Inverse Reinforcement Learning

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Inverse Reinforcement Learning

[equally good titles: Inverse Optimal Control, Inverse Optimal Planning]

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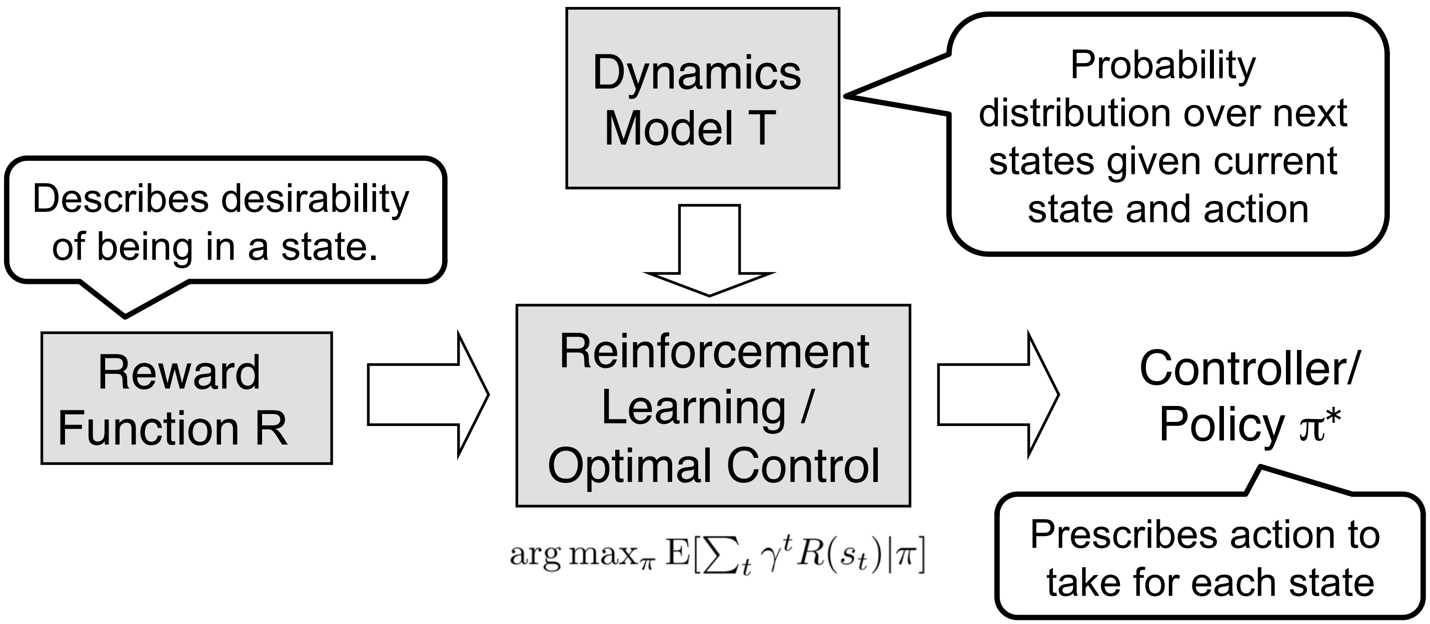
High-level picture



Inverse RL:

Given π^* and T, can we recover R?

More generally, given execution traces, can we recover R?

Motivation for inverse RL

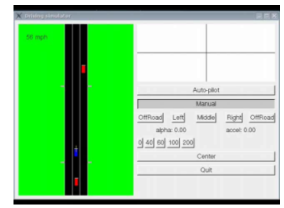
- Scientific inquiry
  - Model animal and human behavior
    - E.g., bee foraging, songbird vocalization. [See intro of Ng and Russell, 2000 for a brief overview.]
- Apprenticeship learning/Imitation learning through inverse RL
  - Presupposition: reward function provides the most succinct and transferable definition of the task
  - Has enabled advancing the state of the art in various robotic domains
- Modeling of other agents, both adversarial and cooperative

Lecture outline

- Example applications
- Inverse RL vs. behavioral cloning
- Historical sketch of inverse RL
- Mathematical formulations for inverse RL
- Case studies

Examples

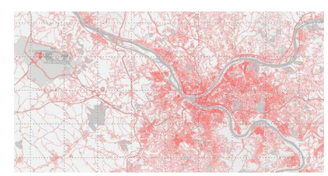
- Simulated highway driving
  - Abbeel and Ng, ICML 2004,
  - Syed and Schapire, NIPS 2007



- Aerial imagery based navigation
  - Ratliff, Bagnell and Zinkevich, ICML 2006

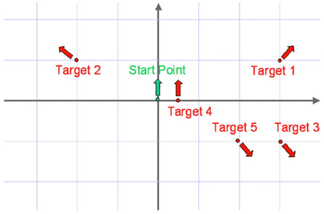
- Parking lot navigation
  - Abbeel, Dolgov, Ng and Thrun, IROS 2008

- Urban navigation
  - Ziebart, Maas, Bagnell and Dey, AAAI 2008

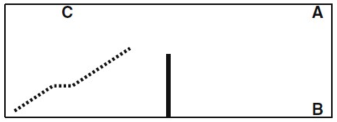


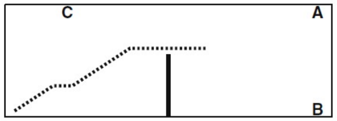
Examples (ctd)

- Human path planning
  - Mombaur, Truong and Laumond, AURO 2009



- Human goal inference
  - Baker, Saxe and Tenenbaum, Cognition 2009





- Quadruped locomotion
  - Ratliff, Bradley, Bagnell and Chestnutt, NIPS 2007
  - Kolter, Abbeel and Ng, NIPS 2008

Urban navigation

- Reward function for urban navigation?

→ destination prediction

Ziebart, Maas, Bagnell and Dey AAAI 2008

Lecture outline

- Example applications
- *Inverse RL vs. behavioral cloning*
- Historical sketch of inverse RL
- Mathematical formulations for inverse RL
- Case studies

Problem setup

- Input:
  - State space, action space
  - Transition model $P_{sa}(s_{t+1} \mid s_t, a_t)$
  - *No* reward function
  - Teacher's demonstration: $s_0, a_0, s_1, a_1, s_2, a_2, \ldots$

    (= trace of the teacher's policy π^*)
- Inverse RL:
  - Can we recover R ?
- Apprenticeship learning via inverse RL
  - Can we then use this R to find a good policy ?
- Behavioral cloning
  - Can we directly learn the teacher's policy using supervised learning?

Behavioral cloning

- Formulate as standard machine learning problem
  - Fix a policy class
    - E.g., support vector machine, neural network, decision tree, deep belief net, …
  - Estimate a policy (=mapping from states to actions) from the training examples $(s_0, a_0), (s_1, a_1), (s_2, a_2), \ldots$

- Two of the most notable success stories:
  - Pomerleau, NIPS 1989: ALVINN
  - Sammut et al., ICML 1992: Learning to fly (flight sim)

Inverse RL vs. Behavioral cloning

- **Which has the most succinct description: π^* vs. R^*?**

- Especially in planning oriented tasks, the reward function is often much more succinct than the optimal policy.

Lecture outline

- Example applications
- Inverse RL vs. behavioral cloning
- *Historical sketch of inverse RL*
- Mathematical formulations for inverse RL
- Case studies

Inverse RL history

- 1964, Kalman posed the inverse optimal control problem and solved it in the 1D input case
- 1994, Boyd+al.: a linear matrix inequality (LMI) characterization for the general linear quadratic setting
- 2000, Ng and Russell: first MDP formulation, reward function ambiguity pointed out and a few solutions suggested
- 2004, Abbeel and Ng: inverse RL for apprenticeship learning---reward feature matching
- 2006, Ratliff+al: max margin formulation

Inverse RL history

- 2007, Ratliff+al: max margin with boosting---enables large vocabulary of reward features
- 2007, Ramachandran and Amir [R&A], and Neu and Szepesvari: reward function as characterization of policy class
- 2008, Kolter, Abbeel and Ng: hierarchical max-margin
- 2008, Syed and Schapire: feature matching + game theoretic formulation
- 2008, Ziebart+al: feature matching + max entropy
- 2008, Abbeel+al: feature matching -- application to learning parking lot navigation style
- 2009, Baker, Saxe, Tenenbaum: same formulation as [R&A], investigation of understanding of human inverse planning inference
- 2009, Mombaur, Truong, Laumond: human path planning
- Active inverse RL? Inverse RL w.r.t. minmax control, partial observability, learning stage (rather than observing optimal policy), … ?

