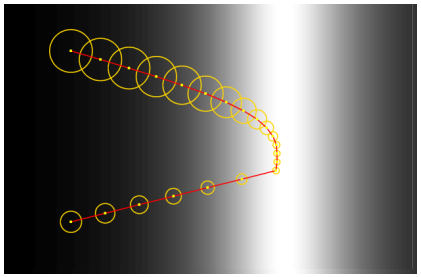


Partially Observable Markov Decision Processes (POMDPs)

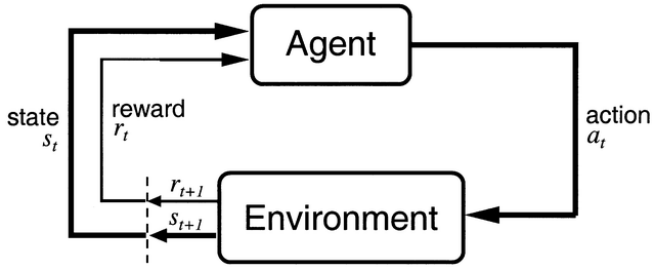
Pieter Abbeel

Outline

- Introduction to POMDPs
- Locally Optimal Solutions for POMDPs
  - Trajectory Optimization in (Gaussian) Belief Space
  - Accounting for Discontinuities in Sensing Domains
- Separation Principle

Markov Decision Process (S, A, H, T, R)

Given



- S: set of states
- A: set of actions
- H: horizon over which the agent will act
- T: S x A x S x {0,1,…,H} → [0,1] , $T_t(s,a,s') = P(s_{t+1} = s' \mid s_t = s, a_t = a)$
- R: S x A x S x {0, 1, …, H} → $\mathbb{R}$, $R_t(s,a,s')$ = reward for ($s_{t+1} = s'$, $s_t = s$, $a_t = a$)

Goal:

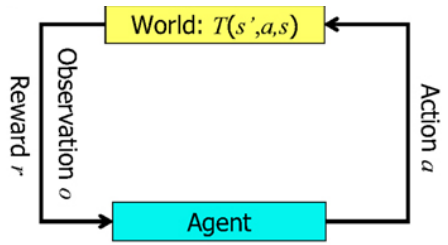
- Find π: S x {0, 1, …, H} → A that maximizes expected sum of rewards, i.e.,

$$\pi^* = \arg\max_{\pi} \mathrm{E}[\sum_{t=0}^{H} R_t(S_t, A_t, S_{t+1})|\pi]$$

POMDP – Partially Observable MDP

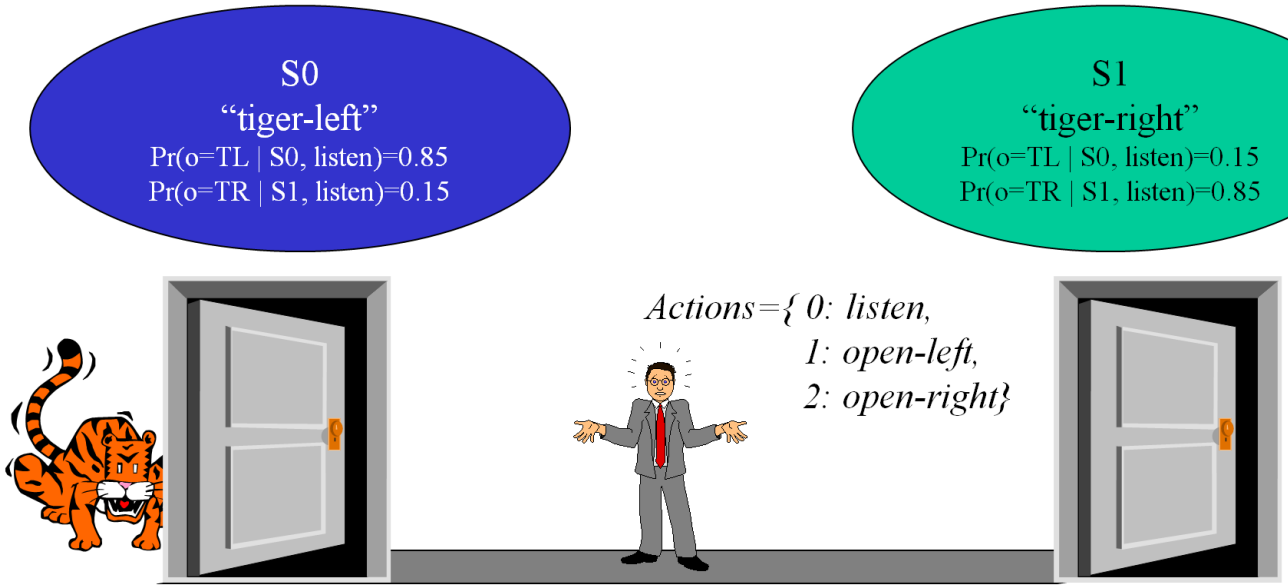
= MDP, BUT

don't get to observe the state itself, instead get sensory measurements



Now: what action to take given current probability distribution rather than given current state.

POMDPs: Tiger Example



Reward Function

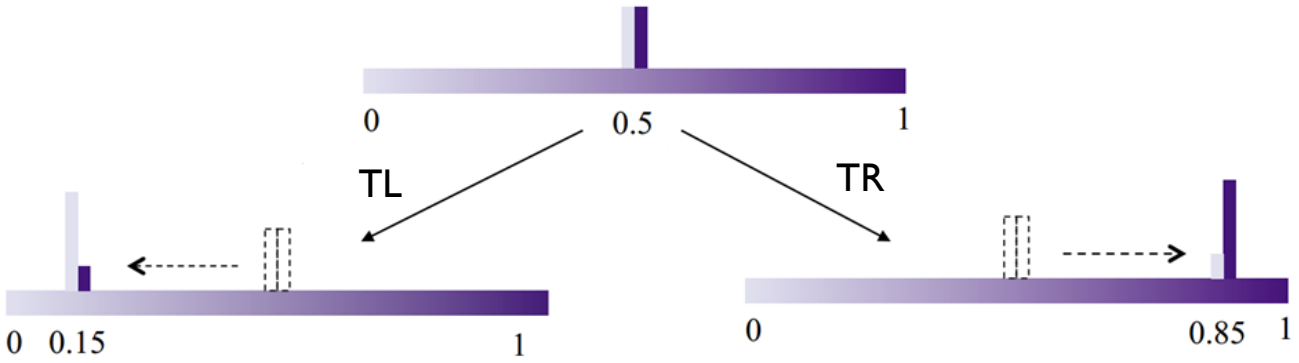
- *Penalty for wrong opening: -100*
- *Reward for correct opening: +10*
- *Cost for listening action: -1*

Observations

- *to hear the tiger on the left (TL)*
- *to hear the tiger on the right(TR)*

Belief State

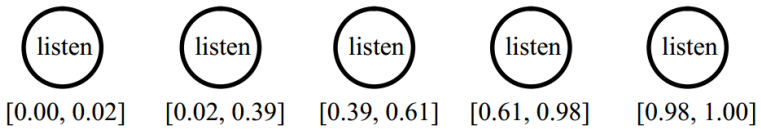
- Probability of S0 vs S1 being true underlying state
- Initial belief state: p(S0)=p(S1)=0.5
- Upon listening, the belief state should change according to the Bayesian update (filtering)



Policy – Tiger Example

- Policy π is a map from [0,1] $\rightarrow$ {listen, open-left, open-right}
- What should the policy be?
  - Roughly: listen until sure, then open
- But where are the cutoffs?

Tiger example optimal policy for $t = 1$



Tiger example optimal policy for $t = 2$

Solving POMDPs

- Canonical solution method 1: Continuous state "belief MDP"
  - Run value iteration, but now the state space is the space of probability distributions
  - → value and optimal action for every possible probability distribution
  - → will automatically trade off information gathering actions versus actions that affect the underlying state
  - Value iteration updates cannot be carried out because uncountable number of belief states – approximation

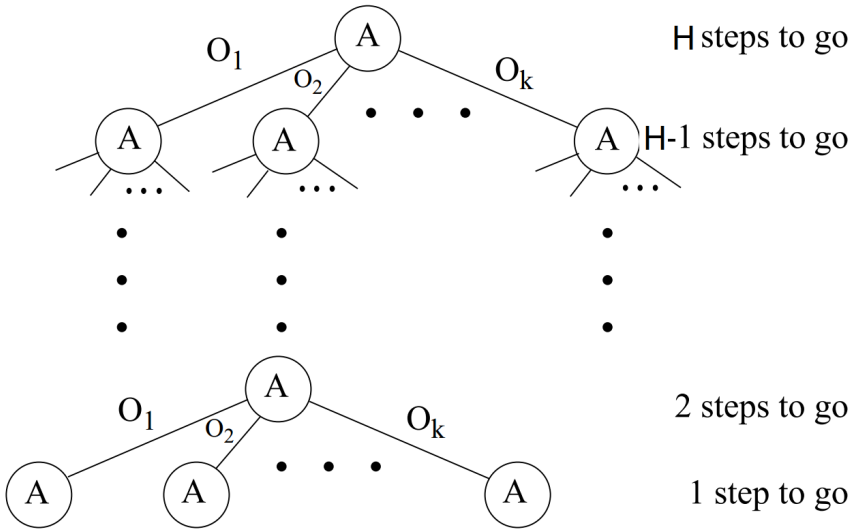
Belief State Update

- Each belief node has |A| action node successors
- Each action node has |O| belief successors
- Each (action, observation) pair (a,o) requires predict/update step
- Matrix/vector formulation:
  - b(s): vector **b** of length |S|
  - p(s'|s,a): set of |S|x|S| matrices T_a
  - p(o|s): vector **o** of length |S|
- $\mathbf{b}_a = T_a\mathbf{b}$ (predict)
- $p(o|a,b) = \mathbf{o}^T \mathbf{b}_a$ (probability of observation)
- $\mathbf{b}_{a,o} = \mathrm{diag}(\mathbf{o})\,\mathbf{b}_a / (\mathbf{o}^T \mathbf{b}_a)$ (update)

$$\equiv b_{a,o}(s') = \frac{p(o \mid s', a)\sum_{s_i \in S} p(s' \mid s_i, a)b(s_i)}{p(o \mid a, b)}$$

Solving POMDPs

- Canonical solution method 2:
  - Search over sequences of actions with limited look-ahead
  - Branching over actions and observations



Finite horizon:

