MATH 110, mock midterm test.

Name
Student ID #

All the necessary work to justify an answer and all the necessary steps of a proof must be shown clearly to obtain full credit. Partial credit **may** be given but only for significant progress towards a solution. Show all relevant work in logical sequence and indicate all answers clearly. Cross out all work you do not wish considered. Books and notes are allowed. Calculators, computers, cell phones, pagers and similar devices are not allowed during the test.

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1. Consider the vector space $P(\mathbb{R})$ and the subsets V consisting of those vectors (polynomials) f for which:

(a) f has degree 3,

(b) $2f(0) = f(1)$,

(c) $f(t) \geq 0$ whenever $t \geq 0$,

(d) $f(t) = f(1-t)$ for all t.

In which of these cases is V a subspace of $P(\mathbb{R})$?

2.

$$\text{Let } A = \begin{bmatrix} 0 & 2 & 3 \\ -1 & 3 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} -2 & 0 \\ 6 & 4 \\ -4 & 6 \end{bmatrix}, \quad v = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}, \quad w = \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}.$$

Do the products Aw, $B^t v^t$, vAw exist? Evaluate those that do. Is the set $\{A, B^t\}$ linearly independent?

3.

$$\text{Let } A = \begin{bmatrix} 0 & 2 \\ 2 & -2 \end{bmatrix}, \quad \beta = \{ \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \}.$$

Find the representation $[L_A]_\beta$, the dual basis β^*, and the matrix $[(L_A)^t]_{\beta^*}$.

4. Let

$$A \; : \; P_n(\mathbb{R}) \to P_n(\mathbb{R}) \quad : \; (Af)(t) := f(t+1).$$

Prove that

$$A = I + \frac{D}{1!} + \frac{D^2}{2!} + \cdots + \frac{D^n}{n!},$$

where D is the differentiation operator on $P_n(\mathbb{R})$.

5. Let $m < n$ and let $\mathbf{f}_1, \ldots, \mathbf{f}_m$ be linear functionals on an n-dimensional space V. Prove that there exists a nonzero vector $x \in V$ such that $\mathbf{f}_j x = 0$ for all $j = 1, \ldots, m$. What does this result say about solutions of linear equations?

6. Reduce the matrix

$$\begin{bmatrix} 1 & -1 & 4 & 3 & -2 & -2 \\ 0 & 2 & 0 & 1 & 1 & 3 \\ -1 & 3 & -4 & -2 & 3 & 5 \end{bmatrix}$$

to its reduced row echelon form. Show all steps.