VII. Smooth Surfaces

24 Curvature
25 Vector space of circles
26 Envelopes and skin
27 Mixed complex

This chapter introduces differential notions used in the study of curves and surfaces. We can view these as part of a simplifying framework that attempts to understand the chaotic world of the discrete in the limit, when the large numbers approach differential structures. Section 24 talks in general about curvature of curves and surfaces. The other three sections are more specialized and construct curves and surfaces from families of circles and spheres. All results are independent of dimension and apply to $(d-1)$-dimensional surfaces in $\mathbb{R}^d$. To simplify language we mostly talk about circles and curves in $\mathbb{R}^2$. Section 25 studies the vector space defined by circles in the plane. Section 26 uses infinite families of circles to construct smooth curves by taking envelopes. Section 27 uses a mix of Voronoi and Delaunay complexes to decompose these envelopes into pieces of algebraic degree two.