

Stat 210B Homework Assignment 5 (due April 24)

1. Calculate the likelihood ratio statistic for testing the null hypothesis $H_0 : \mu_1 = \dots = \mu_k$ based on k independent samples of size n from a $N(\mu_j, \sigma^2)$ distribution. What is the limit distribution under the null hypothesis?
2. Let X_1, \dots, X_n be i.i.d. according to the Pareto distribution with density

$$f_\theta(x) = \theta c^\theta x^{\theta+1},$$

where $0 < \theta$ and $0 < c < x$. Determine the Wald, Rao and likelihood ratio tests of $H_0 : \theta = \theta_0$ against $\theta \neq \theta_0$.

3. Problem 1 on page 226 of van der Vaart.
4. Calculate the asymptotic relative efficiency of the empirical p -quantile and the estimator $\Phi^{-1}(p)S_n + \bar{X}_n$ for estimating the p -th quantile of the distribution of a sample from the univariate normal $N(\mu, \sigma^2)$ distribution (where $S_n^2 = 1/(n-1) \sum_i (X_i - \bar{X}_n)^2$).
5. Find the influence function of the map $F \rightarrow \int_0^t (1-F_-)^{-1} dF$ (the cumulative hazard function).