

Math 121A. Final exam. Dec 16th, 2004.

Name

Student ID #

All the necessary work to justify an answer and all the necessary steps of a proof must be shown clearly to obtain full credit. Partial credit **may** be given but only for significant progress towards a solution. Show all relevant work in logical sequence and indicate all answers clearly. Cross out all work you do not wish considered. This may increase your score.

1. (5 pts.) Find an orthogonal similarity transformation that reduces the matrix

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

to a diagonal form.

2. (8 pts.) Find all zeros and all singularities of the function

$$\frac{\tan z}{z},$$

classify the singularities, indicate the order of each zero and each pole.

3. (10 pts.) Find a two-parameter family of solutions to the equation

$$y'' - xy = x^2$$

as power series in x .

4. (10 pts.) Let f be a twice differentiable real-valued function on $[0, 2\pi]$, with $f(2\pi) = f(0)$ and $\int_0^{2\pi} f(x) dx = 0$. Show that

$$\int_0^{2\pi} (f(x))^2 dx \leq \int_0^{2\pi} (f'(x))^2 dx.$$

5. (10 pts.) Find the Laplace transform of

$$f(t) = \int_0^t \left(e^{-2\tau} \sin^2 \tau + \frac{\sin \tau}{\tau} \right) d\tau.$$

6. (7 pts.) Express the distribution $(e^{2x} + x^2)\delta''(x)$ as a constant-coefficient linear combination of $\delta(x)$ and its derivatives.

7. (10 pts.) Find the Fourier transform of

$$f(x) = \frac{1}{\cosh ax}, \quad a > 0.$$