SEIF, EnKF, EKF SLAM

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Information Filter

- From an analytical point of view == Kalman filter
- Difference: keep track of the inverse covariance rather than the covariance matrix [matter of some linear algebra manipulations to get into this form]
- Why interesting?
  - Inverse covariance matrix = 0 is easier to work with than covariance matrix = infinity (case of complete uncertainty)
  - Inverse covariance matrix is often sparser than the covariance matrix --- for the "insiders": inverse covariance matrix entry (i,j) = 0 if X_i is conditionally independent of X_j given some set $\{X_k, X_l, \ldots\}$
  - Downside: when extended to non-linear setting, need to solve a linear system to find the mean (around which one can then linearize)
  - See Probabilistic Robotics pp. 78-79 for more in-depth pros/cons and Probabilistic Robotics Chapter 12 for its relevance to SLAM (then often referred to as the "sparse extended information filter (SEIF)")

Ensemble Kalman filter (enKF)

- Represent the Gaussian distribution by samples
  - Empirically: even 40 samples can track the atmospheric state with high accuracy with enKF
  - <-> UKF: 2 * n sigma-points, $n = 10^6$ + then still forms covariance matrices for updates
- The intellectual innovation:
  - Transforming the Kalman filter updates into updates which can be computed based upon samples and which produce samples while never explicitly representing the covariance matrix

KF | enKF

Keep track of μ, Σ

Prediction:

$$\bar{\mu}_t = A_t \mu_{t-1} + B_t u_t$$
$$\bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t$$

Correction:

$$K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$$
$$\mu_t = \bar{\mu}_t + K_t (z_t - C_t \bar{\mu}_t)$$
$$\Sigma_t = (I - K_t C_t) \bar{\Sigma}_t$$

Return μ_t, Σ_t

Keep track of ensemble $[x_1, \ldots, x_N]$

Can update the ensemble by simply propagating through the dynamics model + adding sampled noise

?

enKF correction step

- KF:

$$K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$$
$$\mu_t = \bar{\mu}_t + K_t (z_t - C_t \bar{\mu}_t)$$
$$\Sigma_t = (I - K_t C_t) \bar{\Sigma}_t$$

- Current ensemble $X = [x_1, \ldots, x_N]$
- Build observations matrix $Z = [z_t+v_1 \; \ldots \; z_t+v_N]$ where v_i are sampled according to the observation noise model
- Then the columns of

  $X + K_t(Z - C_t X)$

  form a set of random samples from the posterior

Note: when computing K_t, leave Σ_t in the format

$\Sigma_t = [x_1-\mu_t \; \ldots \; x_N-\mu_t] \, [x_1-\mu_t \; \ldots \; x_N-\mu_t]^T$

How about C?

- Indeed, would be expensive to build up C.
- However: careful inspection shows that C only appears as in:
  - C X
  - $C \Sigma C^T = C X X^T C^T$
- → can simply compute h(x) for all columns x of X and compute the empirical covariance matrices required
- [details left as exercise]

Are the columns of $X + K_t(Z - C_t X)$ really samples from $\mathcal{N}(\mu_t, \Sigma_t)$?

one column: $y^{(i)} = x^{(i)} + K_t(z_t + v^{(i)} - C_t x^{(i)})$

where $x^{(i)} \sim \mathcal{N}(\bar{\mu}_t, \bar{\Sigma}_t)$ $\quad v^{(i)} \sim \mathcal{N}(0, Q_t)$

① $E[x^{(i)}] = \bar{\mu}_t + K_t(z_t + 0 - C_t \cdot \bar{\mu}_t)$

$= \bar{\mu}_t + K_t(z_t - C_t\bar{\mu}_t)$

$= \mu_t$ ✓

② $E[(y^{(i)} - Ey^{(i)})(y^{(i)} - Ey^{(i)})^T]$

$= E\left[\left((x^{(i)} + K_t(z_t + v^{(i)} - C_t x^{(i)})) - (\bar{\mu}_t + K_t(z_t - C_t\bar{\mu}_t))\right)\left(\_\_\_\right)^T\right]$

$= E\left[\left((I - K_tC_t)(x^{(i)} - \bar{\mu}_t) + K_t v^{(i)}\right)\left(\_\_\_\right)^T\right]$