$|\mathcal{A}|^{\frac{|\mathcal{O}|^H - 1}{|\mathcal{O}| - 1}}$ nodes

Solving POMDPs

- Approximate solution: becoming tractable for |S| in millions
  - α-vector point-based techniques (belief state)
  - Monte Carlo Tree Search (search over action/observation sequences from current state)
  - …Beyond scope of course…

Solving POMDPs

- Canonical solution method 3:
  - Plan in the ***MDP***
  - Probabilistic inference (filtering) to track probability distribution
  - Choose optimal action for ***MDP*** for currently most likely state

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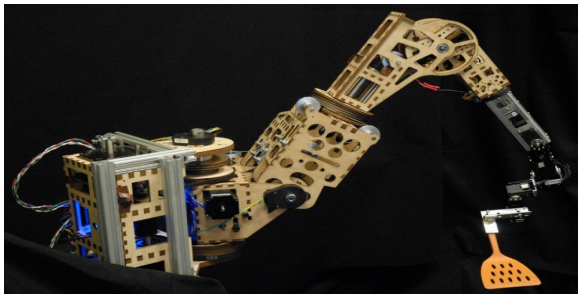
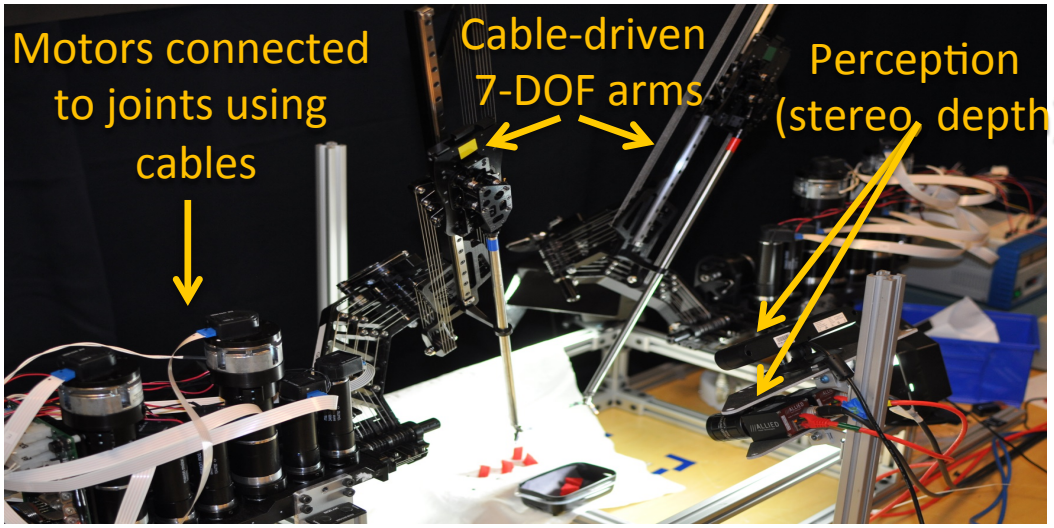
Motivation

Baxter (Rethink)



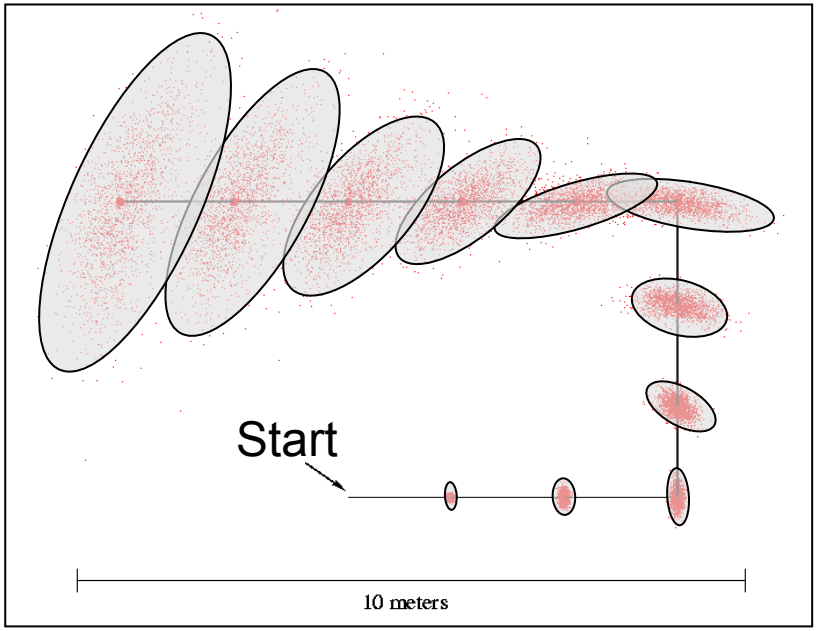
Low-cost arm (Quigley et al.)



Raven surgical robot (Rosen et al.)

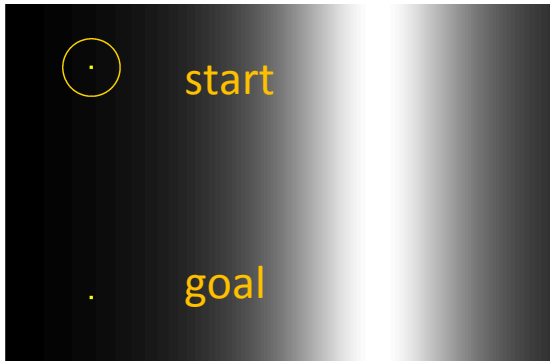
Cost-effective, less precise robots

Model Uncertainty As Gaussians

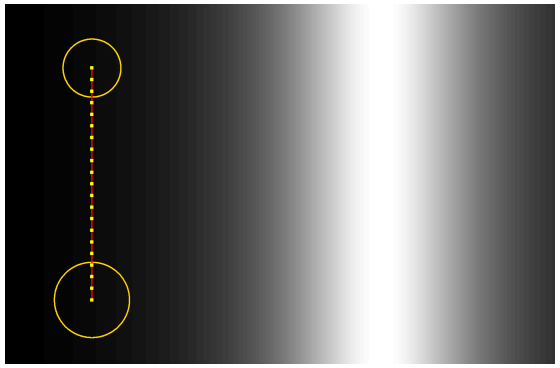


Uncertainty parameterized by
mean and covariance

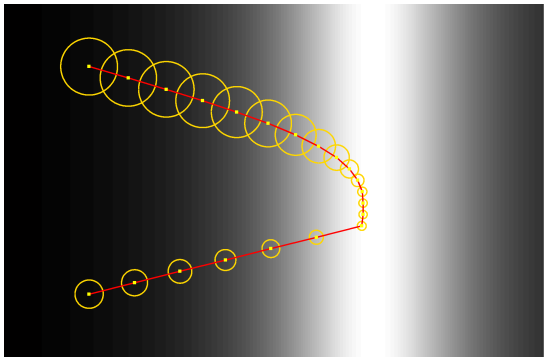
Accounting for Uncertainty



Problem setting



State space plan



Belief space plan

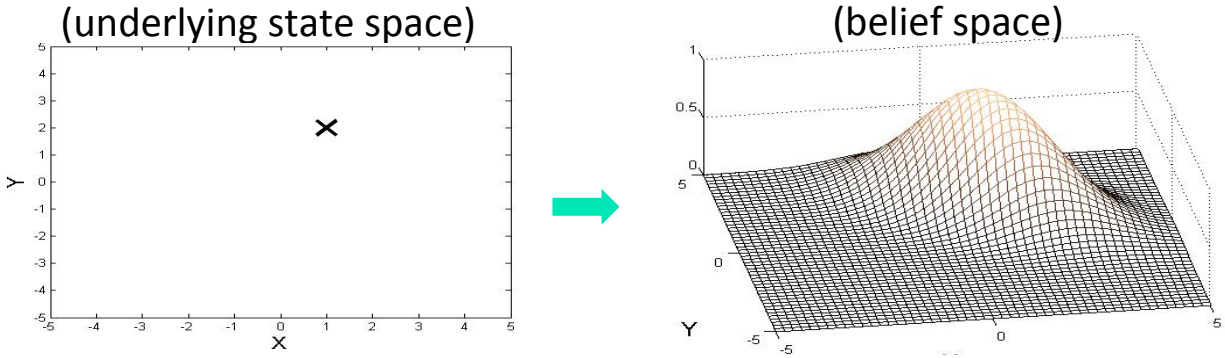
How to find this plan?

[Example from Platt, Tedrake, Kaelbling, Lozano-Perez, 2010]

(Gaussian) Belief Space Planning

- Redefine problem

$$"x_t" = (\mu_t, \Sigma_t)$$



- Convert underlying dynamics to belief space dynamics
  - Bayesian filter (e.g., extended Kalman filter)

Belief Space Planning

- State-space planning through optimization

$$\min_{u,x} \quad \sum_{t=0}^{H} c(x_t, u_t)$$
$$\text{s.t.} \quad x_{t+1} = f_{\text{dynamics}}(x_t, u_t, w_t)$$

Deterministic approximation

$$\min_{u,x} \quad \sum_{t=0}^{H} c(x_t, u_t)$$
$$\text{s.t.} \quad x_{t+1} = f_{\text{dynamics}}(x_t, u_t, 0)$$

- Gaussian belief space planning

$$\min_{u,\mu,\Sigma} \quad \sum_{t=0}^{H} c(\mu_t, \Sigma_t, u_t)$$
$$\text{s.t.} \quad (\mu_{t+1}, \Sigma_{t+1}) = \text{EKF}(\mu_t, \Sigma_t, u_t, z_{t+1})$$

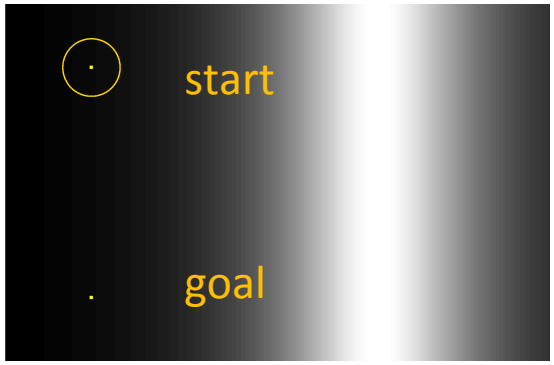
Deterministic approximation (= ML assumption)

$$\min_{u,\mu,\Sigma} \quad \sum_{t=0}^{H} c(\mu_t, \Sigma_t, u_t)$$
$$\text{s.t.} \quad (\mu_{t+1}, \Sigma_{t+1}) = \text{EKF}(\mu_t, \Sigma_t, u_t, h(f(\mu_t, u_t)))$$

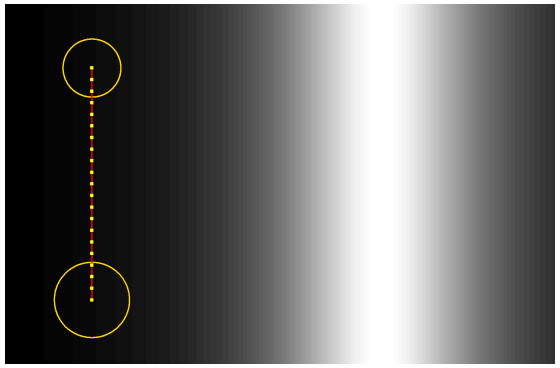
Solved with Sequential Convex Programming

[Platt, Tedrake, Kaelbling, Lozano-Perez, 2010; also Roy et al ; van den Berg et al.]

