CS 174 Quiz
February 25, 2002

Name:

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email:

The quiz is somewhat lengthy and I don't necessarily expect everyone to finish all of the questions. I will be generous with partial credit, however, and it is important that you give at least a *partial* answer to every question if at all possible.

1. [3 pts] Let X be the number of steps until failure of a system component. Suppose that X is a geometric random variable with parameter $p = .1$.

   (a) What is the expected number of steps until failure?

   (b) Provide an upper bound on the probability that the number of steps until failure exceeds 50.

2. [4 pts] Let n balls be tossed at random into n bins.

   (a) Compute the asymptotic probability, as $n \to \infty$, that the first r bins are empty.

   (b) Compute the asymptotic probability, as $n \to \infty$, that the even-numbered bins are empty.

3. [3 pts] I am going to dinner this evening in San Francisco. Let A denote the event that I have trouble finding parking, let B denote the event that I pay too much for my dinner, and let C denote the event that I get food poisoning. I consider my evening a bad evening if any of these events occur. Given $\Pr(A) = \Pr(B) = \Pr(C) = .1$, and given $\Pr(A \cap B) = \Pr(B \cap C) = \Pr(A \cap C) = .05$, give me an upper and a lower bound on the probability that I have a bad evening.

4. [3 pts] I toss a fair 6-sided die repeatedly. Compute the expected number of tosses until two different outcomes are observed.

5. [2 pts] Suppose that n mothers each give birth to a baby and that the n babies are put in a nursery for the night. Unfortunately the hospital forgets to write the names of the mothers on the baby cribs, and the following morning they are forced to assign babies to mothers completely randomly. The asymptotic probability that no mother receives her actual baby is asymptotically:

$$e^{-n} \qquad \frac{e}{n} \qquad 1 - \frac{1}{n} \qquad 1 - e^{-1} \qquad e^{-1} \qquad \frac{1}{n}$$

(Circle the correct answer).

6. [5 pts] Consider a complete graph K_n on n vertices. Let T_n be a particular spanning tree on these same n vertices. (Thus, T_n is a subgraph of K_n). Now consider a random process in which we repeatedly select an edge from K_n and label it "bad." These selections are made independently, and if we choose an edge that is already "bad," it stays "bad." All edges are initially "good."

   (a) What is the probability, after m selections, that all of the edges of T_n are still labeled "good"?

   (b) How big does m have to be, as a function of n, for the expected number of "good" edges in K_n to be equal to 1 (after the m selections)?

7. [4 pts] Suppose that the number of lightning strikes in a given day is a Poisson random variable with parameter $\lambda = 20$. Any given lightning strike has a probability $p = .1$ of hitting a person.

   (a) What is the expected number of lightning strikes per day?

   (b) What is the variance of the number of lightning strikes per day?

   (c) What is the expected number of people hit by lightning per day?

   (d) What is the variance of the number of people hit by lightning per day?

8. [5 pts] Consider the random graph model that we used in Homework 2:

   - Given a set E, initially empty, and a vertex set $V = \{1, 2, \ldots, n\}$
   - For each of the edges $e = \{i, j\}$, independently flip a coin with heads probability p, and if the coin comes up heads, add e to E
   - Output $G = (V, E)$

   (a) For a fixed vertex i, what is the expected number of neighbors of i, as a function of n and p?

   (b) Let X denote the number of edges in G. What is $\mathrm{E}[X]$, as a function of n and p?

   (c) Let X denote the number of edges in G. What is $Var[X]$, as a function of n and p?

9. [5 pts] Consider a particle that starts at the origin on the x-axis. At each time step, the particle takes a step of size one to the right, with probability $p = 0.5$, and it takes a step of size one to the left, also with probability $p = 0.5$. Denote the step at time t as the random variable X_t, where $X_t \in \{-1, 1\}$, and let $Z_T = \sum_{t=1}^{T} X_t$ denote the location of the particle after T steps.

   (a) Compute the expectation $\mathrm{E}[Z_T]$.

   (b) Compute the variance $\mathrm{Var}[Z_T]$.