Answers to homework assignment I

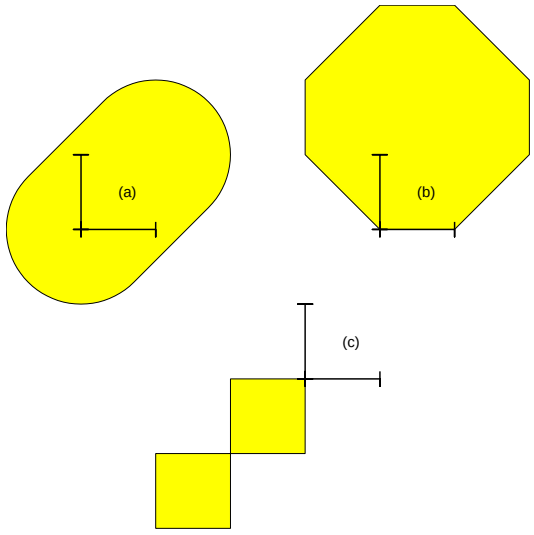
I.5 $A(nx) = A(\sum_{j=1}^n x) = \sum_{j=1}^n (Ax) = n(Ax)$, hence $(m/n)Ax = (1/n)A(n(mx/n)) = A((m/n)x)$.

I.7 While it is necessary, it is not sufficient to verify that $\operatorname{ran} A$ is closed under vector addition and scalar multiplication since $\operatorname{ran} A$ is not, offhand, a subset of a ls. On the other hand, there is no call for *defining* the vector operations on $\operatorname{ran} A$ since they are already defined on all of U. Verification is needed and is straightforward. E.g., $A0$ can serve as the neutral element since, for all $Ax \in \operatorname{ran} A$, $Ax + A0 = A(x+0) = Ax$, while, for every $Ax \in \operatorname{ran} A$, $A(-x) = -Ax$ can serve as the inverse since $Ax + A(-x) = A(x-x) = A0$. Etc.

I.9 Let $f(t-\varepsilon \,..\, t+\varepsilon) := \{f(s) : |s-t| < \varepsilon\}$. $\lambda_t : f \mapsto \operatorname{diam} f(t-\varepsilon \,..\, t+\varepsilon)$, with $\operatorname{diam} M := \sup M - \inf M$, is well defined as extended real, and is subadditive and absolutely homogeneous (since both sup and $-\inf$ are subadditive and positive homogeneous), and is monotone in ε, hence $\lim_{\varepsilon \to 0} \lambda_t(f)$ is well defined, and is an extended seminorm, while f is continuous at t iff $\lambda_t f = 0$.

If you prefer just one seminorm, use $\lambda : f \mapsto \sup_t \lambda_t f$.

I.11 (a) the cylinder spanned by two disks, radius 1, center 0 and center (1,1) ; (b) the octahedron with vertices (0,0), (1,0), (2,1), (2,2), (1,3), (0,3), (-1,2), (-1,1); (c) union of $[0\,..\,1]^2$ shifted by (-1,-1) with that shifted by (-2,-2).



I.12 Unless $X = Y$, the elements of X' have a different domain than the elements of Y', hence are not directly comparable. (However, it is true that every $\lambda \in Y'$ gives rise to an element $\lambda|_X$ of X' and that the resulting map $Y' \to X'$ is onto; see H.P.(18).)

I.13 $AVa = A(\sum_i v_i a(j)) = \sum_j (Av_j)a(j) = [Av_1, \ldots, Av_n]a$, i.e., the two maps agree on their common domain.