Lecture outline

- Example applications
- Inverse RL vs. behavioral cloning
- Historical sketch of inverse RL
- *Mathematical formulations for inverse RL*
- Case studies

Three broad categories of formalizations

- Max margin
- Feature expectation matching
- Interpret reward function as parameterization of a policy class

Basic principle

- Find a reward function R* which explains the expert behavior.
- Find R* such that

$$E[\sum_{t=0}^{\infty} \gamma^t R^*(s_t)|\pi^*] \geq E[\sum_{t=0}^{\infty} \gamma^t R^*(s_t)|\pi] \quad \forall \pi$$

- In fact a convex feasibility problem, but many challenges:
  - R=0 is a solution, more generally: reward function ambiguity
  - We typically only observe expert traces rather than the entire expert policy π* --- how to compute left-hand side?
  - Assumes the expert is indeed optimal --- otherwise infeasible
  - Computationally: assumes we can enumerate all policies

Feature based reward function

- Let $R(s) = w^\top \phi(s)$, where $w \in \mathbb{R}^n$, and $\phi : S \to \mathbb{R}^n$.

$$\begin{aligned}
\mathrm{E}[\sum_{t=0}^{\infty} \gamma^t R(s_t)|\pi] &= \mathrm{E}[\sum_{t=0}^{\infty} \gamma^t w^\top \phi(s_t)|\pi] \\
&= w^\top \mathrm{E}[\sum_{t=0}^{\infty} \gamma^t \phi(s_t)|\pi] \\
&= w^\top \underbrace{\mu(\pi)}_{\text{Expected cumulative discounted sum of feature values or "feature expectations"}}
\end{aligned}$$

- Subbing into $\mathrm{E}[\sum_{t=0}^{\infty} \gamma^t R^*(s_t)|\pi^*] \geq \mathrm{E}[\sum_{t=0}^{\infty} \gamma^t R^*(s_t)|\pi] \quad \forall \pi$

gives us: $\quad$ Find w^* such that $w^{*\top} \mu(\pi^*) \geq w^{*\top} \mu(\pi) \quad \forall \pi$

Feature based reward function

$$\mathrm{E}[\sum_{t=0}^{\infty} \gamma^t R^*(s_t)|\pi^*] \geq \mathrm{E}[\sum_{t=0}^{\infty} \gamma^t R^*(s_t)|\pi] \quad \forall \pi$$

$$\Downarrow$$

Let $R(s) = w^\top \phi(s)$, where $w \in \Re^n$, and $\phi : S \rightarrow \Re^n$.

Find w^* such that $w^{*\top} \mu^{(}\pi^*) \geq w^{*\top} \mu(\pi) \quad \forall \pi$

- Feature expectations can be readily estimated from sample trajectories.
- The number of expert demonstrations required scales with the number of features in the reward function.
- The number of expert demonstration required does *not* depend on
  - Complexity of the expert's optimal policy ¼*
  - Size of the state space

Recap of challenges

Let $R(s) = w^\top \phi(s)$, where $w \in \mathbb{R}^n$, and $\phi : S \rightarrow \mathbb{R}^n$.

$$w^{*\top} \mu(\pi^*) \geq w^{*\top} \mu(\pi) \qquad \forall \pi$$

- Challenges:
  - Assumes we know the entire expert policy π* → assumes we can estimate expert feature expectations
  - R=0 is a solution (now: w=0), more generally: reward function ambiguity
  - Assumes the expert is indeed optimal---became even more of an issue with the more limited reward function expressiveness!
  - Computationally: assumes we can enumerate all policies

Ambiguity

- Standard max margin:

$$\begin{aligned} &\min_w \quad \|w\|_2^2 \\ &\text{s.t.} \quad w^\top \mu(\pi^*) \geq w^\top \mu(\pi) + 1 \qquad \forall \pi \end{aligned}$$

- "Structured prediction" max margin:

$$\begin{aligned} &\min_w \quad \|w\|_2^2 \\ &\text{s.t.} \quad w^\top \mu^{(}\pi^*) \geq w^\top \mu(\pi) + m(\pi^*, \pi) \qquad \forall \pi \end{aligned}$$

- Justification: margin should be larger for policies that are very different from π*.
- Example: m(π, π*) = number of states in which π* was observed and in which π and π* disagree

Expert suboptimality

- Structured prediction max margin with slack variables:

$$\begin{aligned} \min_{w,\xi} \quad & \|w\|_2^2 + C\xi \\ \text{s.t.} \quad & w^\top \mu(\pi^*) \geq w^\top \mu(\pi) + m(\pi^*, \pi) - \xi \qquad \forall \pi \end{aligned}$$

- Can be generalized to multiple MDPs (could also be same MDP with different initial state)

$$\begin{aligned} \min_{w,\xi^{(i)}} \quad & \|w\|_2^2 + C \sum_i \xi^{(i)} \\ \text{s.t.} \quad & w^\top \mu(\pi^{(i)*}) \geq w^\top \mu(\pi^{(i)}) + m(\pi^{(i)*}, \pi^{(i)}) - \xi^{(i)} \qquad \forall i, \pi^{(i)} \end{aligned}$$

Complete max-margin formulation

$$\min_{w} \quad \|w\|_2^2 + C\sum_i \xi^{(i)}$$

$$\text{s.t.} \quad w^\top \mu(\pi^{(i)*}) \geq w^\top \mu(\pi^{(i)}) + m(\pi^{(i)*}, \pi^{(i)}) - \xi^{(i)} \qquad \forall i, \pi^{(i)}$$

[Ratliff, Zinkevich and Bagnell, 2006]

- Resolved: access to π*, ambiguity, expert suboptimality
- One challenge remains: very large number of constraints
  - Ratliff+al use subgradient methods.
  - In this lecture: constraint generation

Constraint generation

Initialize $\Pi^{(i)} = \{\}$ for all i and then iterate

- Solve

$$\min_{w} \quad \|w\|_2^2 + C\sum_i \xi^{(i)}$$
$$\text{s.t.} \quad w^\top \mu(\pi^{(i)*}) \geq w^\top \mu(\pi^{(i)}) + m(\pi^{(i)*}, \pi^{(i)}) - \xi^{(i)} \qquad \forall i, \forall \pi^{(i)} \in \Pi^{(i)}$$

- For current w, find most violated constraint for all i by solving:

$$\max_{\pi^{(i)}} w^\top \mu(\pi^{(i)}) + m(\pi^{(i)*}, \pi^{(i)})$$

  = find optimal policy for current estimate of reward (+ loss augmentation m)