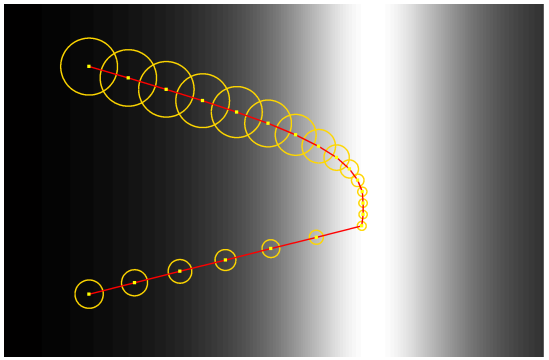
Dealing with Uncertainty



Problem setting



State space plan



Belief space plan

Gaussian Belief Space Planning

Shooting	Partial Collocation	Full Collocation
$\min_{\mathbf{u}_{0:T-1}} \; \mathcal{C}(\hat{\mathbf{x}}_0, \Sigma_0, \mathbf{u}_{0:T-1})$	$\min_{\substack{\mathbf{u}_{0:T-1} \\ \hat{\mathbf{x}}_{0:T}}} \; \mathcal{C}(\hat{\mathbf{x}}_{0:T}, \Sigma_0, \mathbf{u}_{0:T-1})$	$\min_{\substack{\mathbf{u}_{0:T-1} \\ \hat{\mathbf{x}}_{0:T} \\ \Sigma_{0:T}}} \; \mathcal{C}(\hat{\mathbf{x}}_{0:T}, \Sigma_{0:T}, \mathbf{u}_{0:T-1})$
$\tilde{\mathbf{f}}(\hat{\mathbf{x}}_0, \mathbf{u}_{0:t-1}, \mathbf{0}) \in \mathcal{X}_{\text{feasible}}$ $\mathbf{u}_t \in \mathcal{U}_{\text{feasible}}$	$\hat{\mathbf{x}}_T = \hat{\mathbf{x}}_{\text{target}}$, $\hat{\mathbf{x}}_t \in \mathcal{X}_{\text{feasible}}$, $\mathbf{u}_t \in \mathcal{U}_{\text{feasible}}$	s.t $\hat{\mathbf{x}}_{t+1} = \mathbf{f}(\hat{\mathbf{x}}_t, \mathbf{u}_t, \mathbf{0})$, $\Sigma_{t+1} = (I - K_t H_t)\Sigma_{t+1}^{-}$, $\hat{\mathbf{x}}_T = \hat{\mathbf{x}}_{\text{target}}$, $\hat{\mathbf{x}}_t \in \mathcal{X}_{\text{feasible}}$, $\mathbf{u}_t \in \mathcal{U}_{\text{feasible}}$

Gaussian Belief Space Planning

Shooting	Partial Collocation	Full Collocation
$\min_{\mathbf{u}_{0:T-1}} \; \mathcal{C}(\hat{\mathbf{x}}_0, \Sigma_0, \mathbf{u}_{0:T-1})$ $\text{s.t} \quad \tilde{\mathbf{f}}(\hat{\mathbf{x}}_0, \mathbf{u}_{0:T-1}, \mathbf{0}) = \hat{\mathbf{x}}_{\text{target}}$ $\tilde{\mathbf{f}}(\hat{\mathbf{x}}_0, \mathbf{u}_{0:t-1}, \mathbf{0}) \in \mathcal{X}_{\text{feasible}}$ $\mathbf{u}_t \in \mathcal{U}_{\text{feasible}}$	$\min_{\substack{\mathbf{u}_{0:T-1} \\ \hat{\mathbf{x}}_{0:T}}} \; \mathcal{C}(\hat{\mathbf{x}}_{0:T}, \Sigma_0, \mathbf{u}_{0:T-1})$ $\hat{\mathbf{x}}_T = \hat{\mathbf{x}}_{\text{target}},$ $\hat{\mathbf{x}}_t \in \mathcal{X}_{\text{feasible}},$ $\mathbf{u}_t \in \mathcal{U}_{\text{feasible}}$	$\min_{\substack{\mathbf{u}_{0:T-1} \\ \hat{\mathbf{x}}_{0:T} \\ \Sigma_{0:T}}} \; \mathcal{C}(\hat{\mathbf{x}}_{0:T}, \Sigma_{0:T}, \mathbf{u}_{0:T-1})$ $\text{s.t} \quad \hat{\mathbf{x}}_{t+1} = \mathbf{f}(\hat{\mathbf{x}}_t, \mathbf{u}_t, \mathbf{0}),$ $\Sigma_{t+1} = (I - K_t H_t)\Sigma_{t+1}^{-},$ $\hat{\mathbf{x}}_T = \hat{\mathbf{x}}_{\text{target}},$ $\hat{\mathbf{x}}_t \in \mathcal{X}_{\text{feasible}},$ $\mathbf{u}_t \in \mathcal{U}_{\text{feasible}}$

+ Much better scalability
+ No infeasibility issues
- Poorly conditioned / small stepsizes / slow
- Can't constraint mu and Sigma

+ Can constrain states
+ Bends itself into a solution
- Poor scalability
- Infeasible local optima

Gaussian Belief Space Planning

Shooting	Partial Collocation	Full Collocation
$\min_{\mathbf{u}_{0:T-1}} \; \mathcal{C}(\hat{\mathbf{x}}_0, \Sigma_0, \mathbf{u}_{0:T-1})$	$\min_{\substack{\mathbf{u}_{0:T-1} \\ \hat{\mathbf{x}}_{0:T}}} \; \mathcal{C}(\hat{\mathbf{x}}_{0:T}, \Sigma_0, \mathbf{u}_{0:T-1})$	$\min_{\substack{\mathbf{u}_{0:T-1} \\ \hat{\mathbf{x}}_{0:T} \\ \Sigma_{0:T}}} \; \mathcal{C}(\hat{\mathbf{x}}_{0:T}, \Sigma_{0:T}, \mathbf{u}_{0:T-1})$
s.t $\tilde{\mathbf{f}}(\hat{\mathbf{x}}_0, \mathbf{u}_{0:T-1}, \mathbf{0}) = \hat{\mathbf{x}}_{\text{target}}$	s.t $\hat{\mathbf{x}}_{t+1} = \mathbf{f}(\hat{\mathbf{x}}_t, \mathbf{u}_t, \mathbf{0})$	s.t $\hat{\mathbf{x}}_{t+1} = \mathbf{f}(\hat{\mathbf{x}}_t, \mathbf{u}_t, \mathbf{0}),$
$\tilde{\mathbf{f}}(\hat{\mathbf{x}}_0, \mathbf{u}_{0:t-1}, \mathbf{0}) \in \mathcal{X}_{\text{feasible}}$	$\hat{\mathbf{x}}_T = \hat{\mathbf{x}}_{\text{target}},$	$\Sigma_{t+1} = (I - K_t H_t)\Sigma_{t+1}^{-},$
$\mathbf{u}_t \in \mathcal{U}_{\text{feasible}}$	$\hat{\mathbf{x}}_t \in \mathcal{X}_{\text{feasible}},$	$\hat{\mathbf{x}}_T = \hat{\mathbf{x}}_{\text{target}},$
	$\mathbf{u}_t \in \mathcal{U}_{\text{feasible}}$	$\hat{\mathbf{x}}_t \in \mathcal{X}_{\text{feasible}},$
		$\mathbf{u}_t \in \mathcal{U}_{\text{feasible}}$