$v^{(i)}$ and $x^{(i)}$ independent

$= E[(I-K_tC_t)(x^{(i)} - \bar{\mu}_t)(x^{(i)} - \bar{\mu}_t)^T(I-K_tC_t)^T]$

$+ E[K_t v^{(i)} v^{(i)T} K_t^T]$

$= (I-K_tC_t)\bar{\Sigma}_t(I-K_tC_t)^T + K_tQ_tK_t^T$

$= \bar{\Sigma}_t + K_tC_t\bar{\Sigma}_tC_t^TK_t^T - K_tC_t\bar{\Sigma}_t - \bar{\Sigma}_tC_t^TK_t^T + K_tQ_tK_t^T$

$K_t = \bar{\Sigma}_tC_t^T(C_t\bar{\Sigma}_tC_t^T + Q_t)^{-1}$

$= \bar{\Sigma}_t + \bar{\Sigma}_tC_t^TK_t^T - K_tC_t\bar{\Sigma}_t - \bar{\Sigma}_tC_t^TK_t^T$

$= \bar{\Sigma}_t - K_tC_t\bar{\Sigma}_t = \Sigma_t$ Q.E.D. ∎

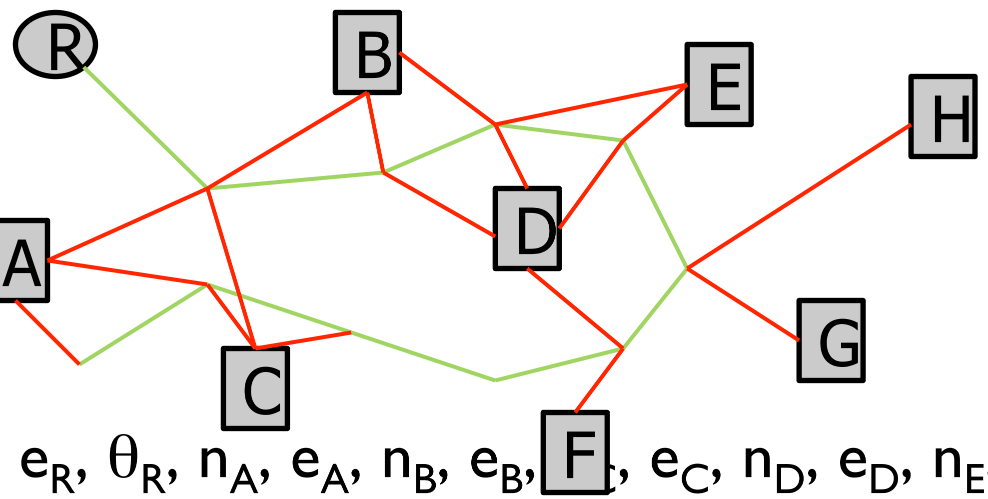
References for enKF

- Mandel, 2007 “A brief tutorial on the Ensemble Kalman Filter”
- Evensen, 2009, “The ensemble Kalman filter for combined state and parameter estimation”

KF Summary

- Kalman filter exact under linear Gaussian assumptions
- Extension to non-linear setting:
  - Extended Kalman filter
  - Unscented Kalman filter
- Extension to extremely large scale settings:
  - Ensemble Kalman filter
  - Sparse Information filter
- Main limitation: restricted to unimodal / Gaussian looking distributions
- Can alleviate by running multiple XKFs + keeping track of the likelihood; but this is still limited in terms of representational power unless we allow a very large number of them

EKF/UKF SLAM



- State: $(n_R, e_R, \theta_R, n_A, e_A, n_B, e_B, n_C, e_C, n_D, e_D, n_E, e_E, n_F, e_F, n_G, e_G, n_H, e_H)$
  - Now map = location of landmarks (vs. gridmaps)
- Transition model:
  - Robot motion model; Landmarks stay in place

Simultaneous Localization and Mapping (SLAM)

- In practice: robot is not aware of all landmarks from the beginning
- Moreover: no use in keeping track of landmarks the robot has not received any measurements about

