

MOCK TEST # 1

1. Find all eigenvalues and eigenvectors of the matrix

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}.$$

(You do not have to normalize eigenvectors.)

2. Show that the vector field

$$\vec{\mathbf{u}} := \begin{bmatrix} x \\ y \\ -2z \end{bmatrix}$$

has zero curl, and find a scalar function ϕ such that

$$\vec{\mathbf{u}} = \vec{\nabla}\phi.$$

3. Determine whether the function $f(x + iy) := xy^2 + i(x + y)^3$ is analytic in \mathbb{C} .
4. Evaluate the integral

$$\int_0^{2\pi} \frac{1 + \cos \theta}{2 - \sin \theta} d\theta.$$