

Stat 210B Homework Assignment 4 (due April 3)

1. Suppose $\|P_n - Q_n\| \rightarrow 0$. Show that P_n and Q_n are mutually contiguous.
2. Suppose P_θ is the uniform distribution on $(0, \theta)$. Let P_θ^n denote the distribution of n iid draws from P_θ . Fix h and determine whether or not P_1^n and $P_{1+h/n}^n$ are mutually contiguous. Consider both $h > 0$ and $h < 0$.
3. Consider estimating the distribution function $P(X \leq x)$ at a fixed point x based on a sample X_1, \dots, X_n from the distribution of X . A nonparametric estimator is $n^{-1} \sum_i 1(X_i \leq x)$. If it is known that the true underlying distribution is $N(\theta, 1)$, another possible estimator is $\Phi(x - \bar{X})$. Calculate the relative efficiency of these estimators.
4. Let X_1, \dots, X_n be a random sample from a density $f(x - \theta)$ where f is symmetric about zero. Calculate the relative efficiency of the t -test and the test that rejects for large values of $\sum \sum_{i < j} 1\{X_i + X_j > 0\}$ for f equal to the logistic, normal, Laplace and uniform shapes.
5. Problem 14.9 in van der Vaart.