Unconstrained Paving and Plastering: Progress Update

Matthew L. Staten, Robert A. Kerr, Steven J. Owen, Ted D. Blacker

Sandia National Laboratories*, Albuquerque, NM, U.S.A.
mlstate@sandia.gov, rakerr@sandia.gov, sjowen@sandia.gov,
tdblack@sandia.gov

Summary: Unconstrained Paving and Plastering [1] were introduced as new methods of generating all-quadrilateral and all-hexahedral finite element meshes. Their introduction was after preliminary conceptual studies. This paper presents an update on Unconstrained Paving and Plastering after significant implementation and conceptual development.

1 Introduction

Modeling and simulation has become an essential step in the engineering design process. Modeling and simulation can be used during either the original design phases, or on assessment of existing designs. In either case, the end result is increased confidence in the design, faster time to market, and reduced engineering cost.

An essential step in modeling and simulation is the creation of a finite element mesh which accurately models the geometric features of the model being analyzed. Meshes generated for three-dimensional models are typically composed of either all-tetrahedral or all-hexahedral elements. Some methods exist for the generation and analysis of hybrid meshes

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which contain a mix of element types. However, for traditional finite element modeling of continuum mechanics, a single element type is most often used.

Quite a debate has emerged over the advantages and disadvantages of hexahedral verses tetrahedral elements. Tetrahedral meshes are typically much easier to generate. On complicated models with complex geometric features, the time savings on generating a tetrahedral mesh rather than a hexahedral mesh can be orders of magnitude with the current meshing technology. However, the benefit of hexahedral elements is that they often perform better in the analysis stage [2,3,4].

Regardless, the bottom line is that customers of finite element meshing software continue to demand improved ability to generate hexahedral elements. This demand is what drives research in hexahedral mesh generation.

Unconstrained Plastering [1] is a new method for the automatic generation of hexahedral meshes. Unconstrained Plastering continues to show promise, although several technical and implementation challenges remain. This paper presents the current status and some of the challenges which are currently being addressed.

2 Previous Research

The work presented in this paper is built upon the work previously presented in [1], which includes an extensive description of previous research done on hexahedral mesh generation at institutions across the world. If the reader is unfamiliar with the existing research in hexahedral mesh generation or the current state-of-the-art in hexahedral mesh generation, they are encouraged to read the previous research section in [1]. Rather than repeating that information here, the previous research described in this paper will be limited to summarizing Unconstrained Paving and Plastering, and other algorithms which directly contributed to their development.

Paving [5] has been shown to be a robust and efficient solution to the quadrilateral surface meshing problem. However, its three-dimensional extension, Plastering [6,7,8], has not done the same for hexahedral mesh generation. Plastering calls for a pre-meshed boundary, which is created without considering global mesh topology. Fronts are then created, from which hexahedral elements are advanced into the solid in an element-by-element fashion. As fronts collide, complex configurations of closely-spaced randomly-oriented quadrilaterals yield complex unmeshed voids which Plastering is rarely able to resolve. As a result, traditional Plastering is able to completely mesh only simple primitive models with carefully pre-meshed boundaries. Plastering's inability to mesh more complex solids stems from its element-by-element geometric approach and the added constraints of a pre-meshed boundary. Like Paving, Plastering considers only local element connectivites, with a high priority placed on incremental nodal placement and single element topology. Although this approach worked well in Paving for two dimensional surface meshing, the extra degree of freedom in three dimensions proves that more global consideration of hexahedral topology is required.

Learning from the experience of Plastering, Whisker Weaving [9,10] was developed with an emphasis on global hexahedral topology. The concept of the dual, or Spatial Twist Continuum [11] was key to the development of Whisker Weaving. Like Plastering, Whisker Weaving also starts from a pre-defined boundary quad mesh. Each quad on the boundary represents a whisker, or incomplete chord in the dual. The topology of the boundary quad mesh is traversed until groups of three or more boundary quadrilaterals are found whose corresponding whiskers could be advanced, or crossed, forming the topology of a single hexahedral element. The spatial locations of interior nodes are not calculated until the topology of the entire mesh is determined. Thus, formation of hexahedral element topology is guided by near-exclusive consideration of mesh topology logic. Geometric characteristics of the solid are considered secondary to the overall mesh topology. This is in stark contrast to Plastering which does nearly the opposite. Whisker Weaving is able to successfully generate hexahedral topology for a wide spectrum of solid geometries. However, because it leaves geometric positioning of interior nodes until after the entire mesh topology has been determined, Whisker Weaving is unable to make any guarantees on reasonable element quality. In practice, the element qualities produced by Whisker Weaving are rarely adequate, and are often inverted.

Research on Plastering and Whisker Weaving has shown that any algorithm which attempts to automatically generate hexahedral meshes must take both model topology as well as geometric model characteristics into consideration. Failure to consider geometric features of a solid will almost always result in poor element quality. Failure to consider global mesh and model topology will almost always result in a failure to generate a valid hexahedral mesh topology.