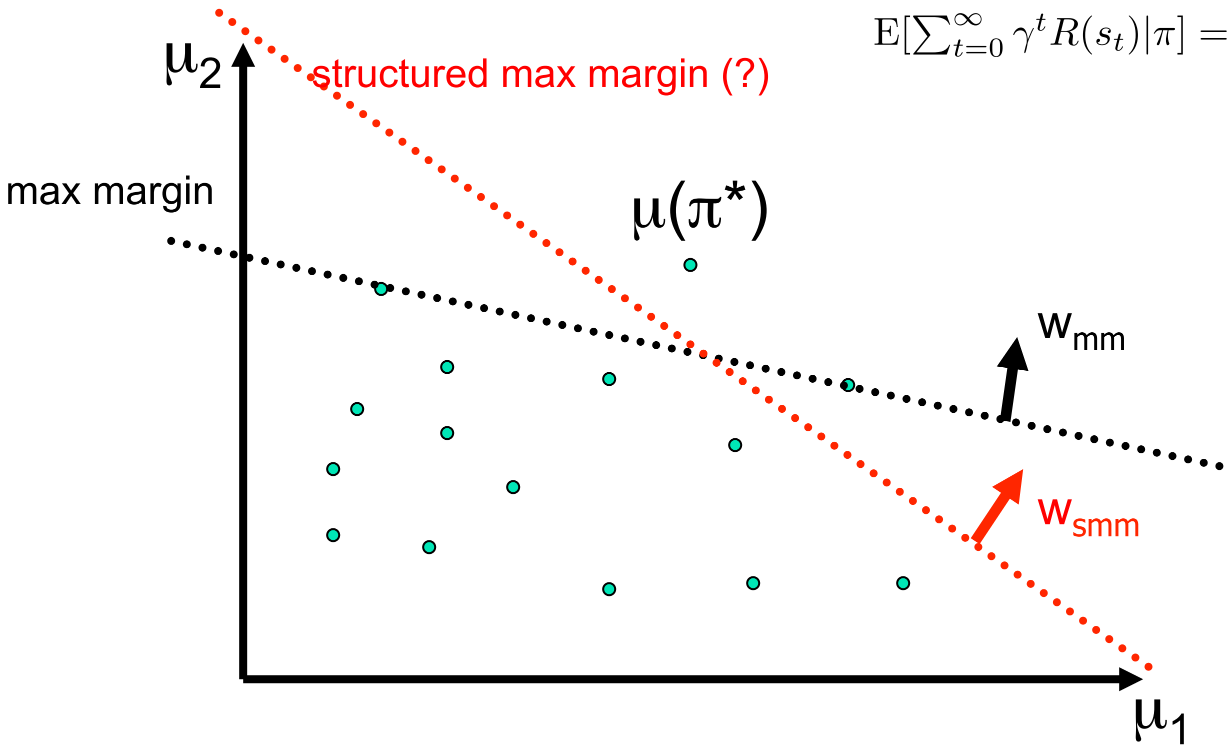
- For all i add $\pi^{(i)}$ to $\Pi^{(i)}$

- If no constraint violations were found, we are done.

Visualization in feature expectation space

- Every policy π has a corresponding feature expectation vector μ(π), which for visualization purposes we assume to be 2D

$$\mathrm{E}[\sum_{t=0}^{\infty} \gamma^t R(s_t)|\pi] = w^\top \mu(\pi)$$



Constraint generation

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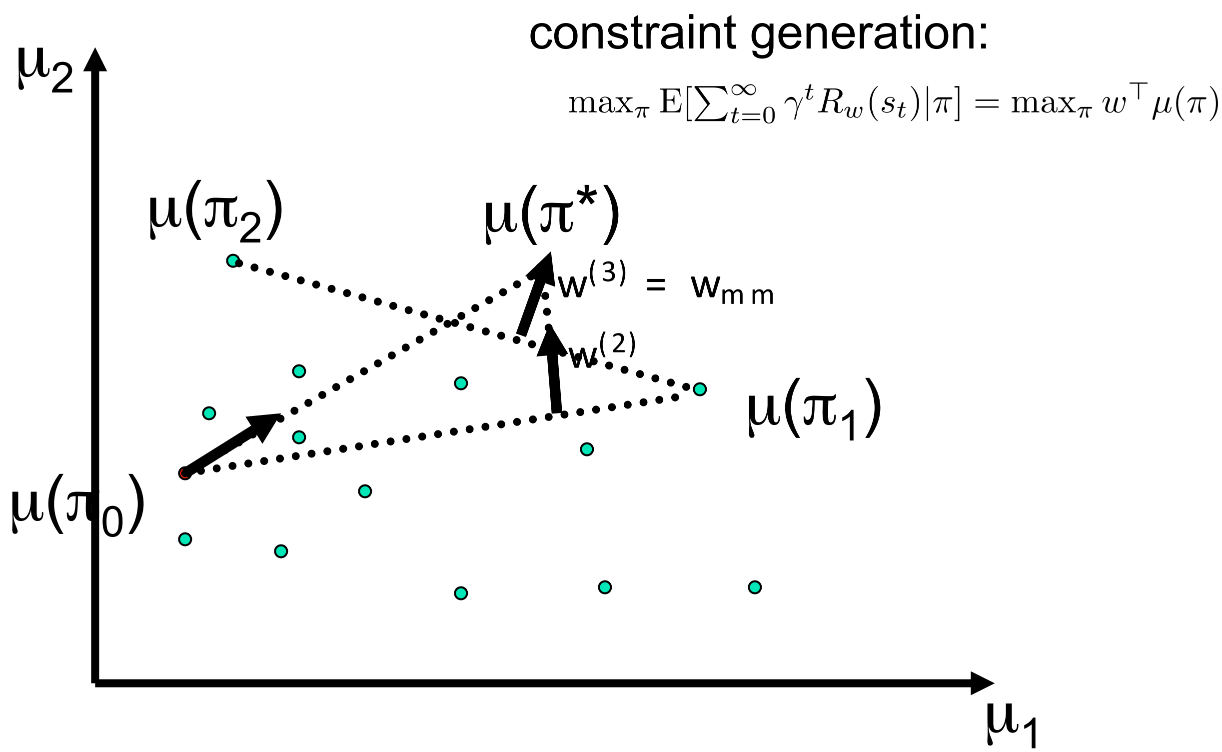


Three broad categories of formalizations

- Max margin (Ratliff+al, 2006)
  - Feature boosting [Ratliff+al, 2007]
  - Hierarchical formulation [Kolter+al, 2008]

- *Feature expectation matching (Abbeel+Ng, 2004)*
  - *Two player game formulation of feature matching (Syed+Schapire, 2008)*
  - *Max entropy formulation of feature matching (Ziebart+al,2008)*

- Interpret reward function as parameterization of a policy class. (Neu +Szepesvari, 2007; Ramachandran+Amir, 2007; Baker, Saxe, Tenenbaum, 2009; Mombaur, Truong, Laumond, 2009)

Feature matching

- Inverse RL starting point: find a reward function such that the expert outperforms other policies

$$\text{Let } R(s) = w^\top \phi(s), \text{ where } w \in \mathbb{R}^n, \text{ and } \phi : S \to \mathbb{R}^n.$$

$$\text{Find } w^* \text{ such that } {w^*}^\top \mu(\pi^*) \geq {w^*}^\top \mu(\pi) \quad \forall \pi$$

- Observation in Abbeel and Ng, 2004: for a policy π to be guaranteed to perform as well as the expert policy μ*, it suffices that the feature expectations match:

$$\|\mu(\pi) - \mu(\pi^*)\|_1 \leq \epsilon$$

implies that for all w with $\|w\|_\infty \leq 1$:

$$|{w^*}^\top \mu(\pi) - {w^*}^\top \mu(\pi^*)| \leq \epsilon$$

Apprenticeship learning [Abbeel & Ng, 2004]

