Grasping

Pieter Abbeel
UC Berkeley EECS

Many figures and equations taken from:
Murray, Li, Sastry, A Mathematical Introduction to Robotic Manipulation, Chapter 5

Outline

- Contacts
  - Frictionless
  - Friction
  - Soft finger
- Grasps
- Grasp quality: force closure
- Contact point selection (John Schulman, ISRR 2011)

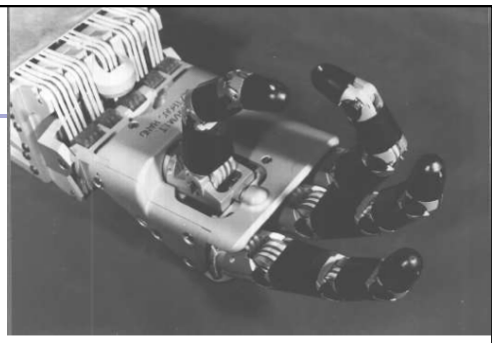


Figure 5.1: The Utah/MIT hand. (Photo courtesy of Sarcos, Inc.)

Frictionless Point Contact

- Forces can only be applied in direction normal to the surface of the object
- Applied wrench $F_{c_i} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} f_{c_i} \quad f_{c_i} \geq 0$

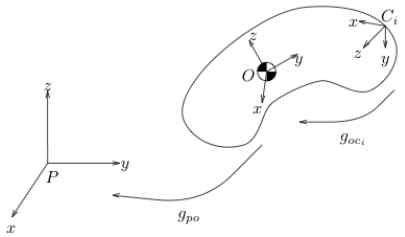


Figure 5.2: Coordinate frames for contact and object forces.

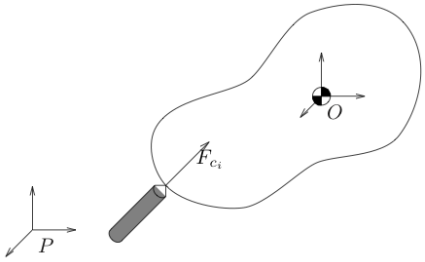


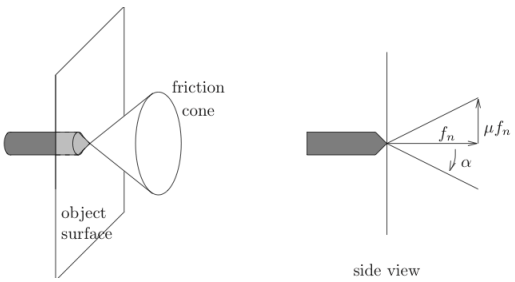
Figure 5.3: Frictionless point contact.

Coulomb Friction Model

- Let f^t denote the tangential force and f^n denote the magnitude of the normal force, then Coulomb's law states that in the static case:

$$|f^t| \leq \mu f^n$$

- μ is the (static) coefficient of friction



Coulomb Friction Model

Table 5.1: Static friction coefficients for some common materials. (Source: CRC Handbook of Chemistry and Physics)

Steel on steel	0.58	Wood on wood	0.25-0.5
Polyethylene on steel	0.3–0.35	Wood on metals	0.2-0.6
Polyethylene on self	0.5	Wood on leather	0.3–0.4
Rubber on solids	1–4	Leather on metal	0.6

Point Contact with Friction

- Space of possible applied wrenches:

$$F_{c_i} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} f_{c_i} \qquad f_{c_i} \in FC_{c_i},$$

$$FC_{c_i} = \{f \in \mathbb{R}^3 : \sqrt{f_1^2 + f_2^2} \leq \mu f_3,\ f_3 \geq 0\}.$$

Soft-Finger Contact

- Also allows for torque around the normal
- Space of possible applied wrenches:

$$F_{c_i} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} f_{c_i} \qquad f_{c_i} \in FC_{c_i}$$

and the friction cone becomes

$$FC_{c_i} = \{f \in \mathbb{R}^4 : \sqrt{f_1^2 + f_2^2} \leq \mu f_3,\ f_3 \geq 0,\ |f_4| \leq \gamma f_3\},$$

where $\gamma > 0$ is the coefficient of torsional friction.

General Contact Model

In general, we model a contact using a *wrench basis*, $B_{c_i} \in \mathbb{R}^{p \times m_i}$, and a friction cone, FC_{c_i}. In all of our examples, we chose $p = 6$, the dimension of the space of generalized forces that can be applied in $SE(3)$. Other choices are possible, the most common being $p = 3$, which is used for planar grasping. The dimension of the wrench basis, m_i, indicates the number of independent forces that can be applied by the contact. We require that FC_{c_i} satisfy the following properties:

1. FC_{c_i} is a closed subset of $\mathbb{R}^{m_i}$ with non-empty interior.
2. $f_1, f_2 \in FC_{c_i} \implies \alpha f_1 + \beta f_2 \in FC_{c_i}$ for $\alpha, \beta > 0$.