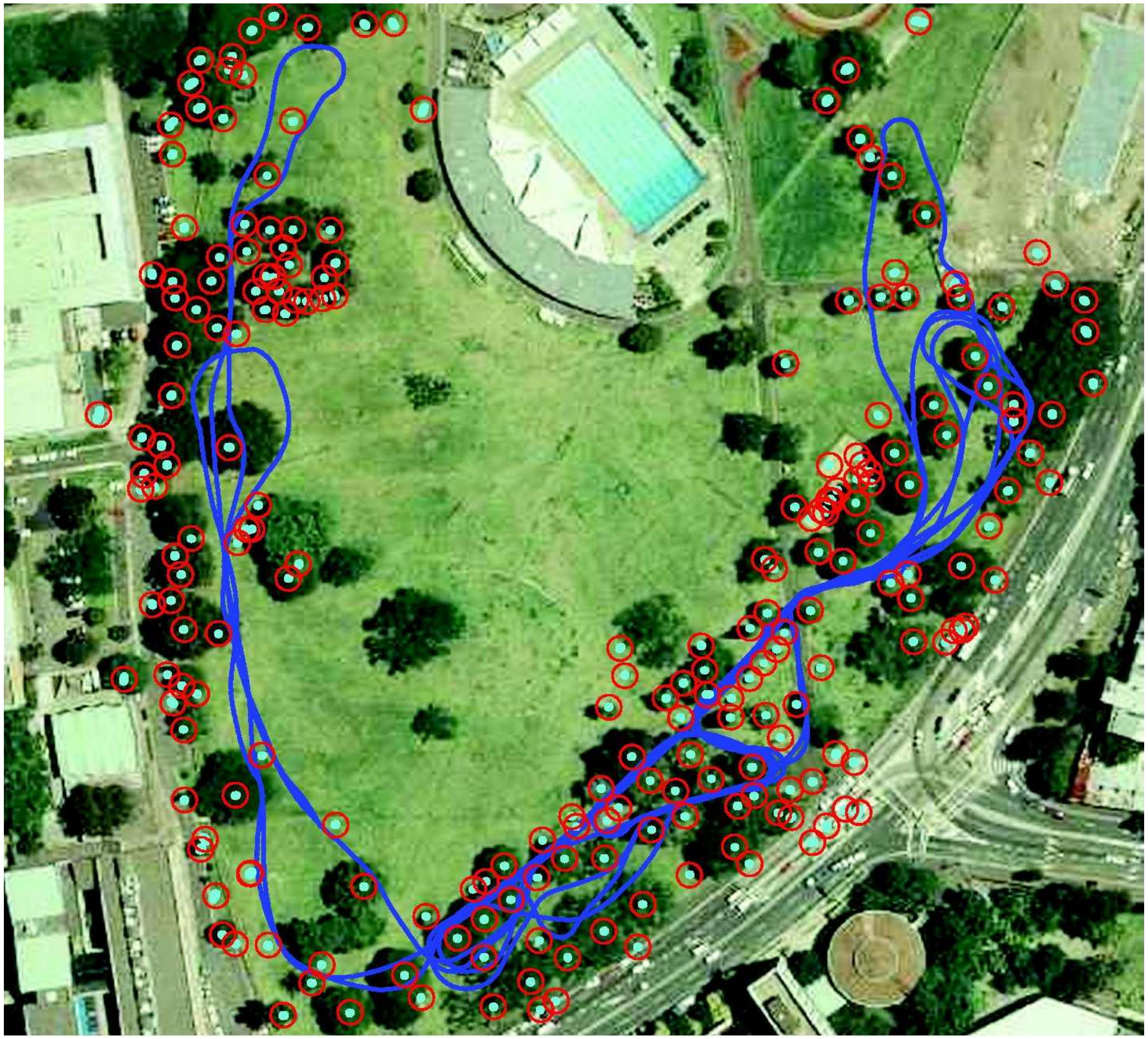
→ Incrementally grow the state when new landmarks get encountered.

Simultaneous Localization and Mapping (SLAM)

- Landmark measurement model: robot measures [x_k; y_k], the position of landmark k expressed in coordinate frame attached to the robot:
  - $h(n_R, e_R, \theta_R, n_k, e_k) = [x_k; y_k] = R(\theta) ([n_k; e_k] - [n_R; e_R])$

- Often also some odometry measurements
  - E.g., wheel encoders
  - As they measure the control input being applied, they are often incorporated directly as control inputs (why?)