It is with this background that research on Unconstrained Paving and Plastering began. The authors introduced the concepts of Unconstrained Paving and Plastering in [1], and briefly summarize them here for clarity. Based on recent research and development efforts, this paper details new discoveries that help move this technology closer to a general all-purpose hexahedral mesh generator.

Unconstrained Paving and Plastering removes the constraint of a pre-meshed boundary. This allows the meshing process to consider more global model topologies without being constrained by local mesh anomalies. The domain is then systematically partitioned through the advancement of fronts. In traditional advancing front methods [5,6,12], individual solid elements are generated by following geometric reasoning to build individual nodes, edges and faces, starting from a predefined boundary mesh and advancing inwards. In contrast, Unconstrained Paving and Plastering advance geometric layers or partitions independent of element distribution. Unconstrained Paving and Plastering delay the final definition of elements until it is absolutely necessary, thus removing any artificial constraints that a pre-meshed boundary imposed.

Unconstrained Paving and Plastering partition the domain into regions classified based on the number of remaining degrees of freedom. The Spatial Twist Continuum [11] defines quadrilateral elements as the intersection of two chords and hexahedral elements as the intersection of three chords. As such, a domain which is to be meshed with quadrilaterals must constrain two degrees of freedom for the entire domain corresponding to two chords required for each quadrilateral. Similarly, a domain which is to be meshed with hexahedra must constrain three degrees of freedom for the entire domain corresponding to the three chords required for each hexahedra.

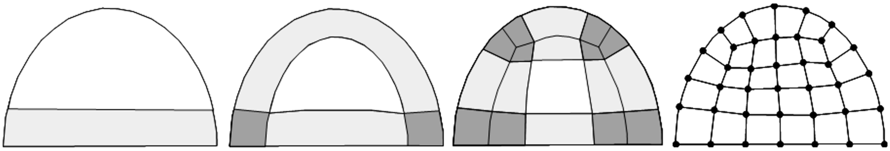
Figure 1 illustrates the meshing of a simple surface with Unconstrained Paving. Unconstrained Paving systematically partitions the surface into sub-regions which are classified as either:

- unmeshed voids (white regions in Figure 1, no degrees of freedom are constrained)
- connecting tubes (light gray regions in Figure 1, one degree of freedom is constrained), or
- final elements (dark gray regions in Figure 1, two degrees of freedom are constrained).

For Unconstrained Plastering, the regions are classified as either:

- unmeshed void (no degrees of freedom are constrained),
- connecting tubes (one degree of freedom is constrained),
- connecting webs (two degrees of freedom are constrained), or
- final elements (three degrees of freedom are constrained).

A front advancement as shown in Figure 1a introduces an unconstrained row of elements with an undetermined number of quadrilaterals. The advancement of a row essential introduces a single new chord to the dual of the eventual mesh. However, the number of quads in this row or chord is left unconstrained at this point. Since a quadrilateral is the intersection of two chords, the insertion of a single new chord through a single front advancement does not uniquely determine any quadrilaterals unless the chord



(a) One row advanced (b) Intermediate step (c) Intermediate step (d) Final mesh

Figure 1. Unconstrained Paving

inserted happens to cross previously inserted chords. In Figure 1b, an additional front is advanced inserting a second chord. At the bottom two corners, this newly inserted row crosses the previously advanced row, thus defining a quadrilateral element at each of the two row crossings.

Figure 1 a, b, and c illustrate that at all times, the unmeshed void (white region) is connected to the boundary either by direct adjacency or through connecting tubes. For example, in Figure 1a, the unmeshed void is connected to one connecting tube on the bottom, and the surface boundary on the top. In Figure 1b, the unmeshed void is connected to two connecting tubes, one on the top and one on the bottom. Likewise in Figure 1c, the unmeshed void is connected to four connecting tubes (i.e top, bottom, left and right). Thus, any of the curves on the unmeshed void can be split into as many mesh edges as needed for resolution of the void. Any splitting of the curves on the unmeshed void can be propagated back to the boundary through the connecting tubes.

Similarly in three dimensions with Unconstrained Plastering, all surfaces of the unmeshed void are connected to the boundary either by direct adjacency or through connecting tubes. As a result, any of the surfaces of the unmeshed void are free to be discretized as required for resolution. Any discretization of the surfaces of the unmeshed void can be propagated back to the boundary through the connecting tubes. This is in contrast to traditional Paving and Plastering where the unmeshed void is completely discretized at all times by either element edges or quadrilateral faces. This discretization proved to be the Achilles heal for traditional Plastering since the unmeshed void is typically discretized with closely spaced randomly oriented quadrilaterals. In general, Unconstrained Paving and Plastering continue advancement of rows and sheets until the unmeshed void can be meshed with Midpoint Subdivision [13].