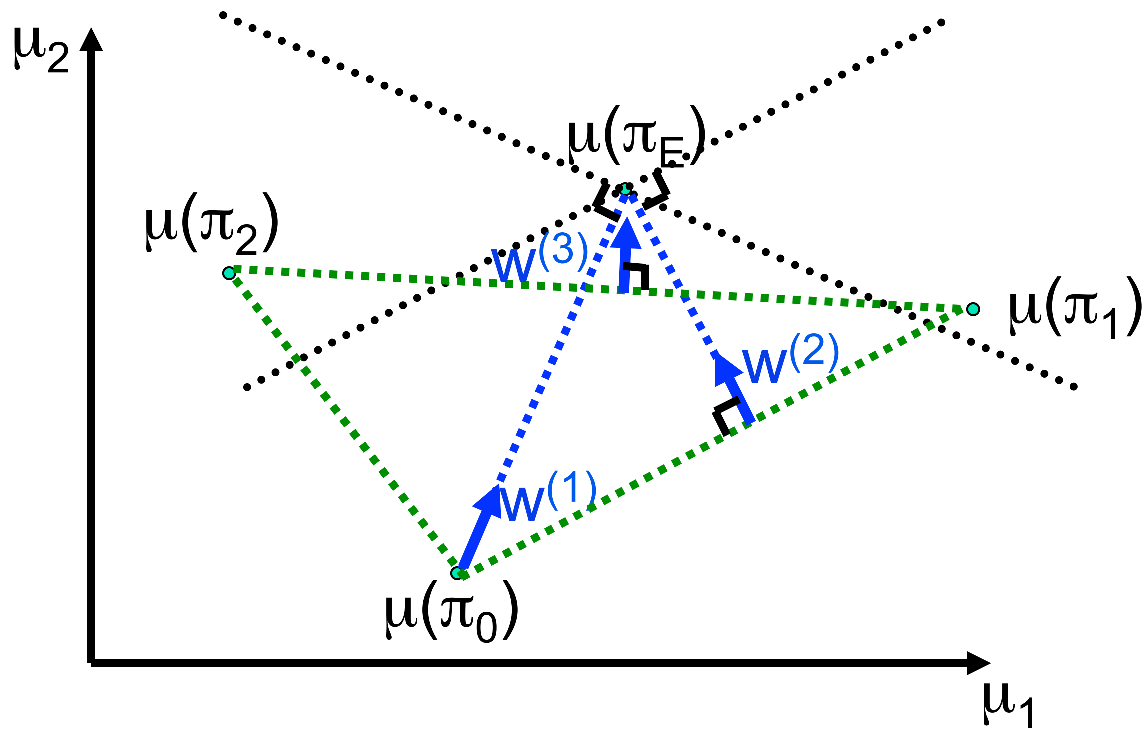
- Assume $R_w(s) = w^\top \phi(s)$ for a feature map $\phi : S \rightarrow \Re^n$.
- Initialize: pick some controller π_0.
- Iterate for i = 1, 2, … :
  - **"Guess" the reward function**:

    Find a reward function such that the teacher maximally outperforms all previously found controllers.

    $$\max_{\gamma, w: \|w\|_2 \leq 1} \gamma$$
    $$s.t. \quad w^\top \mu(\pi^*) \geq w^\top \mu(\pi) + \gamma \quad \forall \pi \in \{\pi_0, \pi_1, \ldots, \pi_{i-1}\}$$

  - **Find optimal control policy** π_i for the current guess of the reward function R_w.
  - $\gamma \leq \varepsilon/2$ exit the algorithm.

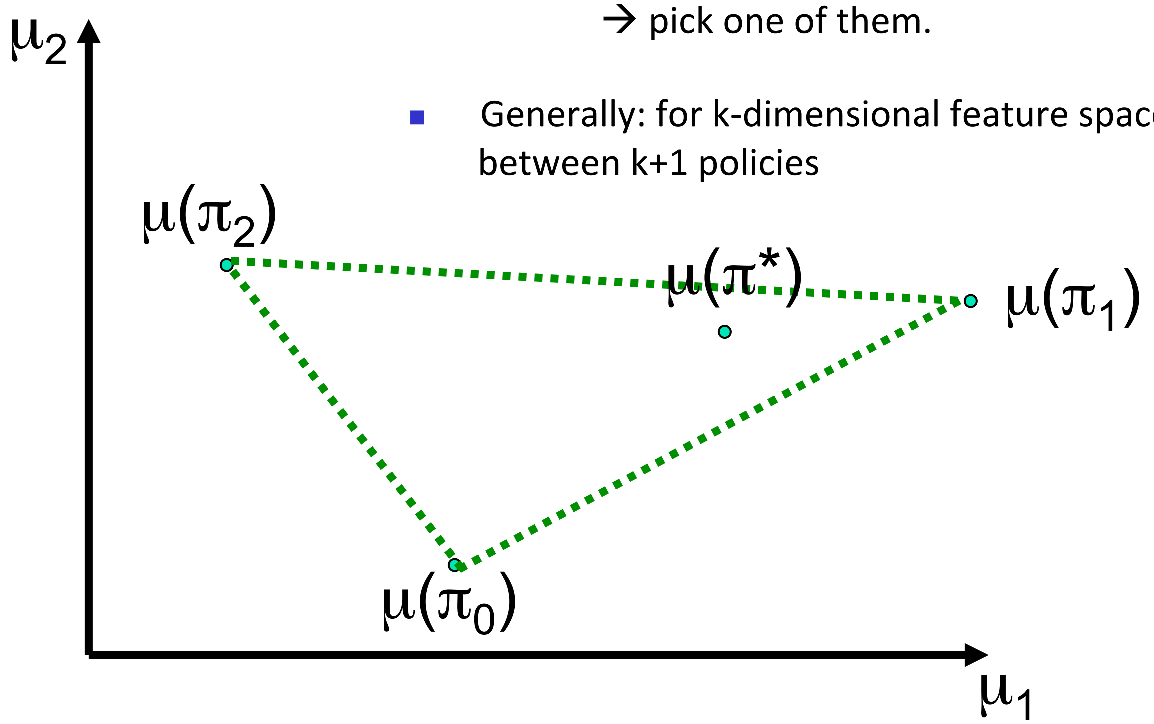
Algorithm example run



Suboptimal expert case

- Can match expert by stochastically mixing between 3 policies
- In practice: for any w* one of μ_0, μ_1, μ_2 outperforms π^*

  → pick one of them.
- Generally: for k-dimensional feature space the user picks between k+1 policies



Feature expectation matching

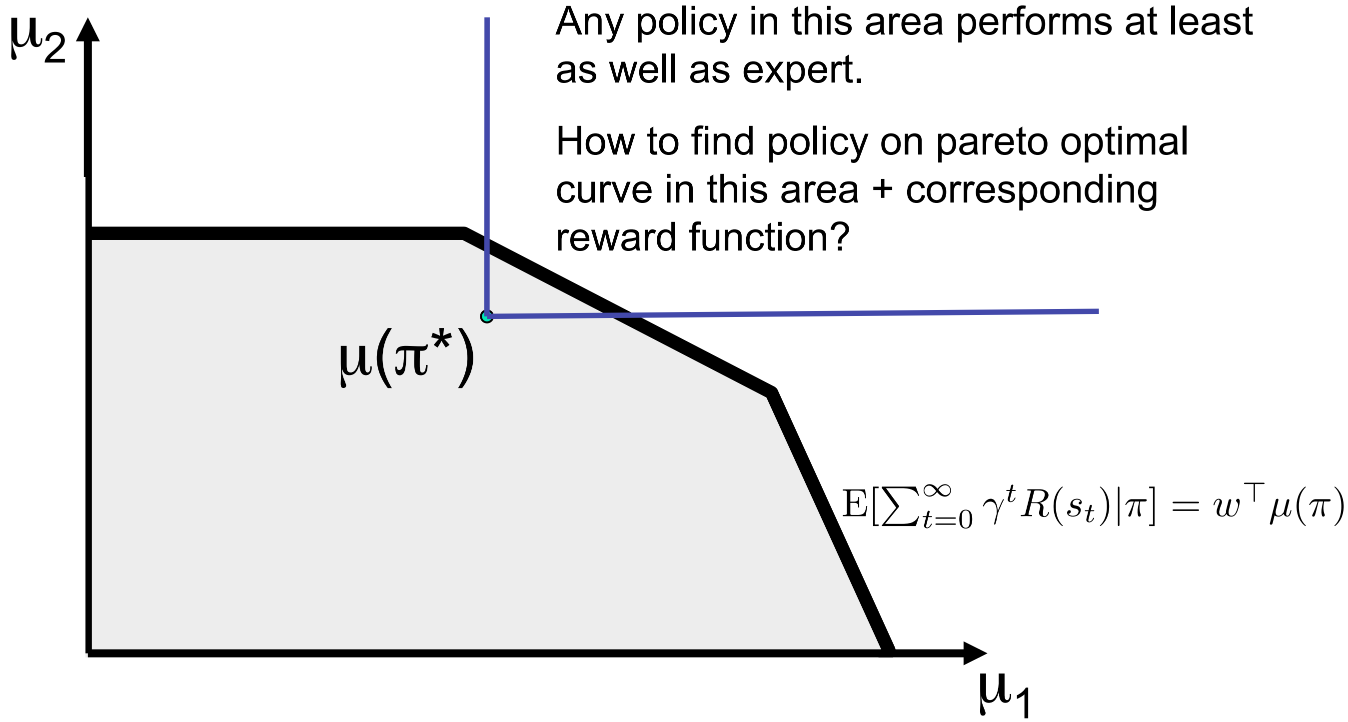
- If expert suboptimal then the resulting policy is a mixture of somewhat arbitrary policies which have expert in their convex hull.