Victoria Park Data Set



[courtesy by E. Nebot]

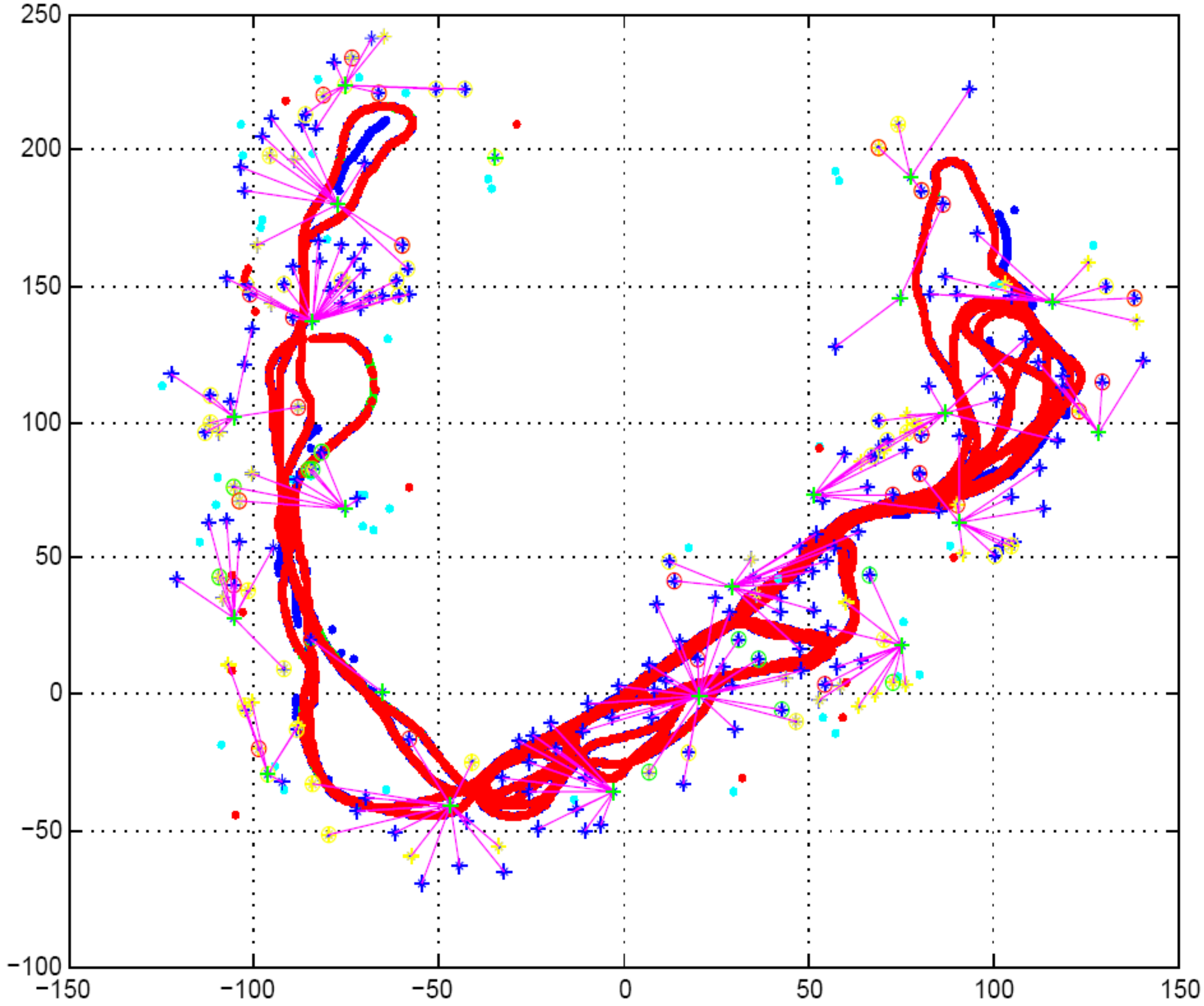
Victoria Park Data Set Vehicle

[courtesy by E. Nebot]

Data Acquisition

[courtesy by E. Nebot]