+ Much better scalability
+ No infeasibility issues
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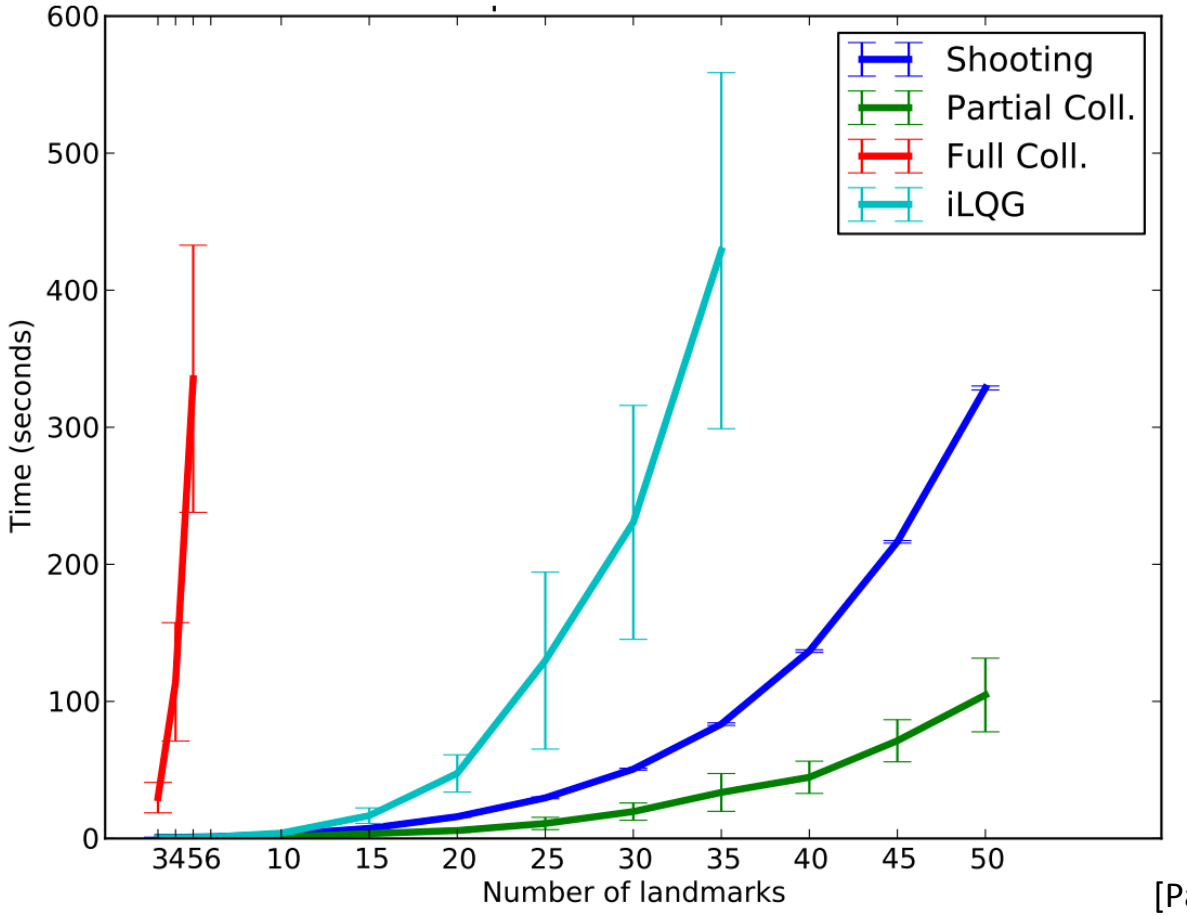
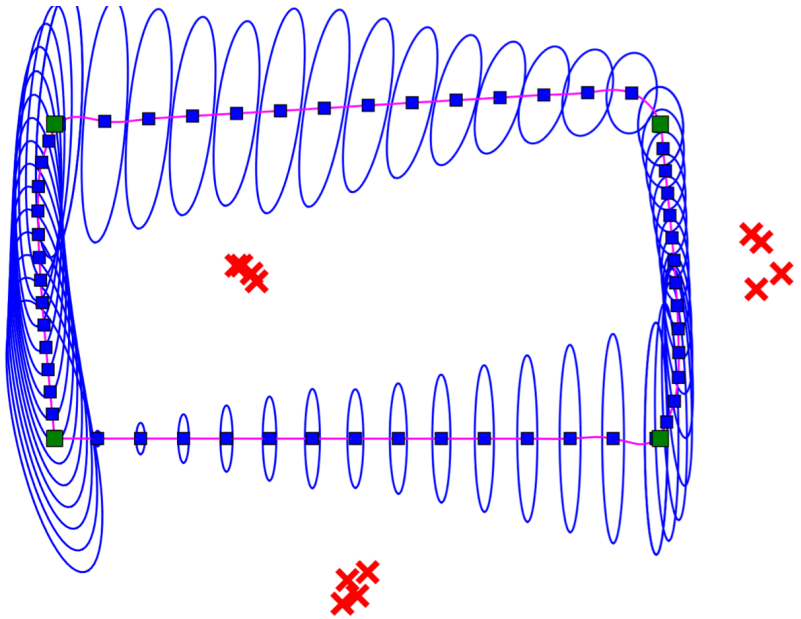
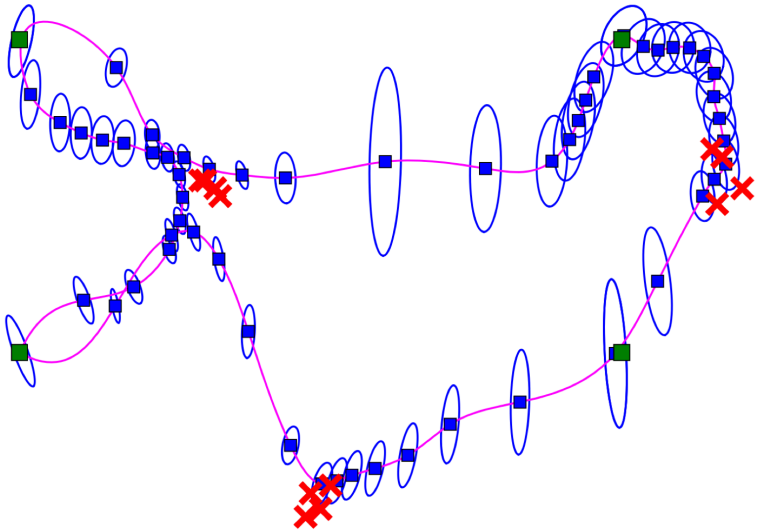
Scalability



[Patil et al., WAFR 2014]

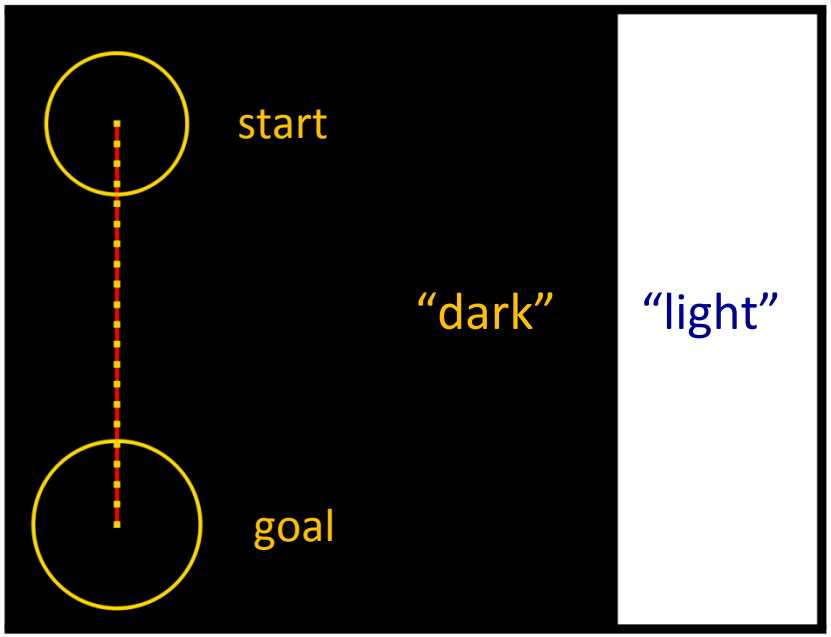
Active SLAM through Gaussian Belief Space Planning





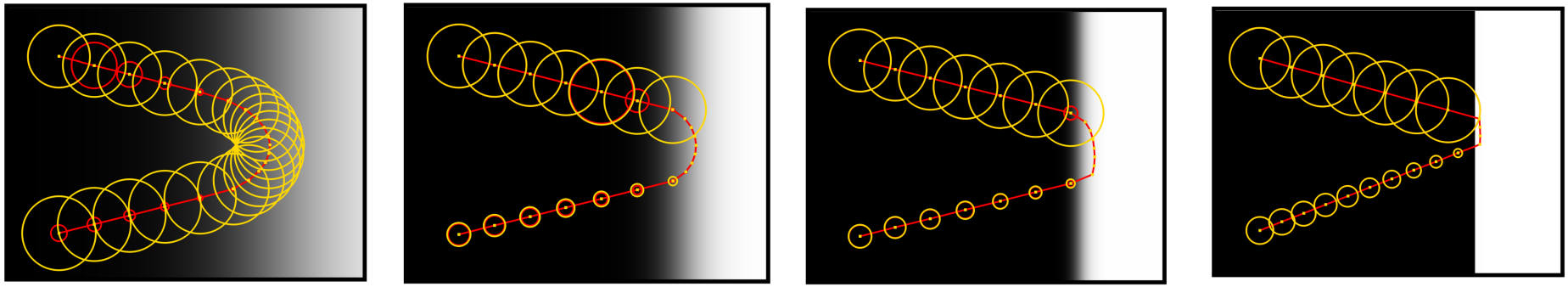
[Patil et al., WAFR 2014]

Dealing with Discontinuities



Zero gradient, hence local optimum

Dealing with Discontinuities



Increasing difficulty

Noise level determined by

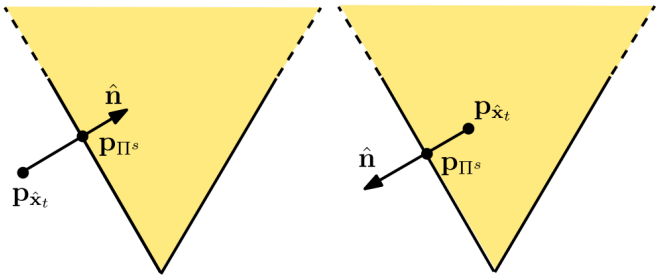
signed distance to sensing region (computed with GJK/EPA)

homotopy iteration

[Patil et al., ICRA 2014]

Signed Distance to Sensing Discontinuity

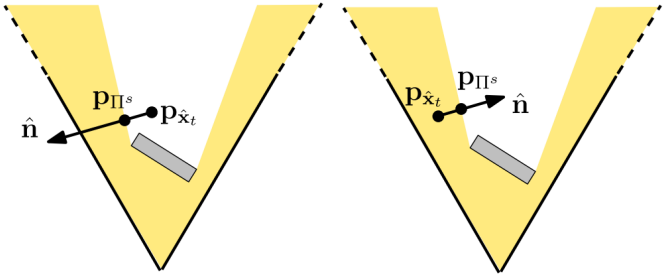
Field of view (FOV) discontinuity



(a) $\mathtt{sd}(\hat{\mathbf{b}}_t, \Pi^s) > 0$
Outside field of view

(b) $\mathtt{sd}(\hat{\mathbf{b}}_t, \Pi^s) < 0$
Inside field of view

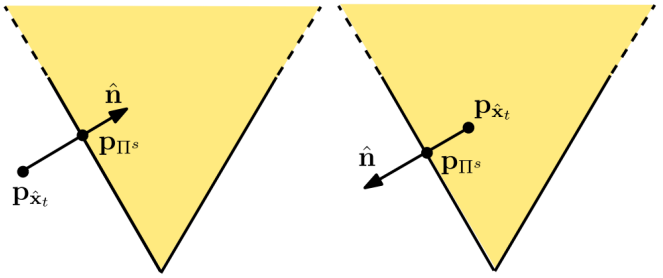
Occlusion discontinuity



(c) $\mathtt{sd}(\hat{\mathbf{b}}_t, \Pi^s) > 0$
Occluded view

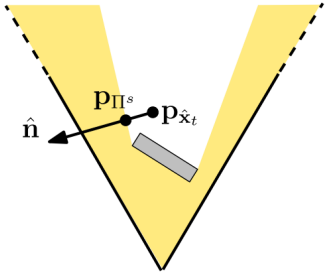
(d) $\mathtt{sd}(\hat{\mathbf{b}}_t, \Pi^s) < 0$
Unoccluded view

