Nonlinear Optimization for Estimation

Pieter Abbeel
UC Berkeley EECS

Problem Setting

- General: find most likely state sequence given observations z:

$$\max_{x,v,w} \log P(v, w)$$

$$\text{s.t.} \quad \forall t \quad x_{t+1} = f(x_t, u_t) + w_t$$

$$z_t = g(x_t) + v_t$$

- For Gaussian v, w:

$$\min_{x,v,w} \sum_t \|w_t\|_2^2 + \sum_t \|v_t\|_2^2$$

$$\text{s.t.} \quad \forall t \quad x_{t+1} = f(x_t, u_t) + w_t$$

$$z_t = g(x_t) + v_t$$

Nonlinear Optimization for Estimation

- Problem:

$$\min_{x,v,w} \sum_t \|w_t\|_2^2 + \sum_t \|v_t\|_2^2$$
$$\text{s.t.} \quad \forall t \quad x_{t+1} = f(x_t, u_t) + w_t$$
$$z_t = g(x_t) + v_t$$

- Initialize by setting some values for $x^{(0)}$, $v^{(0)}$, $w^{(0)}$

- Iterate for i=1, 2, …
  - Linearize around current $x^{(i-1)}$, $u^{(i-1)}$ → A_t, B_t, c_t, G_t, h_t
  - Solve

$$\min_{x,v,w} \sum_t \|w_t\|_2^2 + \sum_t \|v_t\|_2^2$$
$$\text{s.t.} \quad \forall t \quad x_{t+1} = A_t x_t + B_t u_t + c_t + w_t$$
$$z_t = G_t x_t + h_t + v_t$$
$$\|x_t - x_t^{(i-1)}\|_2 \leq \varepsilon$$
$$\|u_t - u_t^{(i-1)}\|_2 \leq \varepsilon$$

"Model Predictive Estimation"

- Given: $\bar{x}_0$
- For k=0, 1, 2, …, T
  - Solve

$$\min_{x,v,w} \sum_{t=0}^{k} \|w_t\|_2^2 + \sum_{t=0}^{k} \|v_t\|_2^2$$

$$\text{s.t.} \quad \forall t : 0 \leq t \leq k \quad x_{t+1} = f(x_t, u_t) + w_t$$

$$z_t = g(x_t) + v_t$$

  - Observe z_{t+1}