Estimated Trajectory

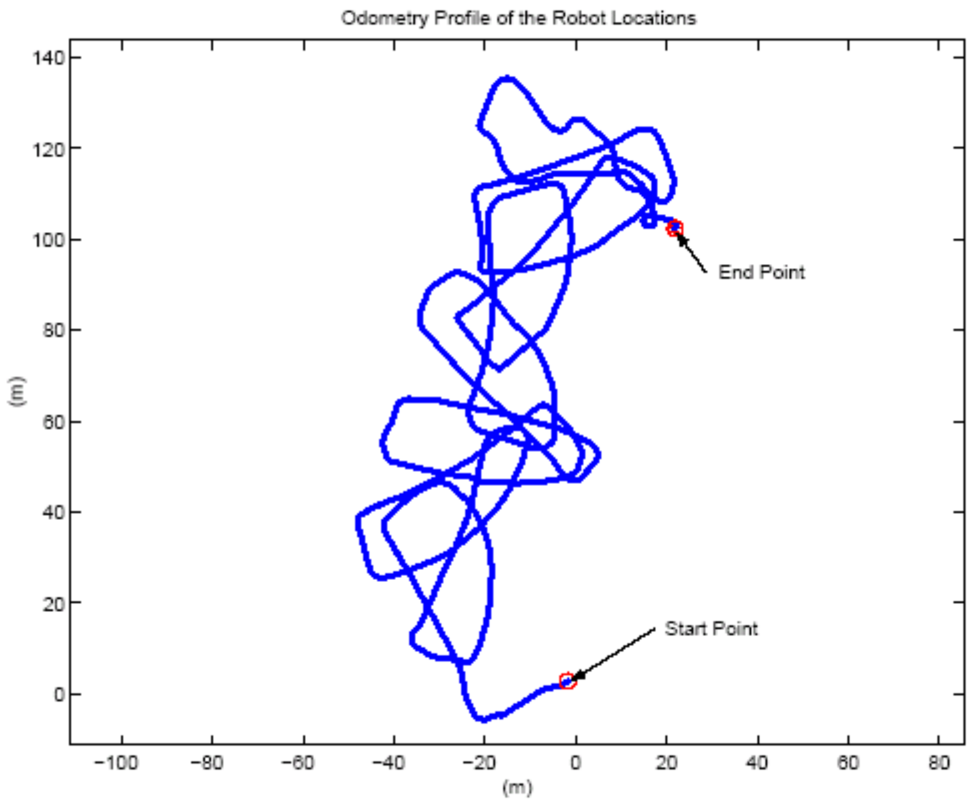


[courtesy by E. Nebot]

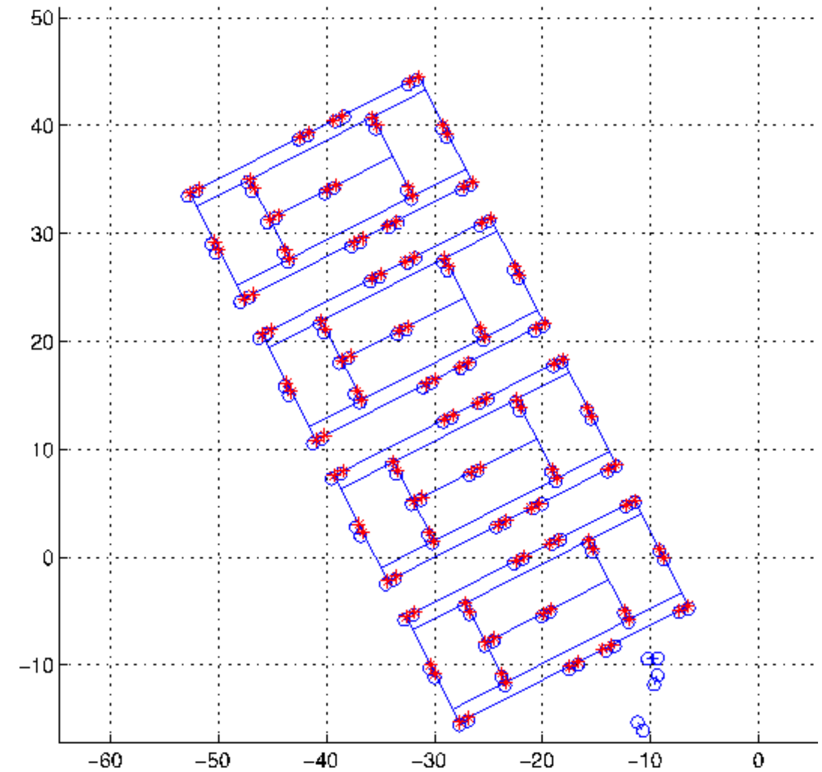
EKF SLAM Application

[courtesy by J. Leonard]