- In practice: pick the best one of this set and pick the corresponding reward function.

- Next:
  - Syed and Schapire, 2008.
  - Ziebart+al, 2008.

Min-Max feature expectation matching [Syed and Schapire (2008)]

Additional assumption: $w \geq 0$, $\sum_i w_i = 1$.



Min-Max feature expectation matching [Syed and Schapire (2008)]

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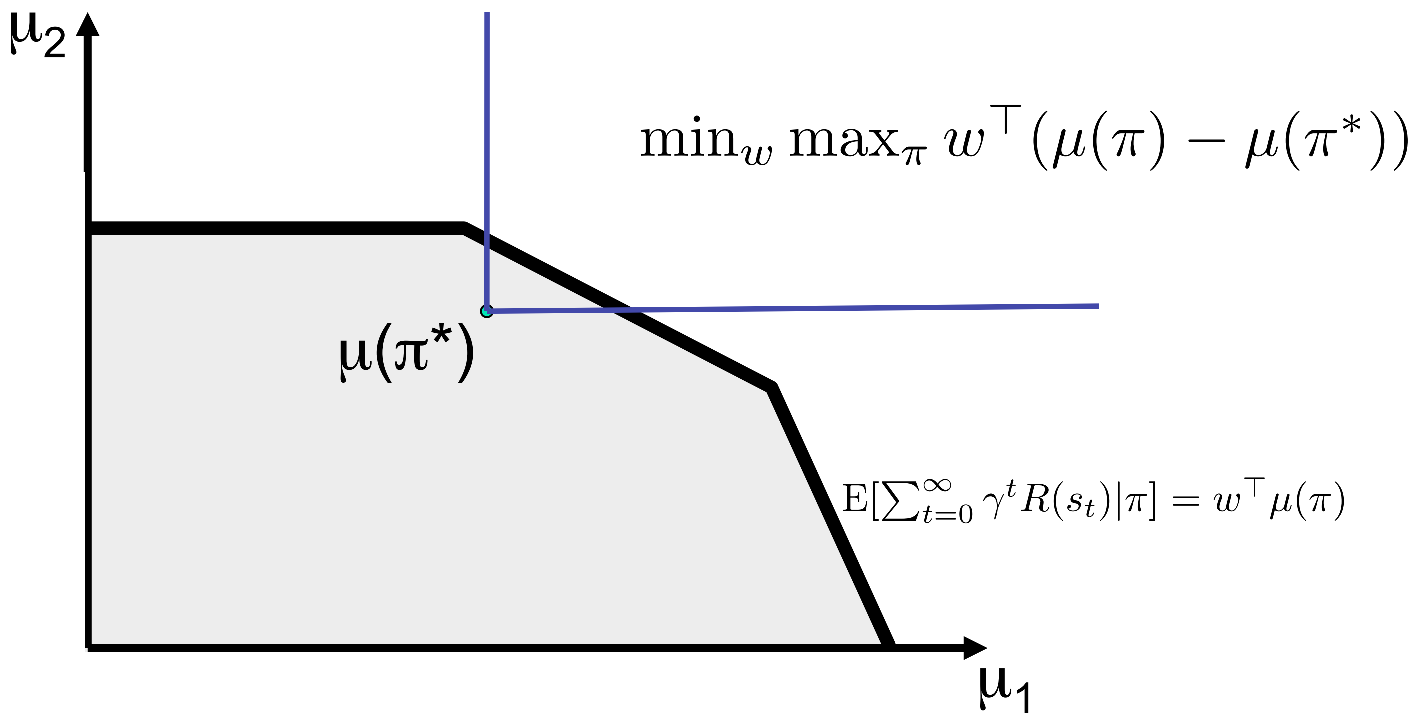


Min max games

- Example of standard min-max game setting:

  rock-paper-scissors pay-off matrix:

		maximizer		
		rock	paper	scissors
minimizer	rock	0	1	-1
	paper	-1	0	1
	scissors	1	-1	0

pay-off matrix G

$$\boxed{\min_{w_m : w_m \geq 0, \|w_m\|_1 = 1} \max_{w_M : w_M \geq 0, \|w_M\|_1 = 1} w_m^\top G w_M}$$

Min-Max feature expectation matching [Syed and Schapire (2008)]

- Standard min-max game:

$$\min_{w_m : w_m \geq 0, \|w_m\|_1 = 1} \max_{w_M : w_M \geq 0, \|w_M\|_1 = 1} w_m^\top G w_M$$

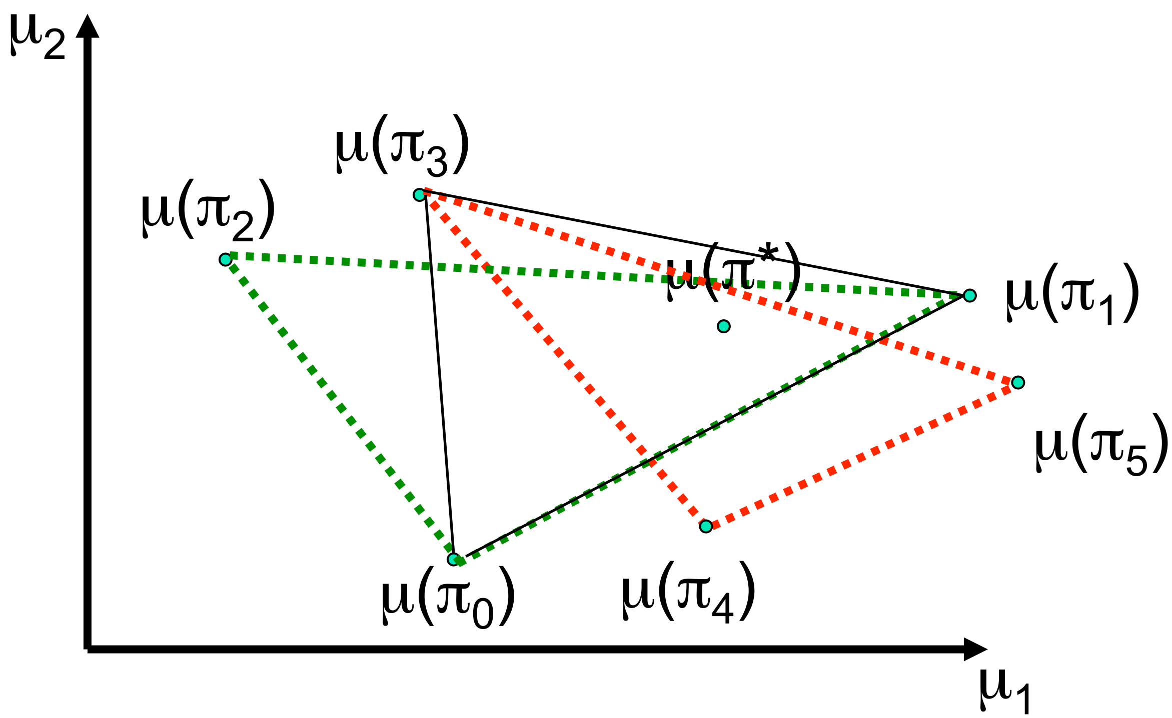
- Min-max inverse RL:

$$\min_{w : \|w\|_1 = 1, w \geq 0} \max_\pi w^\top (\mu(\pi) - \mu(\pi^*))$$

- Solution: maximize over weights ¸ which weigh the contribution of all policies μ_1, μ_2, …, μ_N to the mixed policy.
- Formally: $\min_w \max_\lambda w^\top G \lambda$ $\quad G_{ij} = (\mu(\pi_j) - \mu(\pi^*))_i$

Maximum-entropy feature expectation matching --- [Ziebart+al, 2008]

- Recall feature matching in suboptimal expert case:



Maximum-entropy feature expectation matching --- [Ziebart+al, 2008]

- Maximize entropy of distributions over paths followed while satisfying the constraint of feature expectation matching:

$$
\begin{aligned}
\max_P \quad & -\sum_\zeta P(\zeta) \log P(\zeta) \\
\text{s.t.} \quad & \sum_\zeta P(\zeta)\mu(\zeta) = \mu(\pi^*)
\end{aligned}
$$

- This turns out to imply that P is of the form:

$$
P(\zeta) = \frac{1}{Z(w)} \exp(w^\top \mu(\zeta))
$$

Feature expectation matching

- If expert suboptimal:
  - *Abbeel and Ng, 2004*: resulting policy is a mixture of policies which have expert in their convex hull---In practice: pick the best one of this set and pick the corresponding reward function.
  - *Syed and Schapire, 2008* recast the same problem in game theoretic form which, at cost of adding in some prior knowledge, results in having a unique solution for policy and reward function.
  - *Ziebart+al, 2008* assume the expert stochastically chooses between paths where each path's log probability is given by its expected sum of rewards.

