A2). Furthermore, parameterizations of the fingers/object are given.

A. Constraints for Collision Avoidance

During the course of manipulation, collisions between links of all k-fingers should be prevented. Since each finger is represented by its last link, the constraints can be formulated directly in terms of the finger configuration variables. Consider the hand manipulation system shown in Figure 1. Let "F_i", $i \in \underset{\sim}{k}$, stand for finger i. Let $d : \mathbf{R}^3 \times \mathbf{R}^3 \longrightarrow \mathbf{R}, d^2(x,y) = \sum_{i=1}^{3}|x_i - y_i|^2$, be the Euclidean 2-norm. We define the distance function of "F_i" with "F_j", $j \neq i$ as follows

$$d(F_i, F_j) = \min_{\substack{x \in S_j^r, r \in \underset{\sim}{m}_j \\ y \in S_i^l, l \in \underset{\sim}{m}_i}} d(g_{rj,p}x,\ g_{ri,p}y), \quad (5)$$

where $g_{rj,p}x \triangleq R_{rj,p}x + r_{rj,p}$. According to Canny ([Can88], Ch. 2), for given features of finger i and j, $d(F_i, F_j)$ defines a function on the configuration variables of finger i and finger j. Without loss of generality, we will write, that

$$d(F_i, F_j) : M \longrightarrow \mathbf{R}. \quad (6)$$

For computational advantages, Canny used quaterion coordinates for the orientation space SO(3). But, conversions between quaterion coordinates and orientation matrices are rather straightforward.

Definition 2.4 *Collision between finger i and finger j can be prevented if and only if*

$$d(F_i, F_j) > 0, \quad (7)$$

The subspace of M where finger i is collision free with finger j is denoted by $d(F_i, F_j)^{-1}((0,\infty)) \triangleq \{(g_{r1,p}, \ldots g_{rk,p}) \in M \mid d(F_i, F_j) > 0\}$, and the constraint subspace for collision avoidance of all k-fingers is the intersection:

$$\bigcap_{i<j} d(F_i, F_j)^{-1}((0,\infty)) \subset M. \quad (8)$$

Remark: *It is straightforward, using the kinematic functions of the fingers, to formulate the constraints for collision avoidance between links of all k-fingers, where each finger has more than one link.*

B. Constraints by Finger Kinematics

The second type of basic constraint is the constraint due to the finger kinematic structures. Since the last link is connected to the hand palm by n_i links the set of reachable configurations by the finger is a compact submanifold Q_i of $SE_i(3)$. As was shown in [Can88], Q_i is a semi-algebraic set, and can be expressed by a set of inequalities in terms of the configuration variables, $g_{ri,p}$:

$$Q_i = \{g_{ri,p} \in SE_i(3) : f(g_{ri,p}) \geq 0\}. \quad (9)$$

The subspace of M where finger kinematic constraints is satisfied is the product:

$$(Q_1 \times \ldots \times Q_k) \subset M.$$

Finally, the subspace of M, where all the constraints discussed in this section are satisfied is given by

$$Q_s \triangleq \left\{\cap_{i<j} d(F_i, F_j)^{-1}((0,\infty))\right\} \bigcap \{(Q_1 \times \ldots \times Q_k)\} \subset M. \quad (10)$$

4 The Basic Manipulation Modes

We will decompose dextrous manipulation into the following *manipulation modes: (C) Coordinated manipulation; (D) Rolling motion; (E) Sliding motion and (F) Finger relocation* . Let $[0, t_f]$ be the time interval it takes to reach from the initial state to the final state. $[0, t_f]$ is divided into the union of successive sub-intervals, i.e., $[0, t_f] = \bigcup_{i=0}^{n-1}[t_i, t_{i+1}]$, $0 = t_0 < t_1 < \ldots < t_n = t_f$, such that at each sub-interval $[t_i, t_{i+1}]$ the finger motion is in one of the manipulation modes.

C. Coordinated Manipulation

Coordinated manipulation by a multifingered robot hand has been studied extensively in [LHS88]. It was shown that the fingers can be controlled to move in a coordinated fashion so that the object can be manipulated from one configuration to another. We see that, in addition to satisfying the generic types of constraints discussed in Section 3, the finger motion must also guarantee that the points of contact can in fact stay in contact with the body. We shall now formulate exactly the constraints on finger trajectories for coordinated manipulation mode.