δ_t^s vs. Signed distance

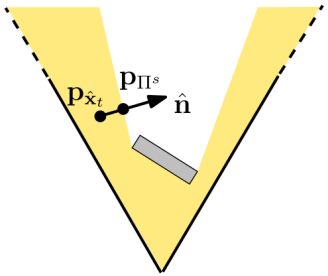


(a) $\mathtt{sd}(\hat{\mathbf{b}}_t, \Pi^s) > 0$
Outside field of view

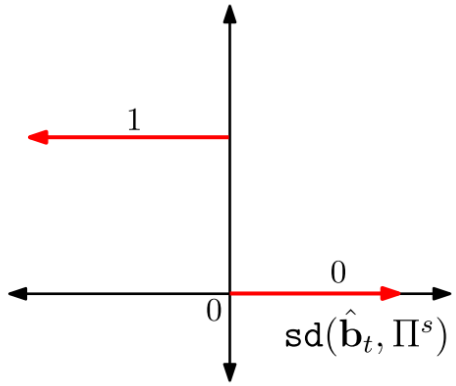
(b) $\mathtt{sd}(\hat{\mathbf{b}}_t, \Pi^s) < 0$
Inside field of view



(c) $\mathtt{sd}(\hat{\mathbf{b}}_t, \Pi^s) > 0$
Occluded view



(d) $\mathtt{sd}(\hat{\mathbf{b}}_t, \Pi^s) < 0$
Unoccluded view



$$\delta_t^s = \chi(\mathtt{sd}(\hat{\mathbf{b}}_t, \Pi^s))$$

Modified Belief Dynamics

$$\mathbf{x}_{t+1} = \mathbf{f}(\mathbf{x}_t, \mathbf{u}_t, \mathbf{q}_t), \quad \mathbf{q}_t \sim \mathcal{N}(\mathbf{0}, I),$$
$$\mathbf{z}_t = \mathbf{h}(\mathbf{x}_t, \mathbf{r}_t), \quad \mathbf{r}_t \sim \mathcal{N}(\mathbf{0}, I),$$

$$\hat{\mathbf{b}}_{t+1} = \mathbf{g}(\hat{\mathbf{b}}_t, \hat{\mathbf{u}}_t) = \begin{bmatrix} \hat{\mathbf{x}}_{t+1} \\ \mathrm{vec}[\sqrt{\Sigma_{t+1}^- - K_t H_t \Sigma_{t+1}^-}] \end{bmatrix}$$

$$\hat{\mathbf{x}}_{t+1} = \mathbf{f}(\hat{\mathbf{x}}_t, \hat{\mathbf{u}}_t, \mathbf{0}), \qquad \Sigma_{t+1}^- = A_t \sqrt{\Sigma_t} (A_t \sqrt{\Sigma_t})^T + Q_t Q_t^T,$$
$$A_t = \frac{\partial \mathbf{f}}{\partial \mathbf{x}}(\hat{\mathbf{x}}_t, \hat{\mathbf{u}}_t, \mathbf{0}), \qquad Q_t = \frac{\partial \mathbf{f}}{\partial \mathbf{q}}(\hat{\mathbf{x}}_t, \hat{\mathbf{u}}_t, \mathbf{0}),$$
$$H_t = \frac{\partial \mathbf{h}}{\partial \mathbf{x}}(\hat{\mathbf{x}}_{t+1}, \mathbf{0}), \qquad R_t = \frac{\partial \mathbf{h}}{\partial \mathbf{r}}(\hat{\mathbf{x}}_{t+1}, \mathbf{0}),$$
$$K_t = \Sigma_{t+1}^- H_t^T \Delta_{t+1} (\Delta_{t+1} H_t \Sigma_{t+1}^- H_t^T \Delta_{t+1} + R_t R_t^T)^{-1} \Delta_{t+1}.$$

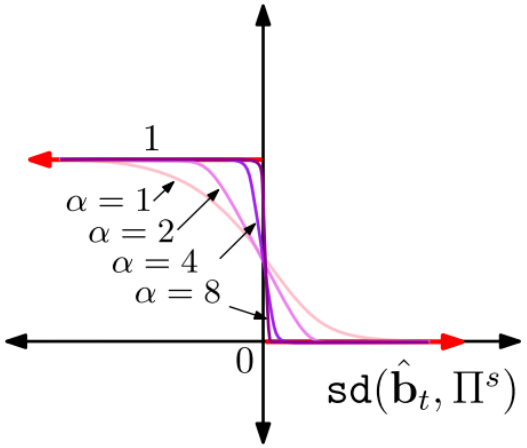
δ_t^s : Binary variable {0,1}

0 -> No measurement

1 -> Measurement

Incorporating δ_t^s in SQP

- Binary non-convex program – difficult to solve
- Solve successively smooth approximations



$$\delta_t^s(\alpha) = \tilde{\chi}(\mathtt{sd}(\hat{\mathbf{b}}_t, \Pi^s), \alpha)$$
$$= 1 - \frac{1}{1+\exp(-\alpha \cdot \mathtt{sd}(\hat{\mathbf{b}}_t, \Pi^s))}$$

Algorithm Overview

- While δ not within desired tolerance
  - Solve optimization problem with current value of α
  - Increase α
  - Re-integrate belief trajectory
  - Update δ

Algorithm

Inputs:

$\bar{\mathcal{B}}_{0:\ell} = [\bar{\mathbf{b}}_0 \dots \bar{\mathbf{b}}_\ell]$, $\bar{\mathcal{U}}_{0:\ell-1} = [\bar{\mathbf{u}}_0 \dots \bar{\mathbf{u}}_{\ell-1}]$: Belief space trajectory initialization

ℓ: Number of time intervals

Cost and constraint definitions (Eq. (4))

Parameters:

α: Approximation parameter for relaxing discrete sensing constraints

k: Coefficient to control rate of increase of α

τ: Execution time step $(0 \leq \tau \leq \ell)$

ε: Convergence tolerance parameter

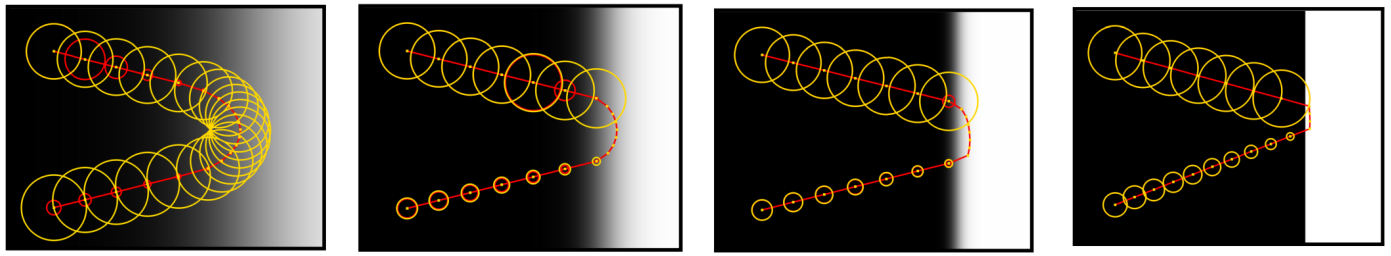
Variables:

$\hat{\mathcal{B}}_{0:\ell} = [\hat{\mathbf{b}}_0 \dots \hat{\mathbf{b}}_\ell]$, $\hat{\mathcal{U}}_{0:\ell-1} = [\hat{\mathbf{u}}_0 \dots \hat{\mathbf{u}}_{\ell-1}]$: Optimization variables

$\boldsymbol{\delta}_{0:\ell}$: Binary vector to track value of continuous approximation for convergence criterion

for $\tau = 0, \dots, \ell-1$ **do** ▷ Re-planning loop following the MPC paradigm
  $\alpha \leftarrow \alpha_{\text{init}}$
  while $\boldsymbol{\delta}_{\tau:\ell}$ not within ε tolerance of true binary values $\{0,1\}$ **do**
    Reset trust region size and penalty coefficient ▷ [25]
    $[\hat{\mathcal{B}}_{\tau:\ell}, \hat{\mathcal{U}}_{\tau:\ell-1}] \leftarrow$ SQP-based optimization of approximation given $[\bar{\mathcal{B}}_{\tau:\ell}, \bar{\mathcal{U}}_{\tau:\ell-1}]$ ▷ [25]
    $\alpha \leftarrow k * \alpha$ ▷ α-update to increase noise outside sensing region
    $\hat{\mathbf{b}}_{t+1} = \mathbf{g}(\hat{\mathbf{b}}_t, \hat{\mathbf{u}}_t) \ \forall \ t = \tau, \dots, \ell-1$ ▷ Integrate belief trajectory after α-update
    Update $\boldsymbol{\delta}_{\tau:\ell} \leftarrow \boldsymbol{\delta}_{\tau:\ell}(\alpha)$ ▷ Eq. (5)
    $[\bar{\mathcal{B}}_{\tau:\ell}, \bar{\mathcal{U}}_{\tau:\ell-1}] \leftarrow [\hat{\mathcal{B}}_{\tau:\ell}, \hat{\mathcal{U}}_{\tau:\ell-1}]$ ▷ Update trajectory initialization
  end while
  Execute $\bar{\mathbf{u}}_\tau$
  Obtain measurement and update $\bar{\mathbf{b}}_{\tau+1}$ using EKF ▷ Eq. (6a)
  Truncate $\bar{\mathbf{b}}_{\tau+1}$ w.r.t sensing region boundary
  Update sensing regions for all sensors
  $\bar{\mathbf{b}}_{t+1} = \mathbf{g}(\bar{\mathbf{b}}_t, \bar{\mathbf{u}}_t) \ \forall \ t = \tau+1, \dots, \ell-1$ ▷ Integrate belief trajectory after Kalman update
    ▷ using previously optimized control inputs $\bar{\mathcal{U}}_{\tau+1:\ell-1}$
end for

