CS 287, Fall 2011 Problem Set #3 LQ Control, Nonlinear Optimization for Optimal Control, Motion Planning

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Deliverable: (1) Reasonable number of pages write-up in pdf format (2) zip file with your source code and clear "main" file for each programming related question. Due date/time: Thursday December 1st, 23:59pm. Email to pabbeel@berkeley.edu.

Please refer to the class webpage for the homework policy.

Various starter files are provided on the course website.

When making your write-up, make sure to answer all questions, and include and discuss plots (and if helpful, snippets of code) which are helpful in demonstrating that your system works and in answering the questions.

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1. LQR for Stabilization

(a) **LQR for a Linear System.** In this question you will implement and evaluate LQR for a linear system. See p1_a_starter.m for detailed instructions.

(b) **LQR-based Stablization for a Nonlinear System, Cartpole.** In this question you will implement and evaluate the application of LQR to stablization of a nonlinear system. See p1_b_starter.m for detailed instructions.

(c) **(Optional / Extra Credit) LQR-based Stabilization of a Helicopter in Hover.** In this question you will implement and evaluate the application of LQR to stablization of a helicopter in hover. See p1_c_starter.m for detailed instructions.

2. LQ-based Trajectory Stabilization

(a) **LQ-based Trajectory Stabilization for a Time-Varying Linear System.** In this question you will implement and evaluate an LQ control design for trajectory stabilization. See p2_a_starter.m for detailed instructions.

(b) **LQ-based Trajectory Stabilization for a Nonlinear System, Cartpole.** In this question you will implement and evaluate the application of LQ based control design for trajectory stabilization of a nonlinear system. See p2_b_starter.m for detailed instructions.

3. Nonlinear Optimization to Find Optimal Open-Loop Trajectories.

(a) **Convex Optimization to Find Optimal Open-Loop Trajectory for a Time-Varying Linear System.** In this question you will implement and evaluate optimization-based generation of an open-loop trajectory. See p3_a_starter.m for detailed instructions.

(b) **Sequential Convex Optimization to Find Optimal Open-Loop Trajectory for a Nonlinear System, Cartpole.** In this question you will implement and evaluate sequential convex optimization based generation of an open-loop trajectory for a nonlinear system. See p3_b_starter.m for detailed instructions.

4. Motion Planning with RRT

(a) **Implementation of RRT in the Absence of Obstacles.** In this question you will implement an RRT for a two-link, two-joint arm. No obstacles are present See p4_a_starter.m for detailed instructions.

(b) **Implementation of RRT in the Absence of Obstacles.** In this question you will implement an RRT for a two-link, two-joint arm. An axis-aligned rectangular obstacle is present See p4_b_starter.m for detailed instructions.

5. LQR in the Presence of Multiplicative Noise

Consider the following optimal control problem:

$$\begin{array}{ll} \min_{x,u} & \mathrm{E}[x_T^\top Q x_T] \\ \text{s.t.} & x_{t+1} = Ax_t + (B + W_t)u_t, \quad \forall t = 0, 1, 2, \ldots, T-1 \end{array}$$

Here $Q \in \mathbb{R}^{n_X \times n_X}, A \in \mathbb{R}^{n_X \times n_X}, B \in \mathbb{R}^{n_X \times n_U}$ are given and fixed. $W_t \in \mathbb{R}^{n_X \times n_U}, t = 0, 1, \ldots, T-1$ are independent random matrices with $\mathrm{E}[W_t] = 0$, and $\mathrm{E}[W_t W_t^\top] = \Sigma_w$.

Find an LQR-like sequence of matrix updates that computes the optimal cost-to-go at all times and the optimal feedback controller at all times.

Hint: there are two complications here: (i) The randomness, and hence the objective involves an expectation. (ii) The noise is multiplicative. If having trouble tackling this problem, first try to solve

$$\begin{array}{ll} \min_{x,u} & \mathrm{E}[\sum_{t=0}^{T-1} x_t^\top Q x_t + u_t^\top R u_t] + \mathrm{E}[x_T^\top Q x_T] \\ \text{s.t.} & x_{t+1} = Ax_t + Bu_t + w_t, \quad \forall t = 0, 1, 2, \ldots, T-1 \end{array}$$

with w_t independent random vectors with $\mathrm{E}[w_t] = 0$, and $\mathrm{E} w_t w_t^\top = \Sigma_w$. As documented in the slides, the solution to this problem has the same feedback controller (though a somewhat higher cost-to-go).

Congratulations!!!!!

You have by now gained a solid understanding of particle filters, Kalman filters, EM, optimal control, and motion planning. And ... you are done with problem sets!!!