Stat 210B Homework Assignment 2 (due February 26)

1. Consider the V-statistic $V_n = 1/n^2 \sum_{i=1}^n \sum_{j=1}^n h(x_i, x_j)$ for a symmetric kernel h such that $Eh^2 < \infty$. Show that V_n is asymptotically normal.

2. Consider the kernel $h(x_1, x_2) = \mathbb{I}_{x_1 + x_2 > 0}$ (where $\mathbb{I}$ denotes an indicator function). Evaluate $\theta = E[h(X_1, X_2)]$ for the mixture $F = (1-\epsilon)N(0,1) + \epsilon N(\alpha, \beta)$.

3. Show that for all $1 \le p < \infty$, we have:

$$H_p(\epsilon, Q, \mathcal{F}) \le H_{p,B}(\epsilon, Q, \mathcal{F})$$

for all ϵ. Show that if Q is a probability measure, we have:

$$H_{p,B}(\epsilon, Q, \mathcal{F}) \le H_\infty(\epsilon/2, \mathcal{F}).$$

4. Suppose that a class $\mathcal{F} \subset \mathcal{L}_1(P)$ satisfies the ULLN. Then also conv($\mathcal{F}$) satisfies the ULLN, where conv($\mathcal{F}$) is the convex hull of $\mathcal{F}$ (the set of arbitrary finite convex combinations of functions from $\mathcal{F}$).