

Mock midterm test.

1. Is the number $\sqrt{2 + \sqrt{2}}$ rational?
3. Are the following subsets of \mathbb{C} (identified with \mathbb{R}^2) (a) open (b) closed (c) perfect (d) connected (e) bounded, with respect to the standard metric $|\cdot|$ on \mathbb{C} ?
 1. $\{z \in \mathbb{Z} : |z^2 - 1| < 1\}$
 2. $\{m + in : m, n \in \mathbb{Z}\}$
 3. $\{1/m + i/n : m, n \in \mathbb{N}\}$
4. A metric space is called *separable* if it contains a countable dense set. Show that \mathbb{R}^k is separable for all $k \in \mathbb{N}$.
5. Determine all the limit points, \liminf and \limsup of the sequence

$$\left(\frac{(-1)^n}{1 + \sin(\pi n/2) + \sin(3\pi n/2)} \right)_{n \in \mathbb{N}}.$$

6. Determine for which $z \in \mathbb{C}$ the power series

$$\sum_{n=0}^{\infty} \frac{z^n}{\sqrt{1+n}}$$

converges.