CS263. Homework Assignment 6
(Solutions due March 14)

March 21, 2007

Exercise 1: Show that the execution always terminates in the call-by-value simply-typed λ-calculus. You need to consider only variables, integer constants, addition, abstraction and application. If you use induction, state precisely on what do you induct.

Hint: You will run into difficulties with the evaluation of an application, because even though you will get by induction that e_1 and e_2 both terminate, you cannot show that the application terminates. The solution is to strengthen the induction hypothesis to say not only that the evaluation terminates, but that it terminates with a value that has certain properties (namely, those that allow you to prove the application case).

Exercise 2: What is the type of the combinators K and S in the polymorphic lambda calculus F_2? Argue that the combinator D does not have a type in F_2.

Exercise 3: What would be a good subtyping rule for continuations? That is, under what conditions it is sound to say that $T_1\ \texttt{cont} \leq T_2\ \texttt{cont}$.

Exercise 4: Consider the following variant of the simply-typed λ-calculus with subtyping:

$$e ::= x \mid e_1 e_2 \mid \lambda x.e$$

Note that the abstraction does not include the type of the formal. The typing rule for abstraction is as follows:

$$\frac{\Gamma, x : \tau_1 \vdash e : \tau_2}{\Gamma \vdash \lambda x.e : \tau_1 \rightarrow \tau_2}$$

There is also a subtyping relation on types, and the rule of subsumption.

Consider now an alternate set of typing rules obtained from the usual one by removing the subsumption rule and adding instead the following restricted form of subsumption:

$$\frac{\Gamma(x) = \tau \quad \tau \leq \tau'}{\Gamma \vdash_0 x : \tau'}$$

(that is, subsumption is applied only on variables).

Let us write $\Gamma \vdash_0 e : \tau$ when expression e can be shown to have type τ in this modified type system. Prove the following theorem:

Theorem 1 *For all* e *and* τ*, if* $\cdot \vdash e : \tau$ *then* $\cdot \vdash_0 e : \tau$

Make sure you state precisely the induction hypothesis.