Discontinuities in Sensing Domains

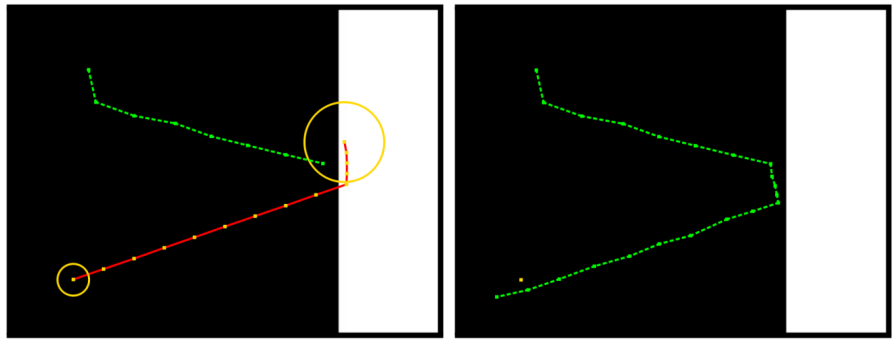


Increasing difficulty

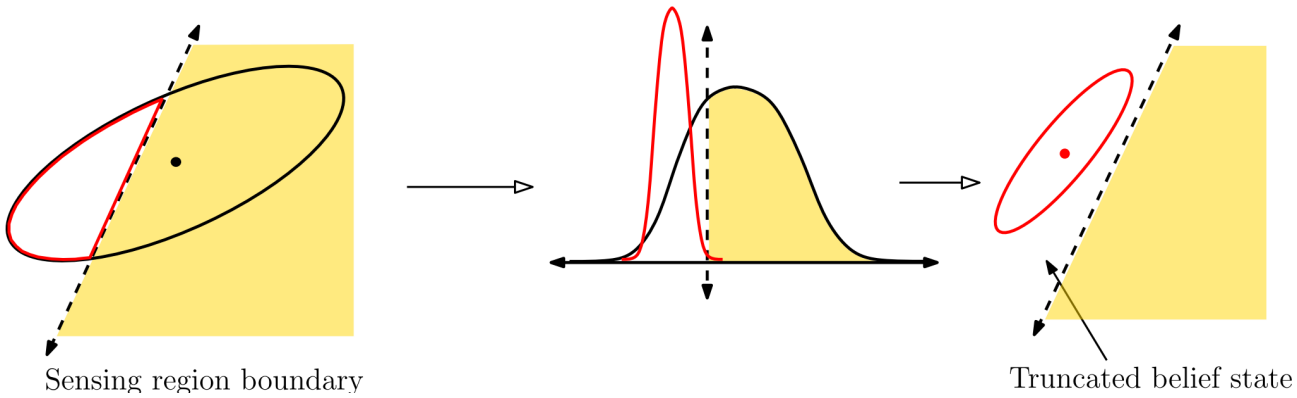
Noise level determined by signed distance to sensing region

* homotopy iteration

However…

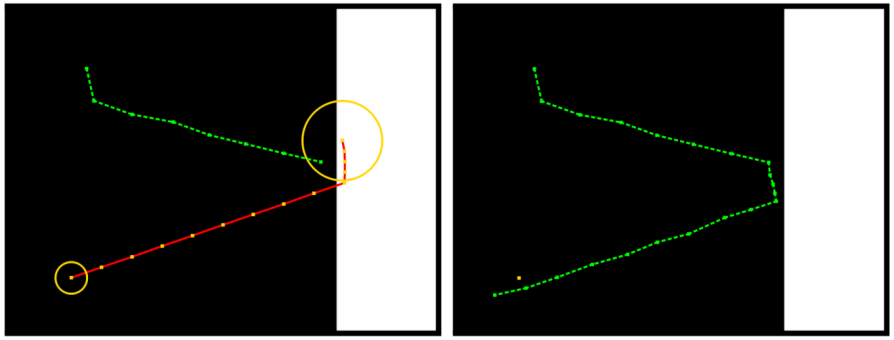


“No measurement” Belief Update

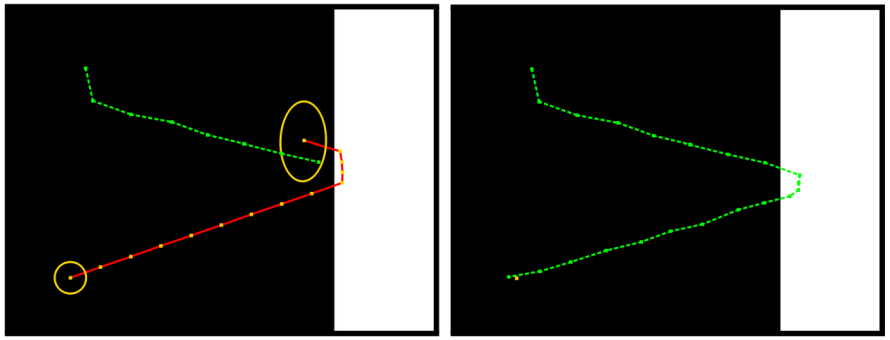


Truncate Gaussian Belief if no measurement obtained

Effect of Truncation

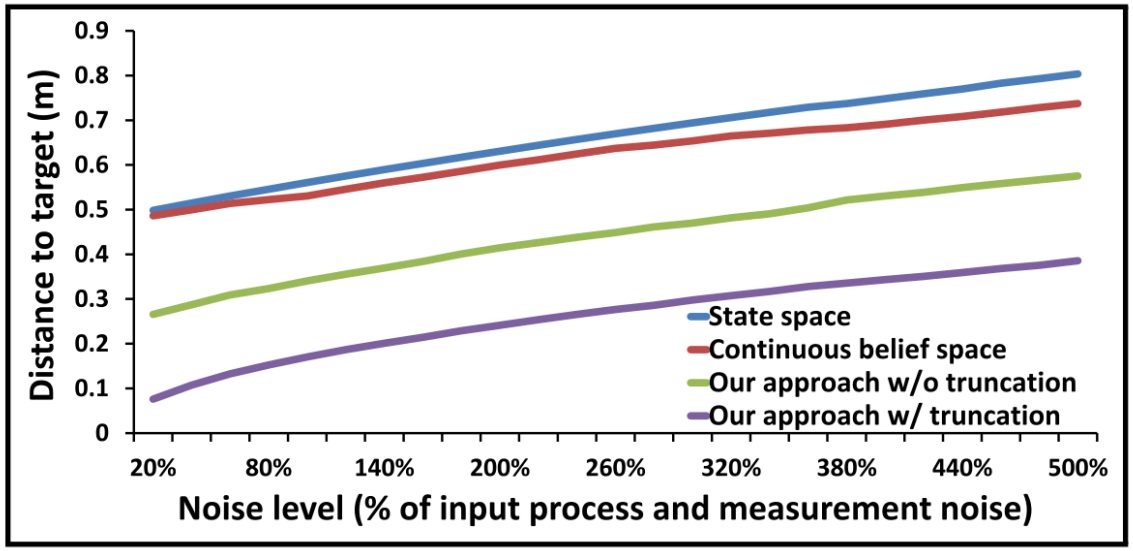


Without "No measurement" Belief Update



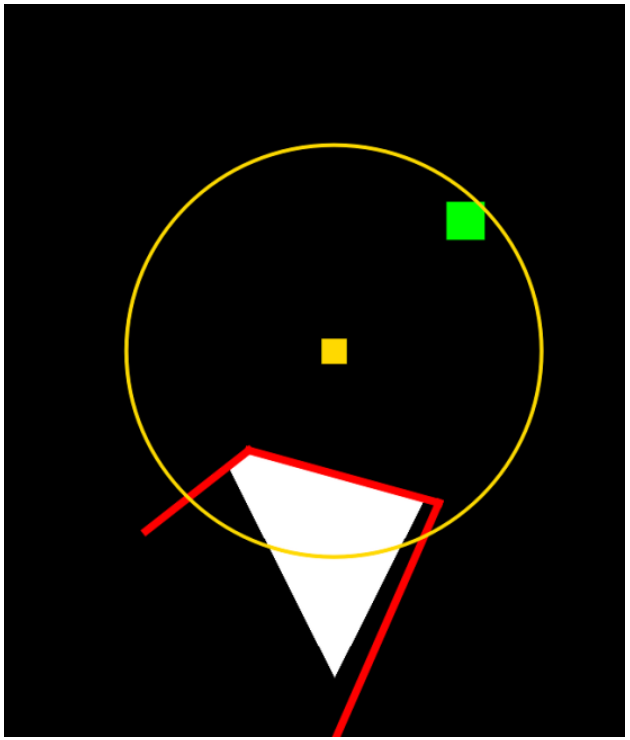
With "No measurement" Belief Update

Experiments

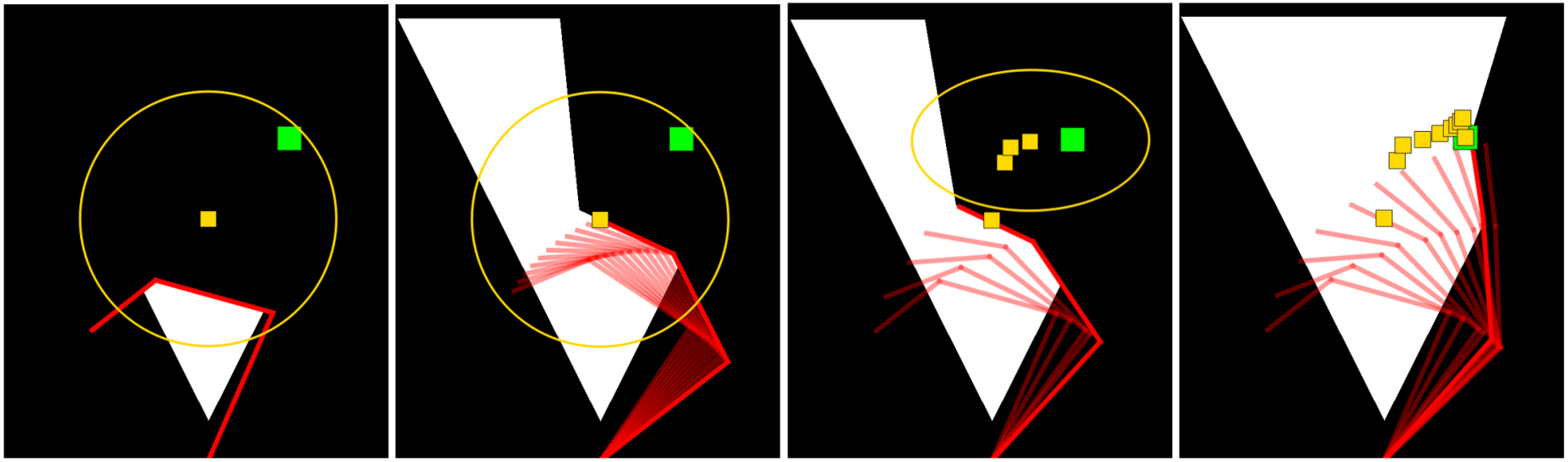


Grasping: Planar 3-link Manipulator

- 6D state space: Arm joint angles + camera orientation + object position
  - 27D belief space
- Objective: Reliably grasp object