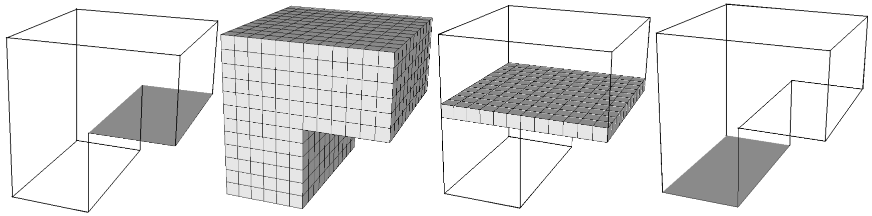
Unconstrained Paving and Plastering rely heavily upon model topology by removing the constraint of the pre-meshed boundary, advancing unconstrained rows and sheets rather than single elements, and by following strict guidelines which consider global ramifications when local dual operations are performed. Unconstrained Plastering also considers geometric characteristics of the model by performing proximity and angle checks between nearby fronts, size checks to make sure that front advancements are consistent with desired element sizes, and layer checks to ensure that advancing fronts are boundary-sensitive. In addition, like traditional Plastering, Unconstrained Plastering advances rows in the primal space which provides access to direct geometric properties of the model and previously advanced rows. In contrast, Whisker Weaving operates in the dual space which is part of the reason geometric features are not considered. It is anticipated that through careful combination of both topological and geometric considerations, Unconstrained Plastering will be successful on arbitrary geometry assemblies.

Although Unconstrained Plastering has matured since its initial introduction, there are still several technical hurdles which must be overcome before success can be declared. Section 3.0 introduces incomplete fronts which Unconstrained Paving and Plastering use to handle model concavities. Model concavities appear in everything except trivial primitive models. Section 4.0 describes the processes of merging and seaming which are used to eliminate proximity problems and small angles between adjacent fronts which occur in nearly every model as fronts collide. Section 5.0 shows some example models which have been meshed with the current implementation. Finally, section 6.0 discusses plans for future research and development, followed by conclusions in section 7.0.

3 Model Concavities – Incomplete Fronts

Concavities are a common occurrence in even simple CAD models. A strict geometric definition of a concavity is anywhere on the model where the interior angle at a point is greater than 180 degrees. However, in a hexahedral meshing sense, a concavity is anywhere that has a large enough interior angle that three hexahedra would more accurately model the geometry than only two hexahedra. Submapping technology defines this condition as a “Corner” [14].

Unconstrained Plastering handles concavities through the definition and advancement of “incomplete fronts.” Figure 2a shows a simple solid with one of the surfaces highlighted. Figure 2b shows what the ideal mesh would look like on this model. The left-most curve on the highlighted surface is a concavity in the model since each mesh edge on it has three adjacent hexahedra. Figure 2c shows the sheet of hexes which is directly adjacent to the highlighted surface. Because of the concavity on the left-most curve, this sheet extends out into the interior of the solid.



(a) simple model with one shaded surface.

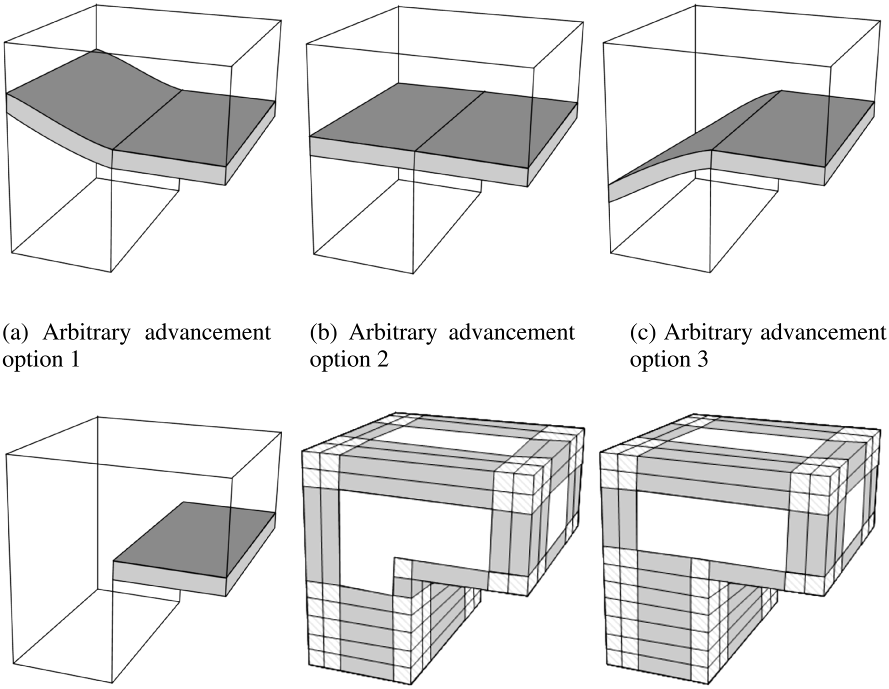
(b) ideal mesh.

