DECISION MAKING UNDER UNCERTAINTY

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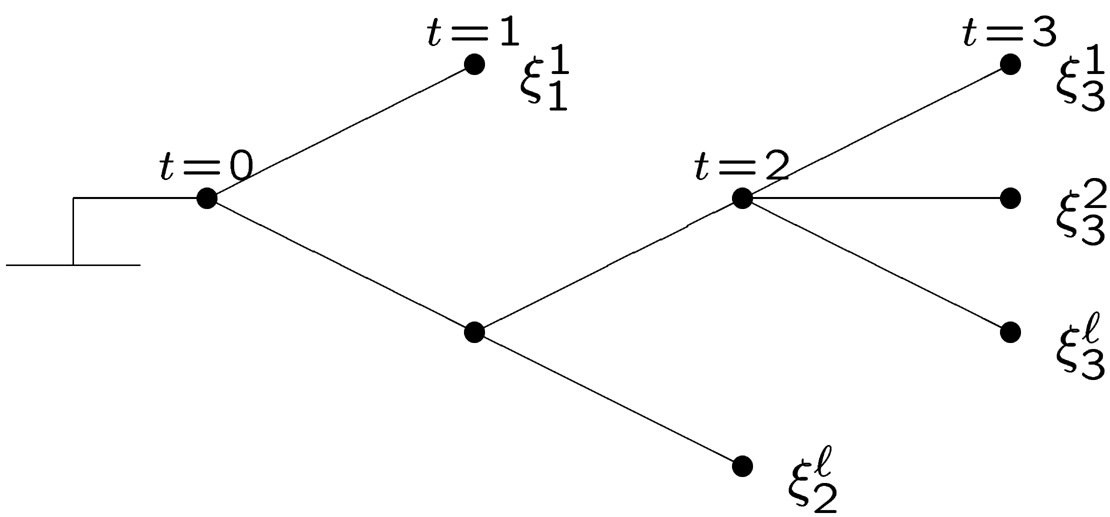
DECISION & INFO. PROCESS

decision: x

$\to$ observation: $\xi \to$ evaluation: $f(\xi, x)$

More generally, (ξ_0 initial state)

$$\xi_0 \to x_0(\xi_0), \quad \xi_1 \to x_1(\xi_0, \xi_1), \quad \ldots$$



Problem (simplest form):

$$\min_{x \in C} E\{f(\xi, x)\} = \int f(\xi, x)\, P(d\xi)$$

INSUFFICIENT INFO./DATA

Actual distribution of present/future states: P
Available information/data/experts

$$\Longrightarrow Q\text{: best estimate of } P$$

Let $(E^P f)(x) = E^P f(x) := \int f(\xi, x)\, P(d\xi)$.

$$x^P \in \operatorname*{argmin}_{x \in C} E^P f(x), \quad x^Q \in \operatorname*{argmin}_{x \in C} E^Q f(x)$$

.

How reliable is the solution x^Q?

- deterministic bounds:

$$\text{dist}(x^Q, x^P) \leq \kappa * \text{dist}(Q, P);$$

- statistical estimates:

$$\text{prob}[\text{dist}(x^Q, x^P) \geq \beta] \leq \eta.$$

DETERMINISTIC BOUNDS

$A(P) := \text{argmin}_{x \in C} E^P f(x)$

Q.no.1: Is $P \mapsto A(P)$ Lipschitz continuous?

$$\exists\, \kappa > 0 : \quad \text{dist}(x^Q, x^P) \leq \kappa \text{dist}(Q, P)$$

Q.no.2: What is the Lipschitz constant κ?

Step 1: (Wets 2001) Let

$$A(u) = \text{argmin}\, \{g(u,x) \,|\, x \in S(u)\}$$

with $u \mapsto S(u)$ (sub)Lipschitz and $g(\cdot,\cdot)$ locally Lipschitz, then $u \mapsto A(u)$ is (sub)Lipschitz.

Step 2: (Wets 1987)

$$(P, x) \mapsto E^P f(x) =: g(P, x)$$

is locally Lipschitz, and (certainly)

$u \mapsto C := S(u)$ is (sub)Lipschitz (constant).

Step 3: (Römisch & Wets 2001?) Lip. constants that depend on choice of the probability distance (Wasserstein, Fortet-Mourier, etc.)

IMPLICATIONS

1. Even with a rough, but reasonable, approximating measure Q we should actually get a pretty good solution; cf. Steps 1 & 2.

2. But, making the problem deterministic, i.e., substituting for P a probability measure Q that assigns probability 1 to a specific event, will result in a solution that could be ‘far’ away from the optimal one, and in some situations it might suggest decisions that are disastrous; the distance between probability measures is essentially maximized.

STATISTICAL BOUNDS

$Q = P^\nu$ where $P^\nu \Rightarrow P$, say, empirical measure

$$x^\nu \in \operatorname*{argmin}_{x \in C} E^{P^\nu} f(x) = \int f(\xi, x)\, P^\nu(d\xi)$$

Q.no.1: $x^\nu \to x^P =: x^*$ (with probability 1)? Q.no.2: How fast?

Step 1. Ergodicity (Wets & . . . ’95-’00)

$$x^\nu \to x^P =: x^* \text{ (w.p.1).}$$

Tools: Variational Analysis [epi-convergence], probability theory for random lower semicontinuous (lsc) functions.

Step 2. Rates $\approx$ CLT for stochastic optimization

Tools: Theory of Random Sets

Result: (Casey/Wets ’01)

CLT for bounded random sets

Needed: CLT for unbounded random sets.

DISTRIBUTED SYSTEMS

Evolution of environment might dependent on multi-factors: time, location (x,y), etc.

state at time 1: $H(\xi_0, \xi_1; x)$, x in fcns-space
initial state ξ_0, initial decision x,
value of F: obtained as solution of PDE
ξ_1 'uncertain' coefficients of PDE.

find $x \in C$ that minimizes $E^P\{f(H(\xi_0, \xi_1; x))\}$.

Challenge (with S. Lucero): Embed a solution technique for PDE in an overall strategy to solve stochastic optimization problem.

Our test case: Flow/transport equations
heterogeneous media $\Rightarrow$
rapidly oscillating stochastic coefficients
x decision: affects boundary conditions