Lecture outline

- Example applications
- Inverse RL vs. behavioral cloning
- Historical sketch of inverse RL
- Mathematical formulations for inverse RL
  - Max-margin
  - Feature matching
  - *Reward function parameterizing the policy class*
- Case studies

Reward function parameterizing the policy class

- Recall:

$$V^*(s;R) = R(s) + \gamma \max_a \sum_{s'} P(s'|s,a) V^*(s;R)$$

$$Q^*(s,a;R) = R(s) + \gamma \sum_{s'} P(s'|s,a) V^*(s;R)$$

- Let's assume our expert acts according to:

$$\pi(a|s;R,\alpha) = \frac{1}{Z(s;R,\alpha)} \exp(\alpha Q^*(s,a;R))$$

- Then for any R and α, we can evaluate the likelihood of seeing a set of state-action pairs as follows:

$$P((s_1,a_1))\dots P((s_m,a_m)) = \frac{1}{Z(s_1;R,\alpha)} \exp(\alpha Q^*(s_1,a_1;R))\dots \frac{1}{Z(s_m;R,\alpha)} \exp(\alpha Q^*(s_m,a_m;R))$$

Reward function parameterizing the policy class

- Assume our expert acts according to:

$$\pi(a|s;R,\alpha) = \frac{1}{Z(s;R,\alpha)} \exp(\alpha Q^*(s,a;R))$$

- Then for any R and α, can evaluate likelihood of set of state-action pairs:

$$P((s_1,a_1))\ldots P((s_m,a_m)) = \frac{1}{Z(s_1;R,\alpha)} \exp(\alpha Q^*(s_1,a_1;R))\ldots \frac{1}{Z(s_m;R,\alpha)} \exp(\alpha Q^*(s_m,a_m;R))$$

- Ramachandran and Amir, AAAI2007: MCMC method to sample from this distribution
- Neu and Szepesvari, UAI2007: gradient method to optimize likelihood [MAP]
- Baker, Saxe and Tenenbaum, Cognition 2009: only 3 possible reward functions → tractable exact Bayesian inference

Reward function parameterizing the policy class -- deterministic systems

- Assume deterministic system $x_{t+1} = f(x_t, u_t)$ and an observed trajectory $(x_0^*, x_1^*, \ldots, x_T^*)$

- Find reward function by solving:

$$
\begin{aligned}
\min_w \ & \textstyle\sum_{t=0}^{T} \|x_t^* - x_t^w\|_2 \\
\text{s.t.} \ & x^w \text{is the solution of:} \\
& \quad \max_x \textstyle\sum_{t=0}^{T} \sum_i w_i \phi_i(x_t) \\
& \quad \text{s.t.} \ \ x_{t+1} = f(x_t, u_t) \\
& \quad \qquad x_0 = x_0^*, \ \ x_T = x_T^*
\end{aligned}
$$

[Mombaur, Truong, Laumond, 2009]

Lecture outline

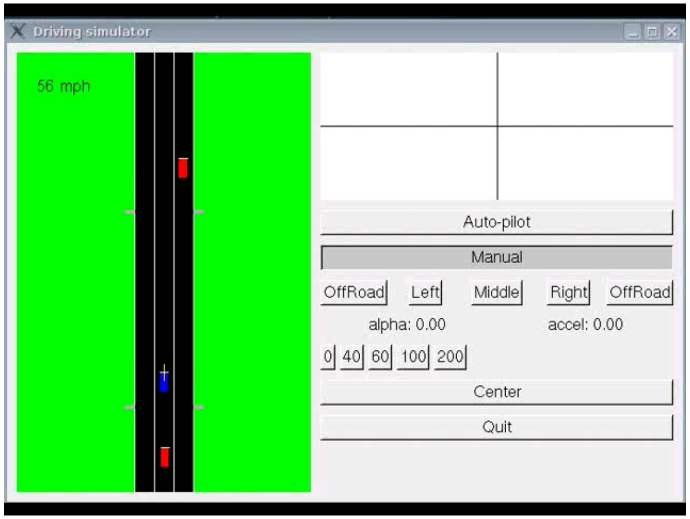
- Example applications
- Inverse RL vs. behavioral cloning
- History of inverse RL
- Mathematical formulations for inverse RL
- *Case studies: (1) Highway driving, (2) Crusher, (3) Parking lot navigation, (4) Route inference, (5) Human path planning, (6) Human inverse planning, (7) Quadruped locomotion*

Simulated highway driving

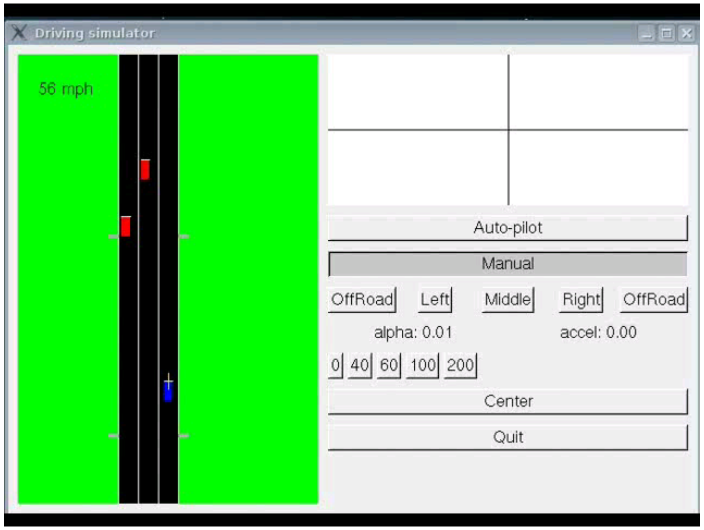
Abbeel and Ng, ICML 2004; Syed and Schapire, NIPS 2007

Highway driving

Teacher in Training World



Learned Policy in Testing World

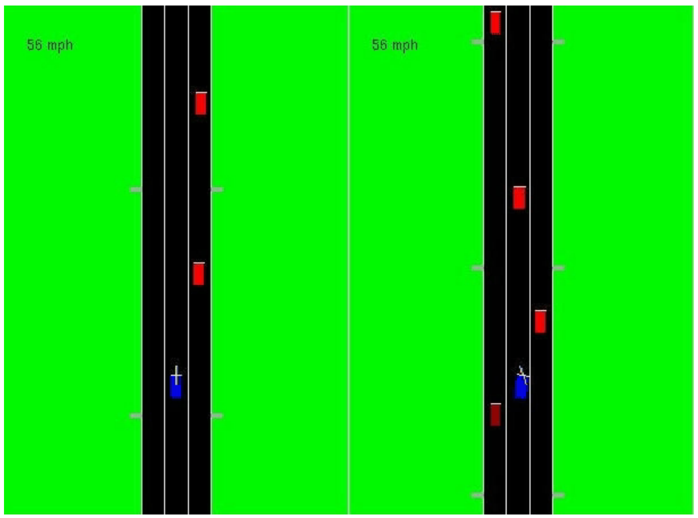


- Input:
  - Dynamics model / Simulator $P_{sa}(s_{t+1} \mid s_t, a_t)$
  - Teacher's demonstration: 1 minute in "training world"
  - Note: R* is unknown.
  - Reward features: 5 features corresponding to lanes/shoulders; 10 features corresponding to presence of other car in current lane at different distances