(c) hex layer adjacent to shaded surface.

(d) shaded surface is a complete front.

Figure 2. Example of an incomplete front

When Unconstrained Plastering begins, it initializes fronts from each of the boundary CAD surfaces. The highlighted surface in Figure 2a would be initialized as a front to advance. However, because of the concavity, this front is marked as an *incomplete* front. In contrast, the highlighted surface in Figure 2d would be initialized as a *complete* front since all of the curves on it are convex. The advancement of this or any other complete front entirely defines a single hexahedral sheet. This is accomplished because the topology of the model completely defines the path the sheet should take. However, the advancement of an incomplete front can only define the portion of the sheet directly in front of the surface(s) comprising the front. Any further advancement of the front would be arbitrary. Figure 3a, b, & c shows three possible arbitrary advancements of a complete sheet from the incomplete front in this example. All three are valid, and the mesh in Figure 2b demonstrates that the sheet in Figure 3b is eventually used. However, choosing between them is not possible until other adjacent fronts are advanced. The recommended procedure for incomplete fronts is to advance the front to form a partial hex sheet, as shown in Figure 3d, followed by advancement of other complete fronts in the model. Figure 3e shows this simple example after several adjacent fronts are advanced. Notice that the incomplete front was advanced only once. Figure 3f shows that by continuing to advance adjacent *complete* fronts, the *incomplete* front can be completed when adjacent fronts are advanced far enough to guide the incomplete hex sheet to completion.



(a) Arbitrary advancement option 1

(b) Arbitrary advancement option 2

(c) Arbitrary advancement option 3

(d) Only known advancement is performed

(e) After several more layers are advanced

(f) Final sheet resolution of incomplete front

Figure 3. Various options and final resolution of incomplete front

Another option would be to completely refrain from advancing this incomplete front until adjacent fronts have been advanced far enough to "complete" the incomplete front. However, doing so would leave the boundary of the solid exposed to direct collisions from other advancing fronts. When fronts collide, seaming and merging is needed. Seaming and merging operations result in nodes and edges with non-optimal valences, which often results in poor element qualities. By performed seaming and merging directly on the model boundary the risk of creating poor elements directly on the model boundary is increased. Since analysis results are often of greater interest on the boundary, care must be taken to ensure as high an element quality as possible on the boundary. As such, the recommendation is that each incomplete front be advanced once in order to form a protective layer directly adjacent to the model boundary. After a single advancement, the incomplete front can wait until adjacent complete fronts are advanced far enough to complete the incomplete front.

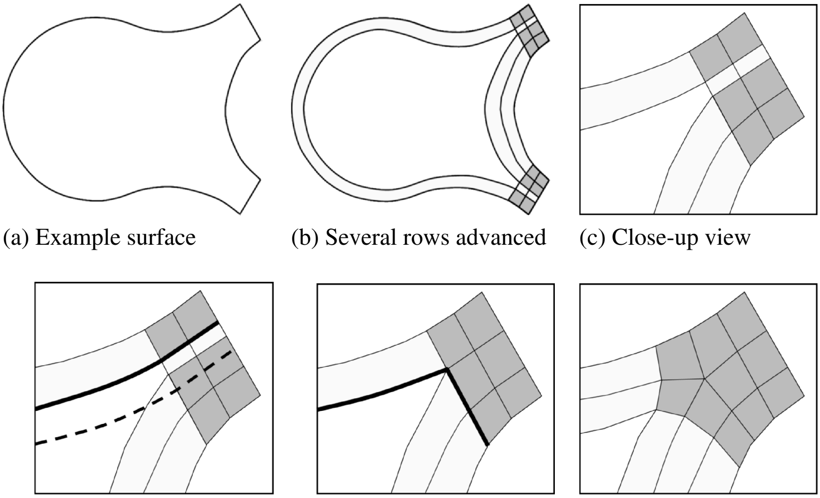
4 Front Collisions

As fronts are advanced in Unconstrained Paving & Plastering, certain operations must be performed. The operations that are performed most often are merging and seaming. Merging is defined as the resolution of small gaps between fronts. Seaming is defined as the resolution of small angles between adjacent fronts. Merging and seaming must be performed iteratively since merging often creates seaming cases; likewise seaming often creates merging cases.

4.1 Merging

During Unconstrained Paving and Plastering, cases requiring merging occur in either the connecting tubes or in the unmeshed void. Figure 4 illustrates the partial meshing of an example surface using Unconstrained Paving. Figure 4c shows a close-up of a connecting tube which is too skinny for an additional advancement. In Figure 4d, the solid dark line represents the front to advance. The dashed dark line represents the desired advanced location of this front, which clearly shows the proximity problem which must be resolved before continuing. Figure 4e shows how the proximity can be resolved by collapsing out the connecting tube, with the solid dark line representing the modified front after merging. Figure 4f shows what the model would look like after the advancement of additional fronts after the merge operation. Essentially, the merge operation inserts a 5-valent node in the quad mesh.