Consider an initial state of the hand manipulation system, given by $z_0 = (g_{o,p}(0), \underline{g}_{f,p}(0)) \in P$. Let the initial contact points be $\underline{c}_o \in \mathbf{R}^{3k}$ and $\underline{c}_f \in \mathbf{R}^{3k}$, respectively, and assume that $\underline{c}_o$ form a grasp.

Let $\underline{g}_{f,p}(t) = (g_{r1,p}(t), \ldots g_{rk,p}(t)) \in M$, $t \in [0, t_1]$, be a set of trajectories of the fingers. The velocity vector of the fingers are denoted by $\xi_{f,p} = (\xi^t_{f1,p}, \ldots \xi^t_{fk,p})^t$. We wish to impose constraints on the finger velocity vector, and therefore the finger trajectories so that coordinated manipulation is well defined. Since the set of contact points initially satisfies the grasp condition, the object velocity can be expressed in terms of the finger velocity as

$$\begin{bmatrix} v_{o,p} \\ w_{o,p} \end{bmatrix} = F(\underline{c}_o, \underline{c}_f, \underline{\psi})\, \xi_{f,p}, \tag{11}$$

Consequently, the rotational components of the contact velocity can be expressed (implicitly) as a function of the finger velocity. This is given by (see [LCS89] for detail).

$$\begin{bmatrix} w^i_x \\ w^i_y \\ w^i_z \end{bmatrix} = -\hat{A}_{\psi_i} A^t_{fi,ri} w_{ri,p} + A_{ri,o} w_{o,p}(\xi_{f,p}),\ i \in \underline{k}, \tag{12}$$

and

$$\dot{\psi}^i = w^i_z,\ i \in \underline{k}. \tag{13}$$

On the other hand, the points of contact (p_{oi}, p_{fi}) evolve as a function of the relative rotational velocity according to Montana's equations.

$$\dot{p}_{oi} = M^{-1}_{oi}(K_{oi} + \hat{K}_{fi})^{-1} \begin{bmatrix} -w^i_y \\ w^i_x \end{bmatrix}, \tag{14}$$

$$\dot{p}_{fi} = M^{-1}_{fi} A_{\psi_i}(K_{oi} + \hat{K}_{fi})^{-1} \begin{bmatrix} -w^i_y \\ w^i_x \end{bmatrix}. \tag{15}$$

Equations (11) $\sim$ (15) constitute a system of differential equations with algebraic constraints. The exogenous input to the system is the velocity vector of the fingers.

Definition 2.5 *(Coordinated Manipulation) We say that a set of finger trajectories $\underline{g}_{f,p}(t) \in M$, $t \in [0, t_1]$, is admissible for coordinated manipulation if the relative rotational velocity $(w^i_x, w^i_y), i \in \underline{k}$ solved from the above system of differential equations with algebraic constraints is identically zero, for all $t \in [0, t_1]$.*

When a set of finger trajectories, $\underline{g}_{f,p}(t)$, satisfies the above admissiblity condition, its lift to the total space P is unqiuely defined. To define the lift of a curve $\underline{g}_{f,p}(t)$, we only need to know the trajectory of the object from that of the fingers.

By the admissiblity condition of the above definition, we have $\dot{p}_{oi}(t) = \dot{p}_{fi}(t) = 0$. Thus, c_{oi} and c_{fi} are all constant. The relation between the object trajectory and the finger trajectory is

$$g_{o,p}(t) c_{oi} = g_{ri,p}(t) c_{fi},\ \forall t \in [0, t_1], i \in \underline{k}$$

Since $\underline{c}_o \in \mathbf{R}^{3k}$ forms a grasp, the object trajectory variable $g_{o,p}(t)$ can be solved uniquely from the above set of equations. Let the solution be $g^*_{o,p}(t), t \in [0, t_1]$, and which defines the lift of $\underline{g}_{o,p}(t)$.

D. Rolling Motion

An efficient manipulation mode for effecting motion of both the object and contact coordinates is *rolling.* Cole et al ([CHS88]) show that when the initial contact points $\underline{c}_o$ is properly chosen then the object can be manipulated with pure rolling constraints. We formulate the admissiblity condition on finger trajectories for rolling motion as follows:

Consider an initial state of the hand manipulation system, with initial contact points $\underline{c}_o(0)$ (or $\underline{p}_o(0)$) and $\underline{c}_f(0)$ (or $\underline{p}_f(0)$), respectively. Assume that $\underline{c}_o(0)$ (or $\underline{p}_o(0)$) forms a grasp.