The set of allowable contact forces applied by a given contact is:

$$F_{c_i} = B_{c_i} f_{c_i} \qquad f_{c_i} \in FC_{c_i}. \tag{5.5}$$

Summary of Common Contact Models

Contact type	Picture	Wrench basis	FC
Frictionless point contact		$\begin{bmatrix}0\\0\\1\\0\\0\\0\end{bmatrix}$	$f_1 \geq 0$
Point contact with friction		$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\\0&0&0\\0&0&0\\0&0&0\end{bmatrix}$	$\sqrt{f_1^2+f_2^2} \leq \mu f_3$ $f_3 \geq 0$
Soft-finger		$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&0\\0&0&0&0\\0&0&0&1\end{bmatrix}$	$\sqrt{f_1^2+f_2^2} \leq \mu f_3$ $f_3 \geq 0$ $\lvert f_4 \rvert \leq \gamma f_3$

Grasp

- = set of wrenches that can be achieved

$$F_o = G_1 f_{c_1} + \cdots + G_k f_{c_k} = \begin{bmatrix} G_1 & \cdots & G_k \end{bmatrix} \begin{bmatrix} f_{c_1} \\ \vdots \\ f_{c_k} \end{bmatrix}$$

$$f_c \in FC$$

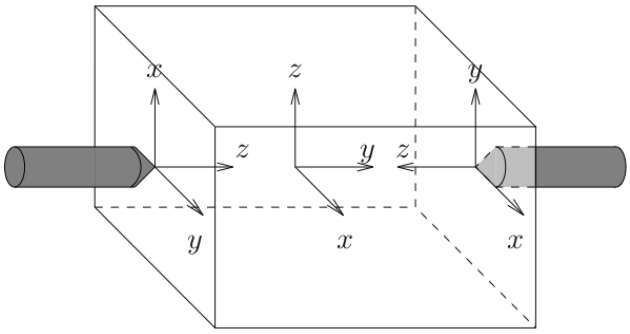
- G_i = wrench basis vectors transformed into single reference coordinate frame
- $G = [G_1 \ldots G_k]$ = grasp map

Example: Grasp Map for Frictionless Point Contacts

$$F_o = \begin{bmatrix} n_{c_1} & \cdots & n_{c_k} \\ p_{c_1} \times n_{c_1} & \cdots & p_{c_k} \times n_{c_k} \end{bmatrix} \begin{bmatrix} f_{c_1} \\ \vdots \\ f_{c_k} \end{bmatrix} = G f_c, \qquad \begin{array}{l} F_o \in \mathbb{R}^6 \\ f_{c_i} \geq 0. \end{array}$$

x = outer product

Example: Soft Finger Grasp of a Box



$$G = \left[\begin{array}{cccc|cccc} 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ \hline -r & 0 & 0 & 0 & 0 & +r & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & +r & 0 & 0 & -r & 0 & 0 & 0 \end{array}\right]$$

$$f_c = (f_{c_1}^1, f_{c_1}^2, f_{c_1}^3, f_{c_1}^4,\ f_{c_2}^1, f_{c_2}^2, f_{c_2}^3, f_{c_2}^4) \in \mathbb{R}^8$$

$$FC = FC_{c_1} \times FC_{c_2}$$

$$FC_{c_1} = \left\{ f_c : \sqrt{(f_{c_1}^1)^2 + (f_{c_1}^2)^2} \leq \mu f_{c_1}^3,\ |f_{c_1}^4| \leq \gamma f_{c_1}^3,\ f_{c_1}^3 \geq 0 \right\}$$

$$FC_{c_2} = \left\{ f_c : \sqrt{(f_{c_2}^1)^2 + (f_{c_2}^2)^2} \leq \mu f_{c_2}^3,\ |f_{c_2}^4| \leq \gamma f_{c_2}^3,\ f_{c_2}^3 \geq 0 \right\}$$

Grasp Quality: Force Closure

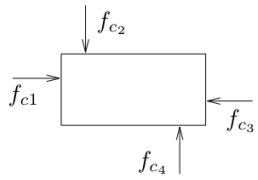
- A grasp is a force-closure grasp IF for any external wrench F_e there exist contact forces $f_c \in FC$ such that

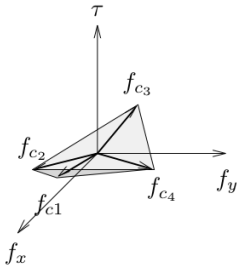
$$G f_c = -F_e$$

i.e., if able to apply sufficient force at each contact, every external wrench can be compensated for.

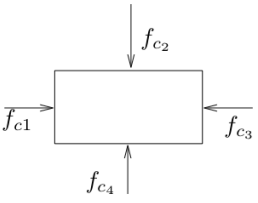
- Example external wrenches:
  - Gravity
  - Held object making contact with another object
    - By accident
    - To perform a task (e.g., insertion, hammering, writing, …)

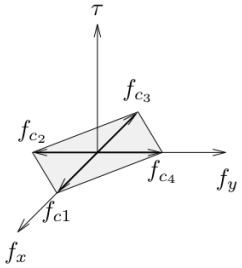
Force Closure





- Yes





- No

Detecting Force Closure

- [make drawing on board]

$$\begin{aligned} \max_{y,\gamma} \quad & \gamma \\ \text{s.t.} \quad & y^\top G_i \geq \gamma \\ & \|y\|_2 \leq 1 \end{aligned}$$

Quality Metric in Case of Force Closure

$$\min_{y:\|y\|_2 \leq 1} \max_{f \in \mathrm{FC}} y^\top G f$$