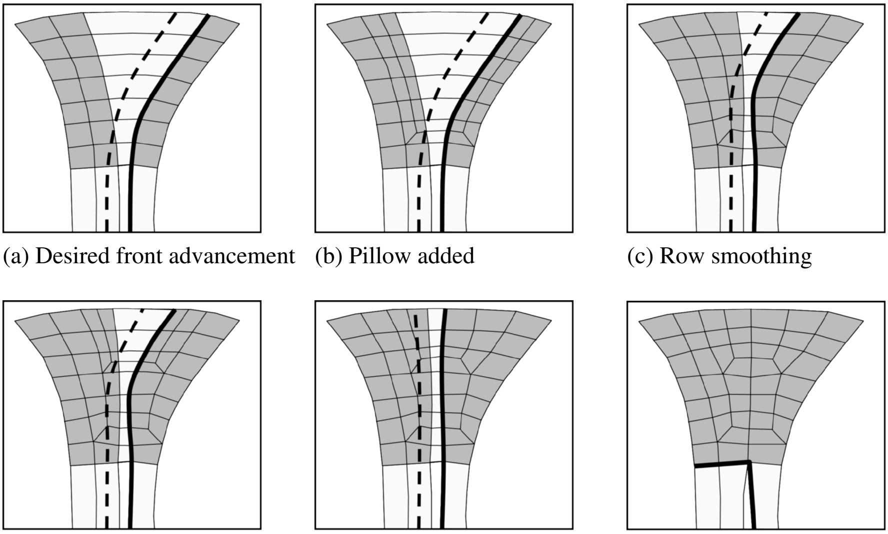
Figure 4c shows the connecting tube which must be collapsed. In this case, proximity problems exist throughout the entire connecting tube. As a result, it makes geometric sense to collapse the entire tube. However, on other geometries, it is possible that the connecting tube might expand in some regions, making proximity only an issue for a portion of the tube. This is illustrated in Figure 5a. However, regardless of the geometric proximity, if part of the tube requires collapsing, then the entire tube must be collapsed in order to keep topological consistency throughout the model. In order to reconcile sizing in cases where only part of the tube has proximity, pillowing [15] can be performed to make the tube sizes more consistent. Figure 5 illustrates that pillowing followed by row smoothing can be performed in the connecting tubes to make the size of the tube consistent so the entire tube can be merged.



(a) Example surface (b) Several rows advanced (c) Close-up view

(d) Desired front advancement (e) Fronts merged (f) Additional fronts advanced

Figure 4. Merging example for connecting tube proximity

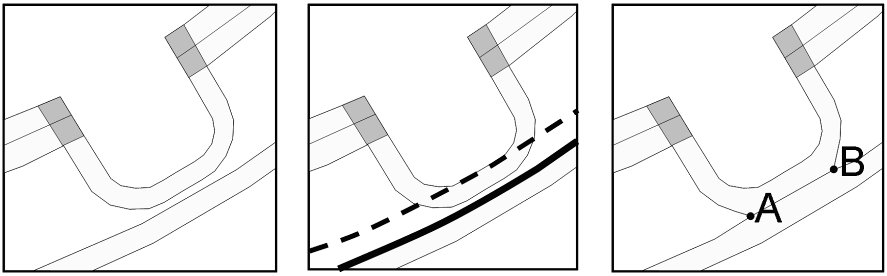


(a) Desired front advancement (b) Pillow added (c) Row smoothing

(d) Pillow added (e) Row smoothing (f) Merge performed

Figure 5. Merging partial tube proximity problems with pillowing

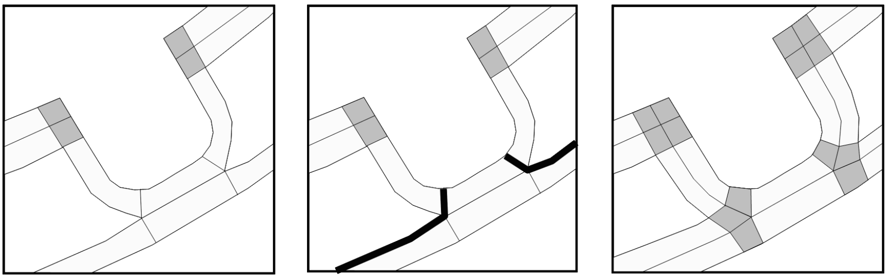
Merge cases can also occur in the unmeshed void as illustrated in Figure 6. The thick dark solid line in Figure 6b is the front to be advanced. The dashed dark line shows the desired advancement which is not possible because of the proximity. Figure 6c shows that the proximity has been merged out. Points A and B are the extremes of the merge and are new topological constraints on the model where nodes must be placed. They will end up being 5-valent nodes in the final mesh. In order to maintain mesh topology consistency, these points must be propagated back to the boundary through the connecting tubes as shown in Figure 6d. The propagation of constraints to the boundary through connecting tubes is called "cutbacks." Cutbacks split the connecting tube into two or more connecting tubes. The original front that had the proximity problem then needs to be updated as shown in Figure 6e. Additional fronts can then be advanced around the proximity as shown in Figure 6f.