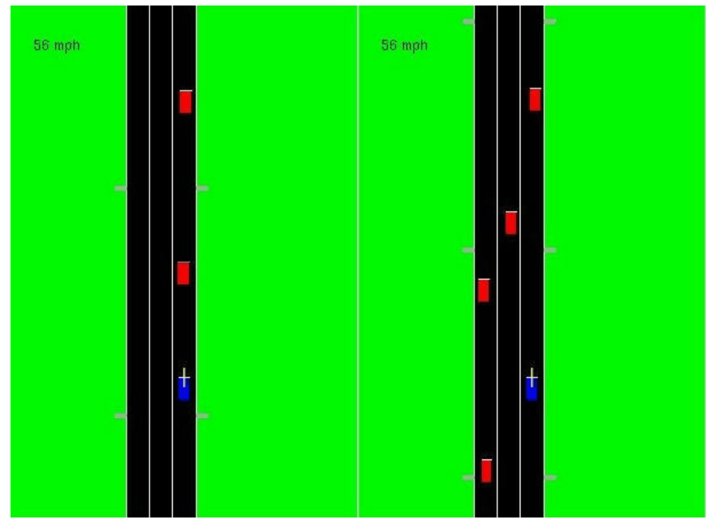
[Abbeel and Ng 2004]

More driving examples

Driving demonstration | Learned behavior



Driving demonstration | Learned behavior



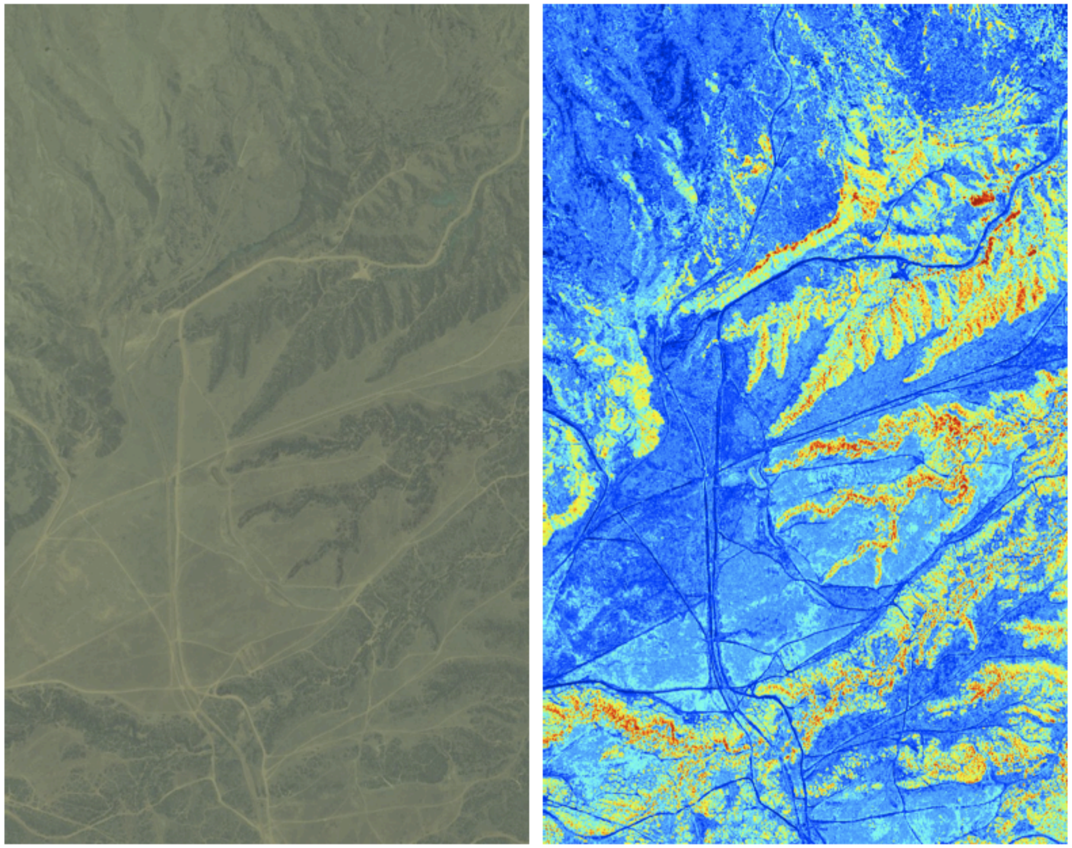
In each video, the left sub-panel shows a demonstration of a different driving "style", and the right sub-panel shows the behavior learned from watching the demonstration.

[Abbeel and Ng 2004]

Crusher

Learning movie

Max margin



[Ratliff + al, 2006/7/8]

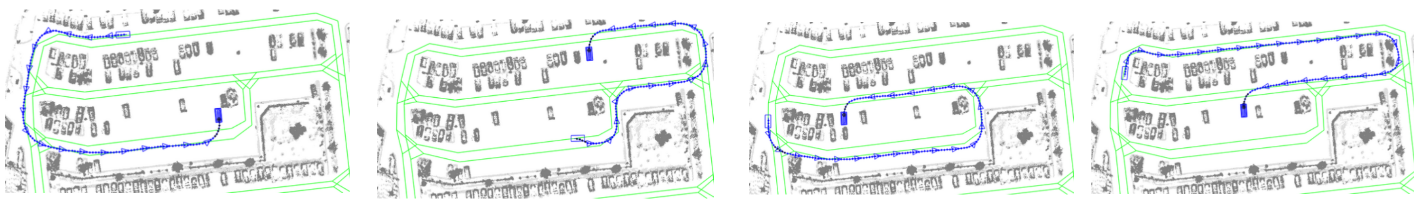
Parking lot navigation

- Reward function trades off:
  - Staying “on-road,”
  - Forward vs. reverse driving,
  - Amount of switching between forward and reverse,
  - Lane keeping,
  - On-road vs. off-road,
  - Curvature of paths.

[Abbeel et al., IROS 08]

Experimental setup

- Demonstrate parking lot navigation on “train parking lots.”