Let $\underline{g}_{f,p}(t) \in M$, $t \in [0, t_1]$, be a set of finger trajectories and consider the following system of differential equations with algebraic constraints ($i \in \underline{k}$):

$$\begin{cases} \begin{bmatrix} v_{o,p} \\ w_{o,p} \end{bmatrix} = F(\underline{p}_o(t), \underline{p}_f(t), \underline{\psi})\, \xi_{f,p}, \\ \begin{bmatrix} w^i_x \\ w^i_y \\ w^i_z \end{bmatrix} = -\hat{A}_{\psi_i} A^t_{fi,ri} w_{ri,p} + A^t_{oi,o} w_{o,p}(\xi_{f,p}), \\ \dot{p}_{oi} = M^{-1}_{oi}(K_{oi} + \hat{K}_{fi})^{-1} \begin{bmatrix} -w^i_y \\ w^i_x \end{bmatrix}, \\ \dot{p}_{fi} = M^{-1}_{fi} A_{\psi_i}(K_{oi} + \hat{K}_{fi})^{-1} \begin{bmatrix} -w^i_y \\ w^i_x \end{bmatrix}, \\ \dot{\psi}_i = w^i_z + T_{oi} M_{oi} \dot{p}_{oi} + T_{fi} M_{fi} \dot{p}_{fi} \end{cases} \tag{16}$$

Here, the first two equations are algebraic, and $\xi_{f,p}$ is considered as an input term. The initial conditions are given by the initial state of the hand system. Let $p_{oi}(t)$ and $p_{fi}(t), i \in \underline{k}$, be the solutions of (16), and $c_{oi}(t)$ the corresponding point of contact.

Definition 2.6 *(Rolling Motion) We say that a set of finger trajectories $g_{f,p}(t) \in M$, $t \in [0, t_1]$, constitutes a set of admissible trajectories for rolling motion if: (i) $w^i_z, i \in \underline{k}$, from (16) is identically* *zero for all $t \in [0, t_1]$; and (ii) the set of contact points $\underline{c}_o(t) \in \mathbf{R}^{3k}$ forms a grasp for all $t \in [0, t_1]$.*

The solution of the object trajectory from the above system of differential equations with algebraic constraints defines the lift of the base curve $\underline{g}_{f,p}(t), t \in [0, t_1]$.

E. Sliding Motion

When a finger or a group of m fingers ($1 \leq m \leq k$) are commanded to slide along the object surface, the remaining (non-sliding) fingers, together with contact wrenches from the sliding fingers constrained to the boundaries of the friction cones, should be able to held the object in the same configuration. The control algorithm presented in [LHS88] can be modified for this purpose. Constraint formulation for sliding motion is given here.

Assume that gravity is the only external force to be balanced during the course of sliding. Let $\hat{g}_p$ denote the gravity vector relative to C_p, and $Ad^t_{g^{-1}_{o,p}}(\hat{g}^t_p, 0)^t$ is then the equivalent wrench on the object relative to C_o.

Let $1 \leq m \leq k$ be the number of fingers to be slid simultaneously. Let $\pi_m = \{(\pi^i_m)^{i=m}_{i=1}, \pi^i_m \in \underline{k}\}$ define a permutation of m fingers to be slid. For example, if $\pi_3 = \{1, 3, 4\}$, then finger 1, 3 and 4 will be slid simultaneously. Note that for a given m, there are $\begin{pmatrix} k \\ m \end{pmatrix} = \frac{k!}{(k-m)!m!}$ different ways that a total number of m fingers can possibly slide. Thus, we have to perform $(2^k - 1)$ tests for all possible sliding motions.