(a) Proximity case in unmeshed void

(b) Desired front advancement

(c) Merged model, A & B are new model constraints



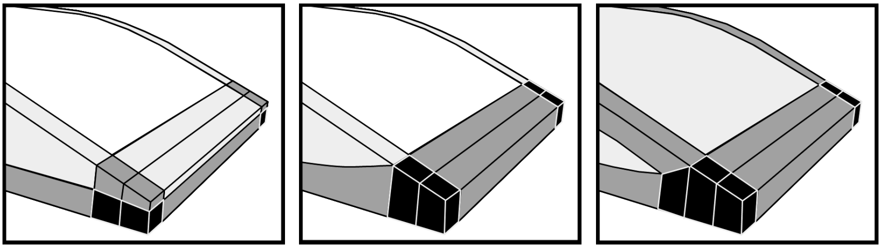
(d) Cutbacks added

(e) New fronts for advancement

(f) Several additional fronts are advanced

Figure 6. Merging example for unmeshed void proximity

Figure 7 illustrates a merge case in the connecting tubes during Unconstrained Plastering. The unmeshed void is drawn in white, the connecting tubes in light gray, the connecting webs in dark gray, and the final elements in black. Proximity in the connecting tube on the right stops additional sheets from advancing from both the top and bottom of the model. The proximity is resolved by merging out the connecting tube and the corresponding connecting webs in Figure 7b. An additional front from the top of the model can now be advanced as illustrated in Figure 7c.



(a) 3D Merge Case (b) Merge Performed (c) Additional sheet advanced

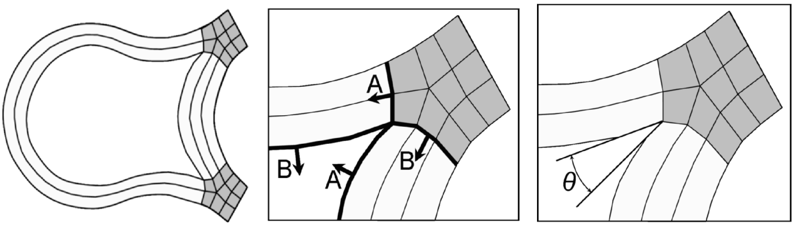
Figure 7. Unconstrained Plastering merge case

4.2 Seaming

During Unconstrained Paving and Plastering, cases requiring seaming occur where two fronts intersect. The need for seaming is based on the angle at which the fronts intersect. Figure 8a shows the model from Figure 4 after the merging was performed. Figure 8b shows a close up showing the intersection between fronts A and B. If the angle of intersection θ, is less than a specified tolerance β, then seaming is required. Seaming is performed, as shown in Figure 8d, by merging the fronts together until the θ increases above $\beta * w$, where w is a weighting factor greater than 1.0. If $w = 1.0$, then the angle will only increase to exactly the seaming tolerance. Subsequent small perturbations nearby could cause the angle to decrease which would cause seaming to be required again. If a larger w is used, such as 1.1, then θ will have increased enough over β that nearby changes will be less likely to drop θ enough to require additional seaming.

Figure 8d illustrates the completion of the seam. Point C is the point where the seaming stops. Point C is a new constraint on the model where a five-valent node will be located. Similar to point A and B in Figure 6c, point C must be propagated back to the boundary with cutbacks. Figure 8e shows the addition of the cutbacks and the modifications to fronts A and B. Figure 8f shows the model after fronts A and B have both been advanced one additional row.

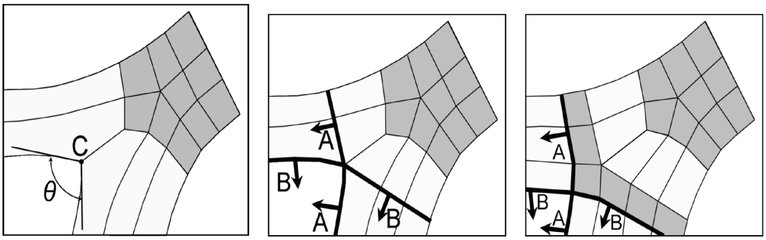
The highlighted region of Figure 8g illustrates that cutbacks in seaming often create a set of connecting tubes which are no longer connected to an unmeshed void. These connecting tubes are like all other connecting tubes in that they still have one degree of freedom remaining for Unconstrained



(a) Example surface

(b) Check for seaming at intersection of fronts

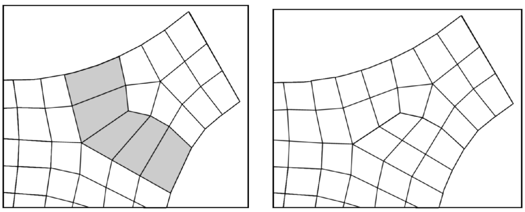
(c) Angle of intersection determines if seaming is required