Robot Arm Occluding Object from Camera View



Initial belief

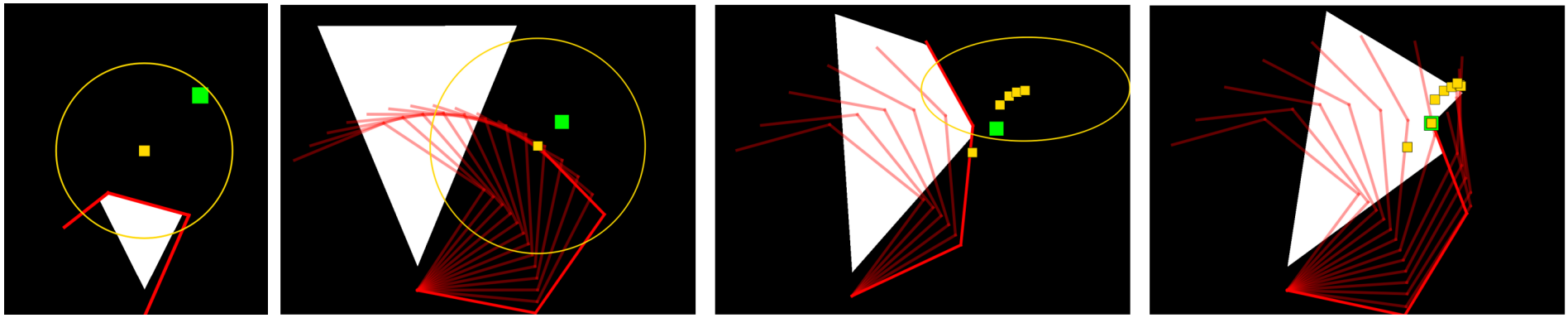
State space
plan execution

(way-point)

(end)

Belief space plan execution

Same Scenario but Movable Camera



Initial belief

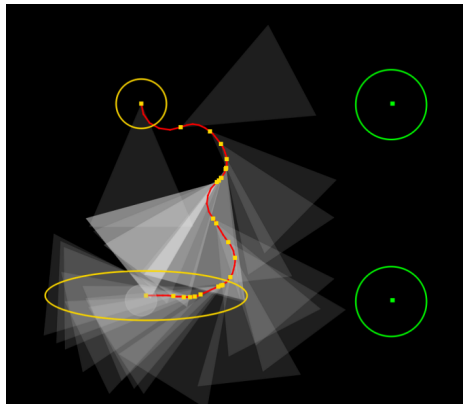
State space
plan execution

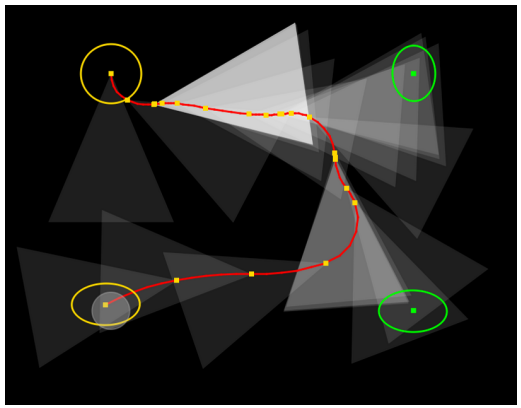
(way-point)

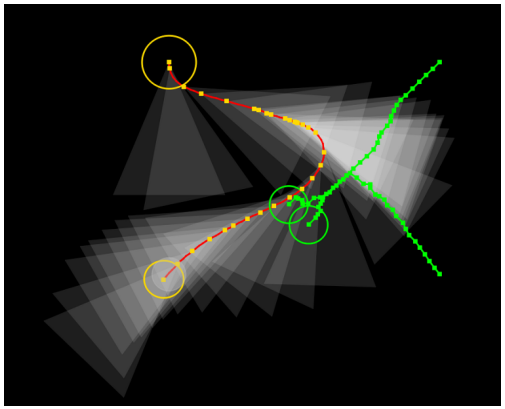
(end)

Belief space plan execution

Car and Landmarks (Active Exploration)

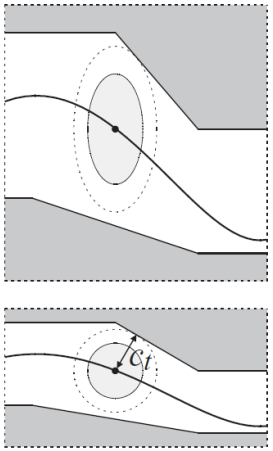






Collision Avoidance

- So far approximating robot geometry as points or spheres

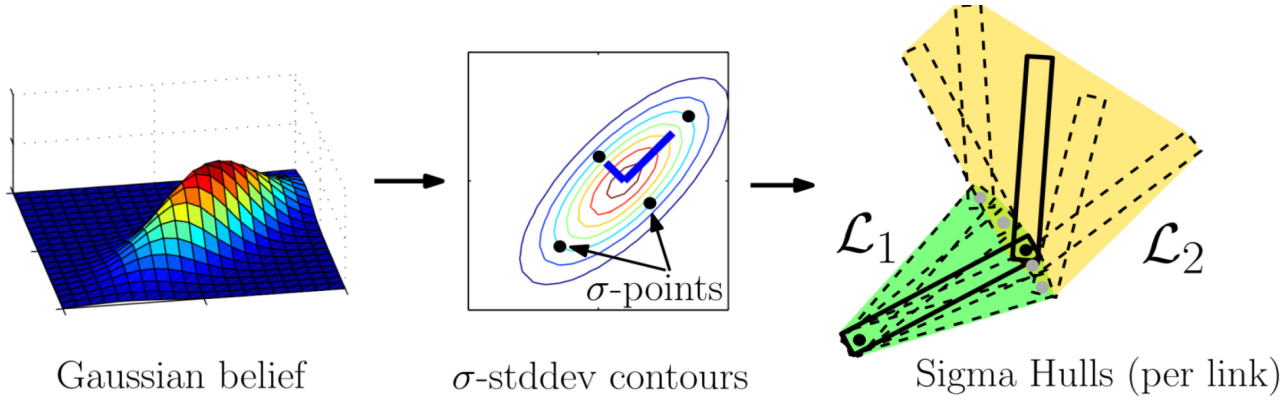


Van den Berg et al.

- Articulated robots cannot be approximated as points/spheres
  - Gaussian noise in joint space
  - Need probabilistic collision avoidance w.r.t robot links

Sigma Hulls

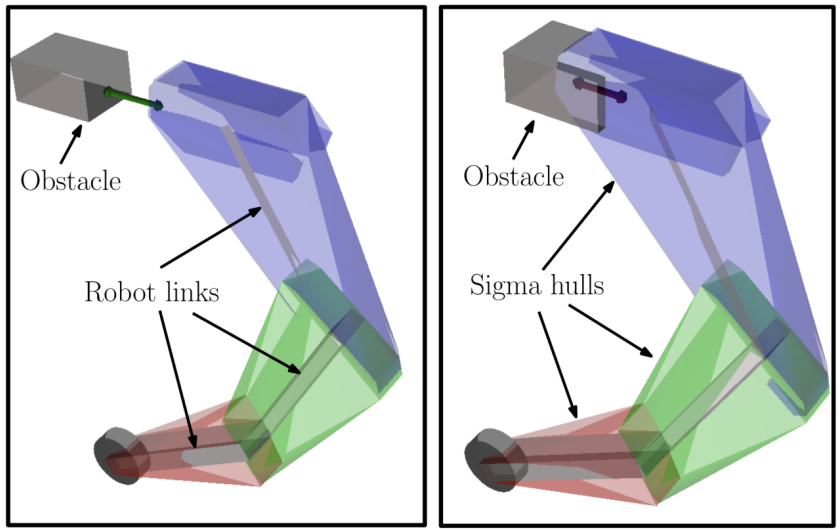
- Definition: Convex hull of a robot link transformed (in joint space) according to sigma points
- Consider sigma points lying on the -standard deviation contour of uncertainty covariance (UKF)



$$\mathcal{X} = [\hat{\mathbf{x}} \dots \hat{\mathbf{x}}] + \lambda[\mathbf{0} \ \sqrt{\Sigma} \ -\sqrt{\Sigma}]$$

Collision Avoidance Constraint

Consider signed distance between obstacle and sigma hulls



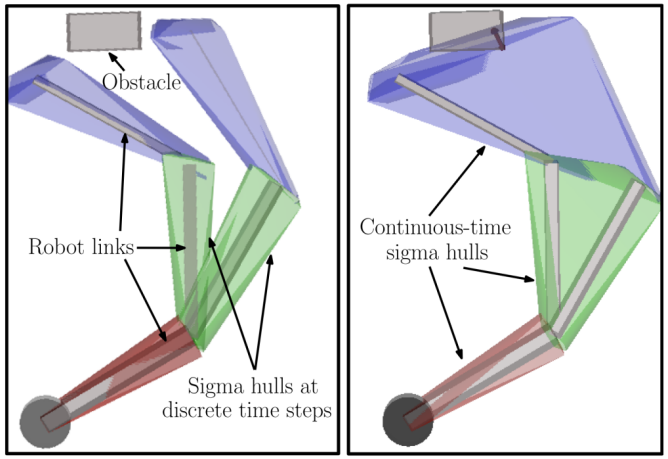
(a) Obstacle outside sigma hulls (b) Obstacle overlaps sigma hulls

Continuous Collision Avoidance Constraint

- Discrete collision avoidance can lead to trajectories that collide with obstacles in between time steps
- Use convex hull of sigma hulls between consecutive time steps

$$\texttt{sd}(\texttt{convhull}(\mathcal{A}_t, \mathcal{A}_{t+1}), O) \geq d_{\text{safe}} \quad \forall\, O \in \mathcal{O}$$

- Advantages:
  - Solutions are collision-free in between time-steps
  - Discretized trajectory can have less time-steps