EKF SLAM Application



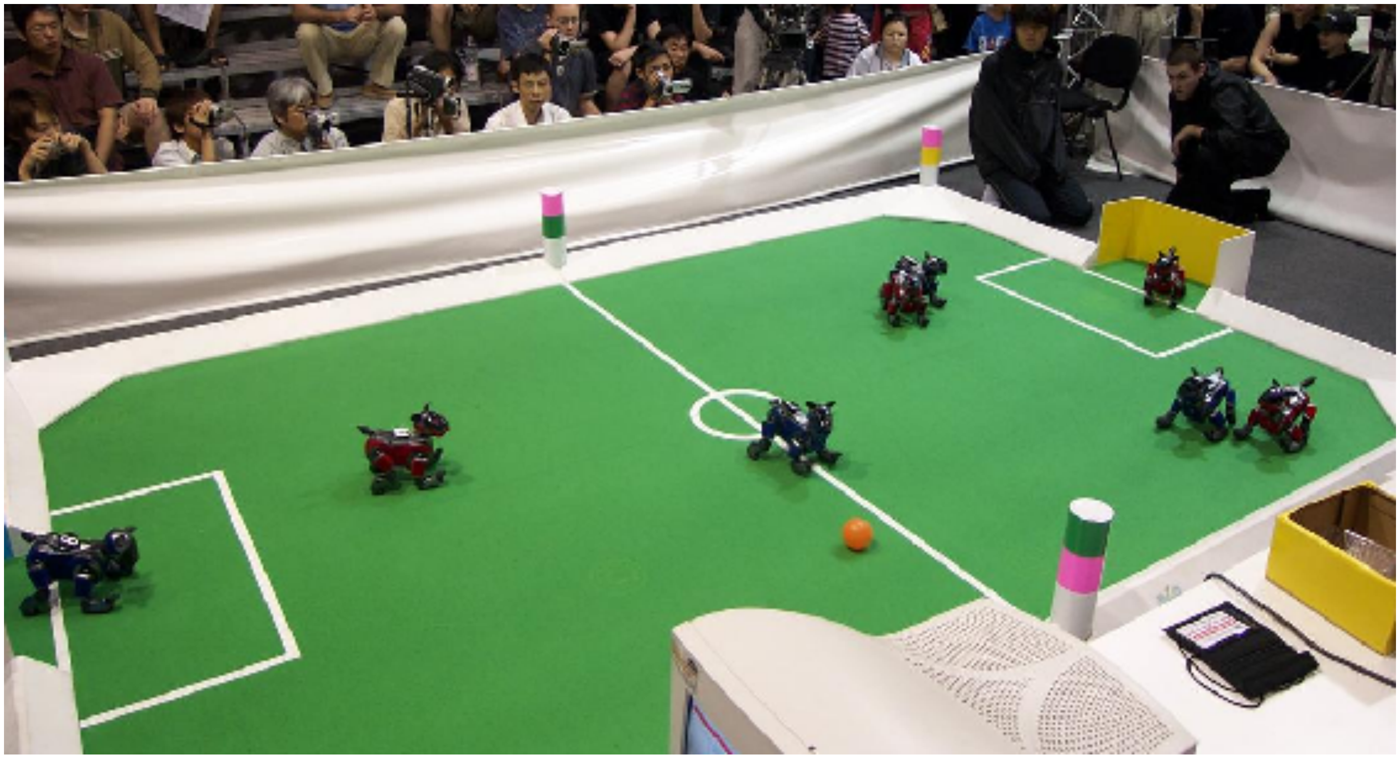
odometry



estimated trajectory

[courtesy by John Leonard]

Landmark-based Localization



EKF-SLAM: practical challenges

- Defining landmarks
  - Laser range finder: Distinct geometric features (e.g. use RANSAC to find lines, then use corners as features)
  - Camera: “interest point detectors”, textures, color, …
- Often need to track multiple hypotheses
  - Data association/Correspondence problem: when seeing features that constitute a landmark --- Which landmark is it?
  - Closing the loop problem: how to know you are closing a loop?
  - → Can split off multiple EKFs whenever there is ambiguity;
  - → Keep track of the likelihood score of each EKF and discard the ones with low likelihood score
- Computational complexity with large numbers of landmarks.