(d) Seaming is performed until angle increases

(e) Cutbacks are added and fronts updated

(f) Additional fronts are advanced



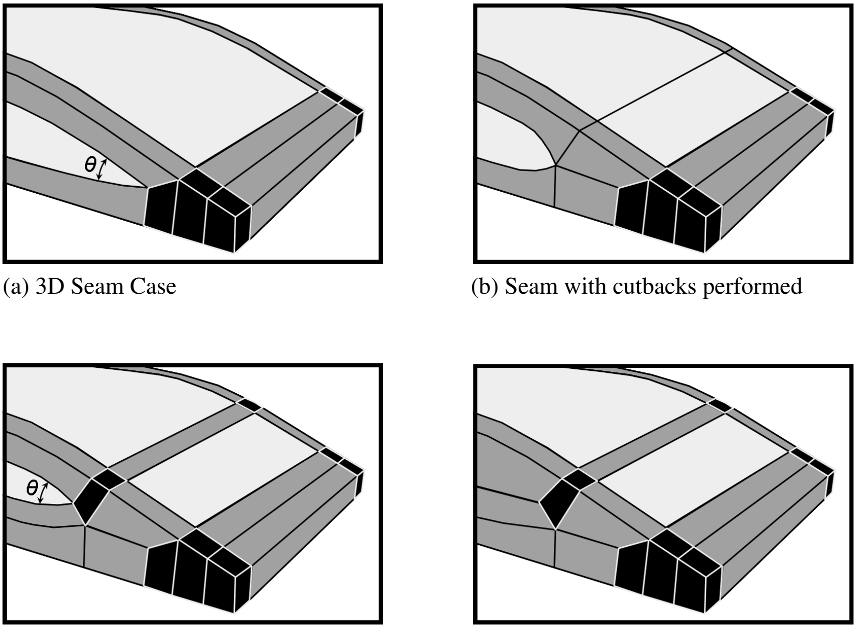
(g) Connecting tube behind seam no longer touched unmeshed void

(h) Final mesh in seaming region

Figure 8. Unconstrained Paving seaming example

Paving and two degrees of freedom for Unconstrained Plastering. For Unconstrained Paving, these connecting tubes can be diced as many times as required to get the proper element resolution as shown in Figure 8h. For Unconstrained Plastering, these connecting tubes which are no longer connected to an unmeshed void represent a partition of the solid which can be meshed with traditional paving and sweeping [16,17,18].

Figure 9 Illustrates a seaming case in Unconstrained Plastering which is a result of the merge operation performed in Figure 7. After the seaming operation is performed, an additional sheet is advanced as illustrated in Figure 9c. This sheet advancement creates an additional seaming case which is seamed as illustrated in Figure 9d. As illustrated in Figure 8g & h, connecting tubes that no longer touch an unmeshed void are often created during seaming. This is also the case in three dimensions with Unconstrained Plastering. The two light gray surfaces on the top of the model in Figure 9d represent two such connecting tubes. These two light gray surfaces can be meshed with quadrilaterals and swept [16,17,18] through the corresponding connecting tube to obtain the final mesh. By definition, the topology of connecting tubes in three dimensions has only a single source and a single target, which simplifies the sweeping process to 1-1 sweeping.



(a) 3D Seam Case (b) Seam with cutbacks performed

(c) Additional front advanced (d) Another seaming operation performed

Figure 9. Unconstrained Plastering Seaming example

5 Unconstrained Plastering Examples

Conceptually, Unconstrained Paving and Plastering can handle a wide variety of model complexity. The logic is available to handle concavities, small model angles, collisions between fronts, seaming of adjacent fronts, and assembly models. However, as is often the case, implementation of the logic lags the conceptual progression. At the time of its initial introduction [1], no solids had yet been successfully meshed with Unconstrained Plastering. Since then implementation has progressed to successfully mesh numerous models of simple complexity.

Figure 10 shows a seemingly simple model meshed with Unconstrained Plastering. However, the concavity requires the use of incomplete fronts. In addition, merging in both connecting tubes and in the unmeshed void were required. As expected, the resulting mesh topology is that of a submapped mesh [14]. The minimum scaled Jacobian in this mesh is 0.92 on a scaled from 0.0 to 1.0 where 1.0 is the perfect hexahedral element.

Unconstrained Plastering meshed the model in Figure 11 with a minimum scaled Jacobian of 0.62. The topology of the model is that of a simple brick, however, the top surface has the shape of a rhombus, which results in non-perpendicular angles leading to lower quality elements and some irregular transitioning nodes on the top and bottom surface. The mesh topology of the mesh is that of a swept mesh.

