Stat 210B Homework Assignment 3 (due March 13)

1. Show that the Cramér-von Mises statistic, $n\int(F_n - F)^2 dF$, is a continuous function of the empirical process (with respect to the supremum norm). Show that the distribution of this statistic is independent of F, for all continuous F.

2. Suppose that $X_1, ..., X_m$ and $Y_1, ..., Y_n$ are independent samples from distribution functions F and G, respectively. The Kolmogorov-Smirnov statistic for testing the null hypothesis $H_0 : F = G$ is the supremum distance $K_{m,n} = ||\hat{F}_m - \hat{G}_n||_\infty$ between the empirical distribution functions of the two samples.

   (a) Find the limit distribution of $K_{m,n}$ under the null hypothesis. (Express your result as a functional of a Brownian bridge).

   (b) Show that the Kolmogorov-Smirnov test is asyptotically consistent against every alternative $F \neq G$.

3. Given densities p_1 and p_2 with respect to some σ-finite measure μ, define that the *Hellinger distance* as follows:

$$h(p_1, p_2) = \left(\frac{1}{2}\int\left(\sqrt{p_1} - \sqrt{p_2}\right)^2 d\mu\right)^{1/2}$$

   and define the *variation distance* as follows:

$$\|p_1 - p_2\|_1 = \int |p_1 - p_2| d\mu.$$

   Establish the following two inequalities:

$$h^2(p_1, p_2) \leq \frac{1}{2}\|p_1 - p_2\|_1 \leq h(p_1, p_2)[2 - h^2(p_1, p_2)]^{1/2}.$$

4. Consider two probability measures P_1 and P_2 on a measurable space $(\mathcal{X}, \mathcal{A})$, dominated by a σ-finite measure μ and let p_1 and p_2 be the densities of P_1 and P_2 with respect to μ, respectively. Show that the Hellinger distance $h(p_1, p_2)$ does not depend on the dominating measure μ.

5. Problem 5.24 in van der Vaart.