(a) Obstacle does not collide with discrete-time sigma hulls

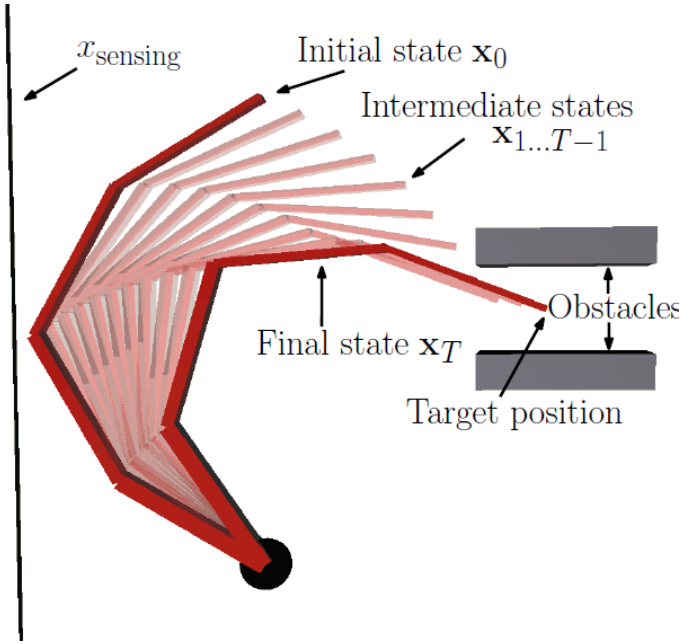
(b) Obstacle overlaps with continuous-time sigma hulls

Belief Space Model Predictive Control

- During execution, update the belief state based on the actual observation
- Re-plan after every belief state update
- Effective feedback control, provided one can re-plan sufficiently fast

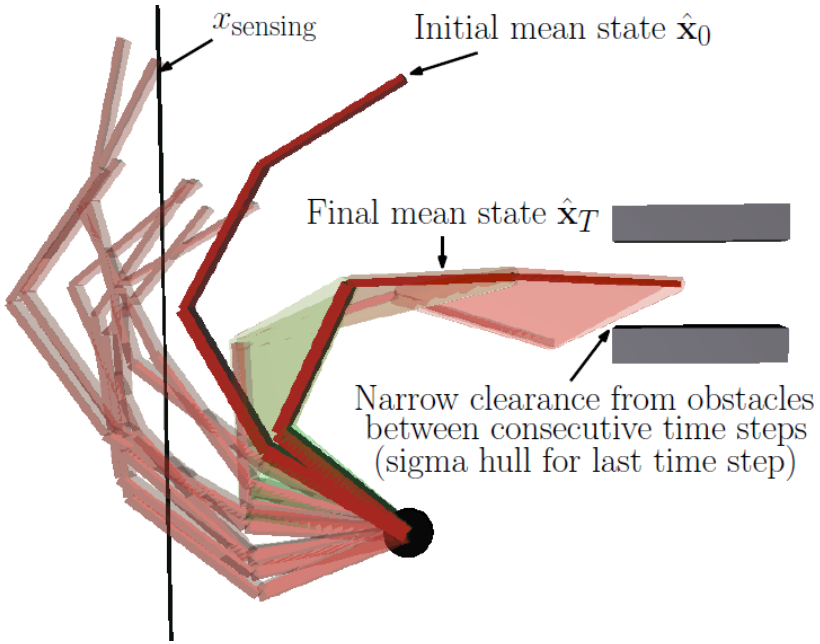
Example: 4-DOF planar robot

State space trajectory



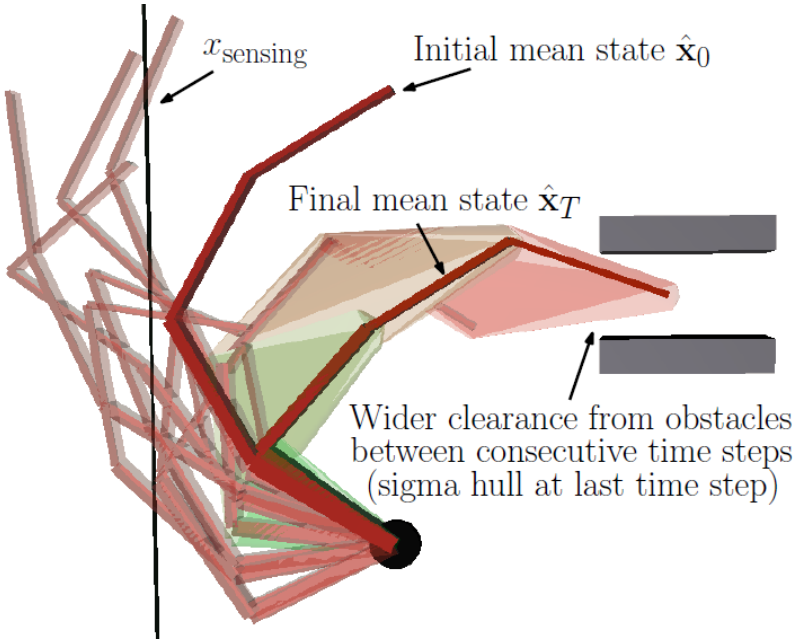
Example: 4-DOF planar robot

1-standard deviation belief space trajectory

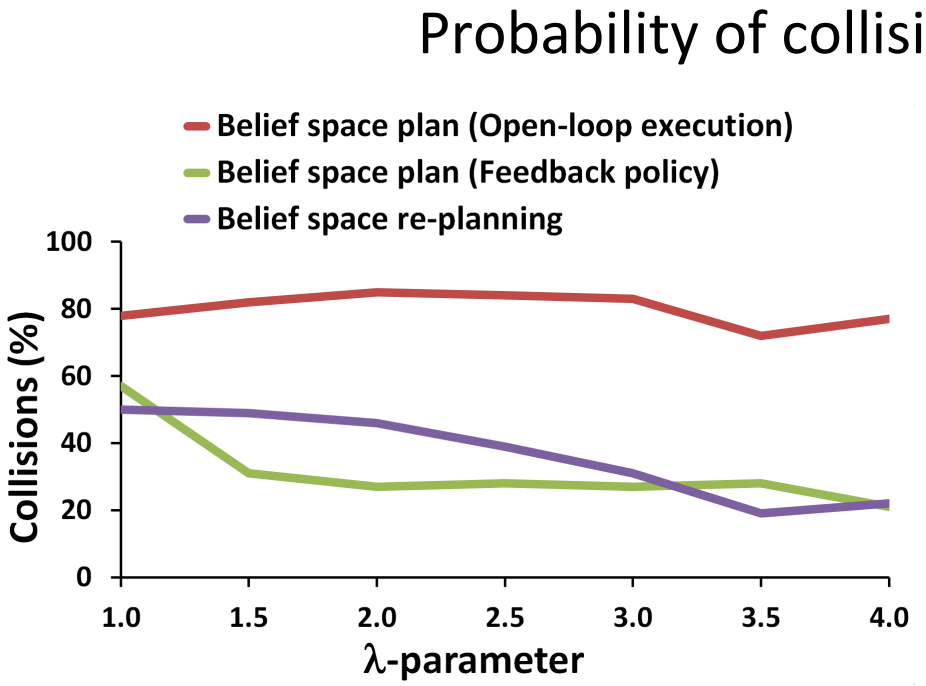


Example: 4-DOF planar robot

4-standard deviation belief space trajectory

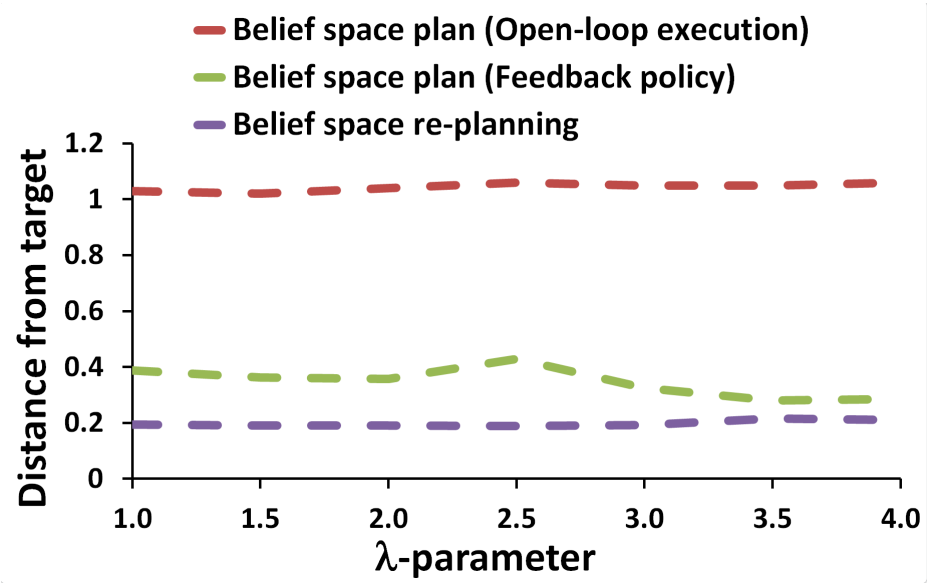


Example: 4-DOF planar robot



Example: 4-DOF planar robot

Mean distance from target



Outline

- Introduction to POMDPs
- Locally Optimal Solutions for POMDPs
  - Trajectory Optimization in (Gaussian) Belief Space
  - Accounting for Discontinuities in Sensing Domains
- Separation Principle

Separation Principle

- Assume: $x_{t+1} = Ax_t + Bu_t + w_t \quad w_t \sim \mathcal{N}(0, Q_t)$

  $z_t = Cx_t + v_t \quad v_t \sim \mathcal{N}(0, R_t)$

- Goal: $\text{minimize } \mathrm{E}\left[\sum_{t=0}^{H} u_t^\top U_t u_t + x_t^\top X_t x_t\right]$

- Then, optimal control policy consists of:

  1. Offline/Ahead of time: Run LQR to find optimal control policy for fully observed case, which gives sequence of feedback matrices

  $$K_1, K_2, \ldots$$

  2. Online: Run Kalman filter to estimate state, and apply control

  $$u_t = K_t \mu_{t|0:t}$$

Recap

- POMDP = MDP but sensory measurements instead of exact state knowledge
- Locally optimal solutions in Gaussian belief spaces
  - Augmented state vector (mean, covariance)
  - Trajectory optimization
- Homotopy methods for dealing with discontinuities in sensing domains
- Sigma Hulls for probabilistic collision avoidance
- Separation principle