Figure 12 illustrates a model which is also relatively simple. The top surface is a curved nurb. The tapered end caused the merging and seaming cases illustrated in Figure 7 and Figure 9 to arise which resulted in the irregular nodes on the side of the volume. The irregular nodes on the top of the model are introduced by the quad mesher which meshes the top surface of a connecting tube which is then swept as described in section 4.2. The minimum scaled Jacobian in this mesh is 0.577. The mesh topology is neither mapped or swept, but a true unstructured mesh topology. Although the model topology of this model could yield a swept mesh topology, this example illustrates that Unconstrained Plastering is not restricted to such simple mesh topologies. Rather, Unconstrained Plastering is free to insert hexahedral sheets into the solid as required to adequately model the geometric complexity.

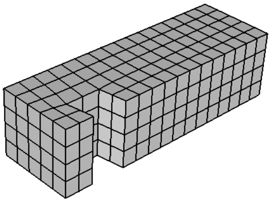


Figure 10. Example model 1

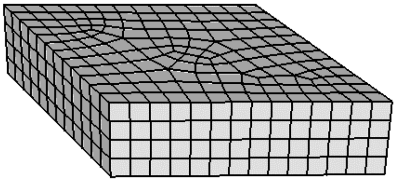


Figure 11. Example model 2

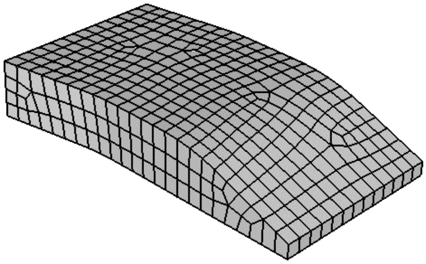
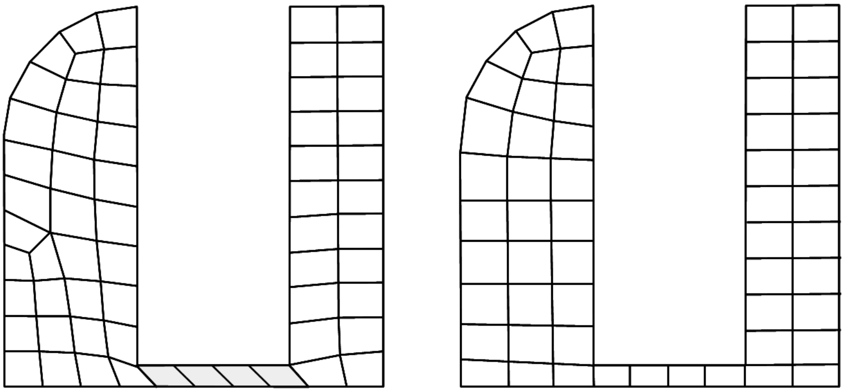


Figure 12. Example model 3

6 Future Research

As mentioned previously, implementation of Unconstrained Plastering lags the conceptual development. As a result, resources are currently allocated to implementation of the currently available logic. Current implementation resources are focused on more complicated concavities, models with very thin sections, boundary topology which intersects at extreme angles, and assembly models. It is anticipated that as implementation progresses, additional cases will be encountered which will require additional logic and operations which are not yet considered.

At the time of publication of [1], implementation on Unconstrained Plastering had begun, but implementation of Unconstrained Paving was not a priority. Priorities and resources have since been modified. As a result, implementation of Unconstrained Paving has now begun. It is anticipated that Unconstrained Paving will behave much better than traditional Paving on surfaces that have skinny regions as illustrated in Figure 13. Figure 13a shows the result from traditional Paving. Since the boundary edges are meshed apriori, the nodes are placed without considering proximity to other curves. As a result, the gray elements illustrate that skewed elements often result in thin sections such as this. Figure 13b shows what a more desirable mesh would be where the nodes opposite the thin section line up. Since Unconstrained Paving is not constrained by an apriori boundary mesh, it is anticipated that the cutback process resulting from the merging in this region will result in a mesh similar to that in Figure 13b.



(a) Traditional Paving result (b) Unconstrained Paving result

Figure 13. Mesh comparision between traditional Paving and Unconstrained Paving

7 Conclusions

The conceptual understanding of Unconstrained Paving and Plastering has progressed significantly since its initial introduction. Conceptually, many complex models can be handled. Implementation lags the conceptual development, but is the focus of current resources.

Triangular and tetrahedral meshing algorithms are accompanied by long accepted mathematical proofs and theorems [19]. In contrast, Unconstrained Paving and Plastering, like traditional Paving [5], are quite heuristic and currently lack complete mathematical verification. Regardless, traditional Paving is accepted as a robust solution to the quadrilateral meshing problem because it has been implemented dozens of times at both academic and commercial institutions resulting in robust and efficient mesh generation software. Likewise, the current implementation of Unconstrained Plastering indicates that once a complete implementation is in place, Unconstrained Paving and Plastering may also provide efficient and robust mesh generation tools. Future advances in mathematics and mesh generation may provide the mathematical backing they currently lack.

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