Consider an initial state of the hand manipulation system given by $(g_{o,p}(0), \underline{g}_{f,p}(0)) \in P$, assume that the corresponding contact points $\underline{c}_o(0)$ forms a grasp. Let $\mathbf{g}_{f,p}(t) \in M$, $t \in [0, t_1]$, be a set of trajectories of the fingers such that

$$g_{ri,p}(t) = g_{ri,p}(0),\ \forall i \in \underline{k} \backslash \pi_m.$$

In other words, the trajectories of the non-sliding fingers stay constant. Let

$$G(K) \backslash \pi_m \triangleq \sum_{i \in \underline{k} \backslash \pi_m} G_i(K_i) \tag{17}$$

denote the set of contact wrenches from the nonsliding fingers, and $\sum_{i \in \pi_m} G_i(\partial K_i)$ denote the set of contact wrenches from the sliding fingers, where ∂K_i stands for the boundary of K_i. Then, the object can be held stationary under gravity force while simultaneously sliding fingers in π_m if

$$Ad^t_{g^{-1}_{o,p}} \begin{bmatrix} \hat{g}_p \\ 0 \end{bmatrix} \in G(K) \backslash \pi_m + \sum_{j \in \pi_m} G_i(\partial K_i). \tag{18}$$

With the object configuration stays constant, the velocity of the sliding fingers relative to the object is simply the velocity of the fingers, i.e.,

$$\begin{bmatrix} v^1_x \\ v^i_y \\ v^i_z \\ w^i_x \\ w^i_y \\ w^i_z \end{bmatrix} = -\begin{bmatrix} \hat{A}_{\psi_i} & 0 \\ 0 & \hat{A}_{\psi_i} \end{bmatrix} Ad_{g^{-1}_{fi,ri}} \begin{bmatrix} v_{ri,p} \\ w_{ri,p} \end{bmatrix}, \quad \forall i \in \pi_m. \tag{19}$$

If the relative rotational velocity is zero, the contact coordinates for the sliding fingers evolve according to

$$\begin{cases} \dot{p}_{oi} = -M^{-1}_{oi}(K_{oi} + \hat{K}_{fi})^{-1}\hat{K}_{fi} \begin{bmatrix} v^i_x \\ v^i_y \end{bmatrix}, \quad \forall i \in \pi_m, \\ \dot{p}_{fi} = M^{-1}_{fi} A_{\psi_i}(K_{oi} + \hat{K}_{fi})^{-1} K_{oi} \begin{bmatrix} v^i_x \\ v^i_y \end{bmatrix}, \quad \forall i \in \pi_m, \\ \dot{\psi}_i = T_{oi} M_{oi} \dot{p}_{oi} + T_{fi} M_{fi} \dot{p}_{fi}, \quad \forall i \in \pi_m. \end{cases} \tag{20}$$

(19) and (20) together constitute a system of differential equations with algebraic constraints, and is denoted by (*).

Definition 2.7 *(Sliding Motion) We say that a set of finger trajectories $\underline{g}_{f,p}(t) \in M$, $t \in [0, t_1]$, constitutes a set of admissible trajectories for sliding motion if: (i) (w^i_x, w^i_y, w^i_z) and v^i_z defined by (*) are identically zero for all $t \in [0, t_1]$ and $i \in \pi_m$; (ii) (18) is satisfied for all $t \in [0, t_1]$, and (iii) the set of contact points $\underline{c}_o(t) \in \mathbf{R}^{3k}$ forms a grasp for all $t \in [0, t_1]$.*

The object motion is constant during the course of sliding motion and this defines the lift.

F. Finger Relocation

Finally, we conclude this section by defining constraints for finger relocation. In a finger relocation mode, a group of m fingers ($1 \leq m \leq k$) are allowed to break contacts with the object and they will be positioned at other locations, provided that the set of contact points by the remaining fingers still forms a grasp.

Again, let the initial state of the hand manipulation system be $(\underline{g}_{f,p}(0), g_{o,p}(o)) \in P$ and let π_m be defined as before.

Definition 2.8 *(Finger Relocation) We say that a set of finger trajectories $\underline{g}_{f,p}(t) \in M$, $t \in [0, t_1]$, such that $g_{ri,p}(t) = g_{ri,p}(0), \forall i \in \underline{k} \backslash \pi_m$ constitutes a set of admissible trajectories for finger relocation if*

$$G(K) \backslash \pi_m = \mathbf{R}^6. \tag{21}$$

In other words, the set of contact points by the remaining fingers still forms a grasp.

The object motion is again constant and the lift is defined.

5. Conclusions

In this paper, we have formulated the dextrous manipulation problem for a robot hand. The configuration space of the system is defined and a bundle picture which explains how dextrous manipulation would work is given. On the other hand, we have to confess that the admissiblity conditions defined here are rather complicated and we are still in the process of simplifying them. Fortunately, some very elegant results have been obtained in [LC89].

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