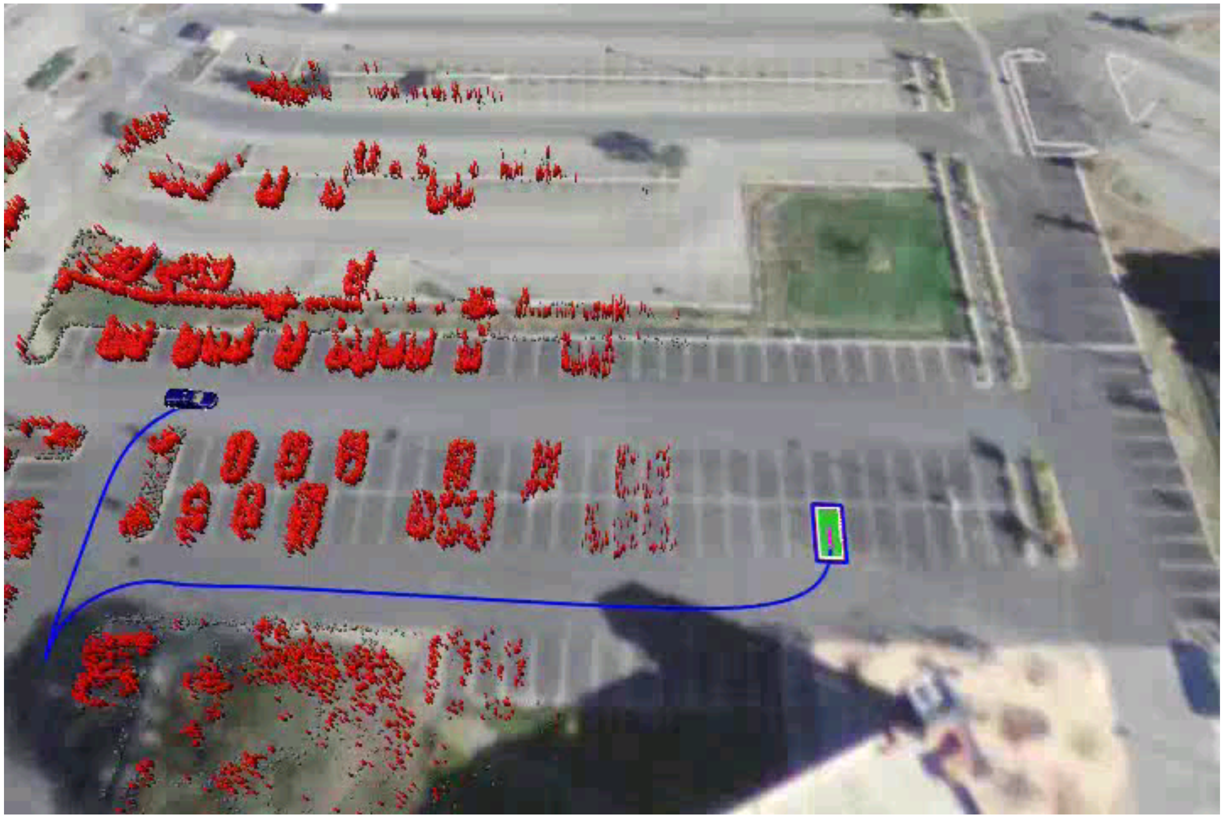


- Run our apprenticeship learning algorithm to find reward function.
- Receive “test parking lot” map + starting point and destination.
- Find the trajectory that maximizes the *learned reward function* for navigating the test parking lot.

Nice driving style

Sloppy driving-style

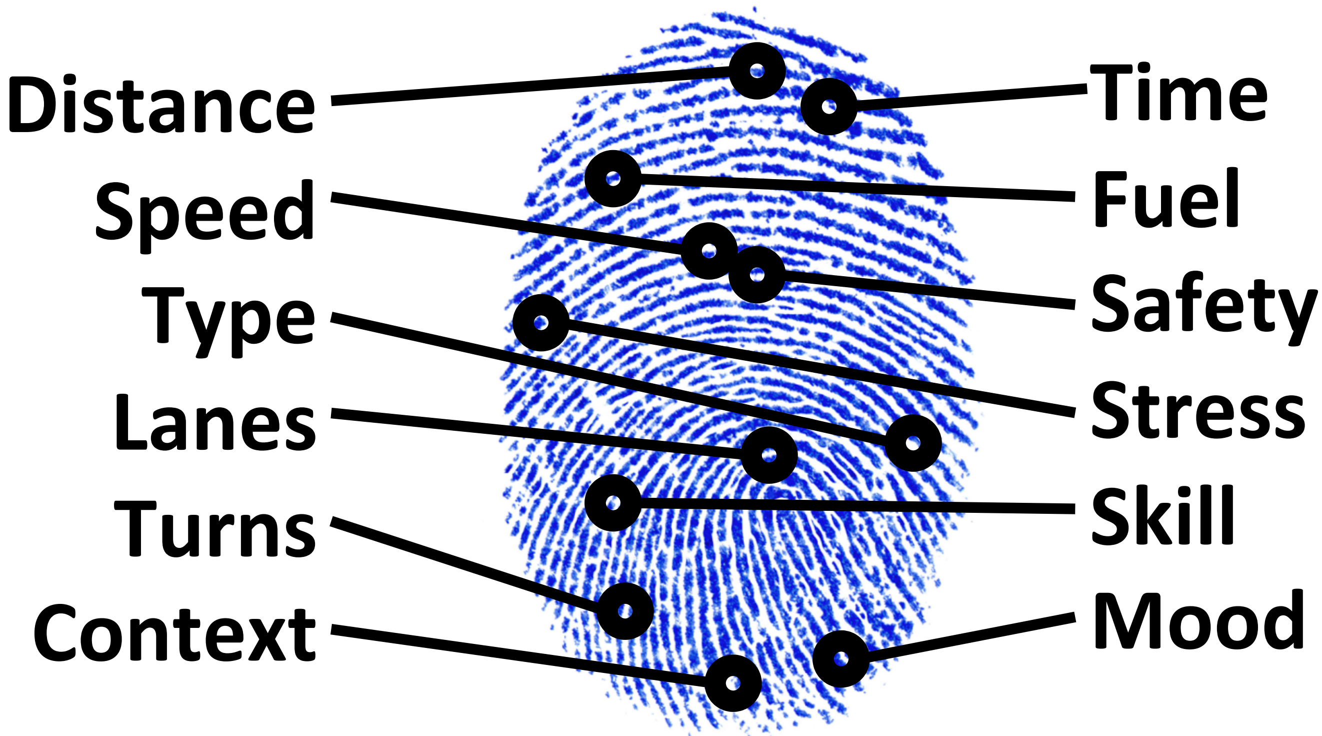
“Don’t mind reverse” driving-style



Only 35% of routes are "fastest"

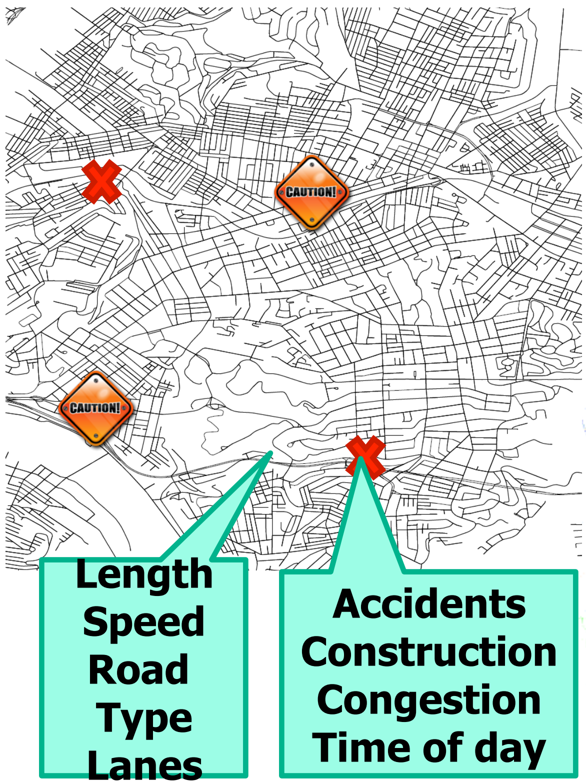
(Letchner, Krumm, & Horvitz 2006)


Time
Money
Stress
Skill
Ziebart+al, 2007/8/9

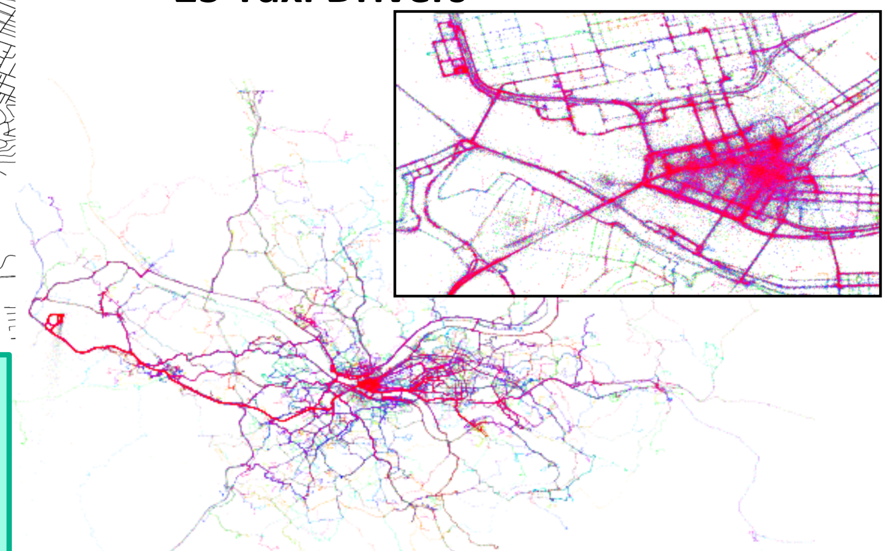


Ziebart+al, 2007/8/9

Data Collection



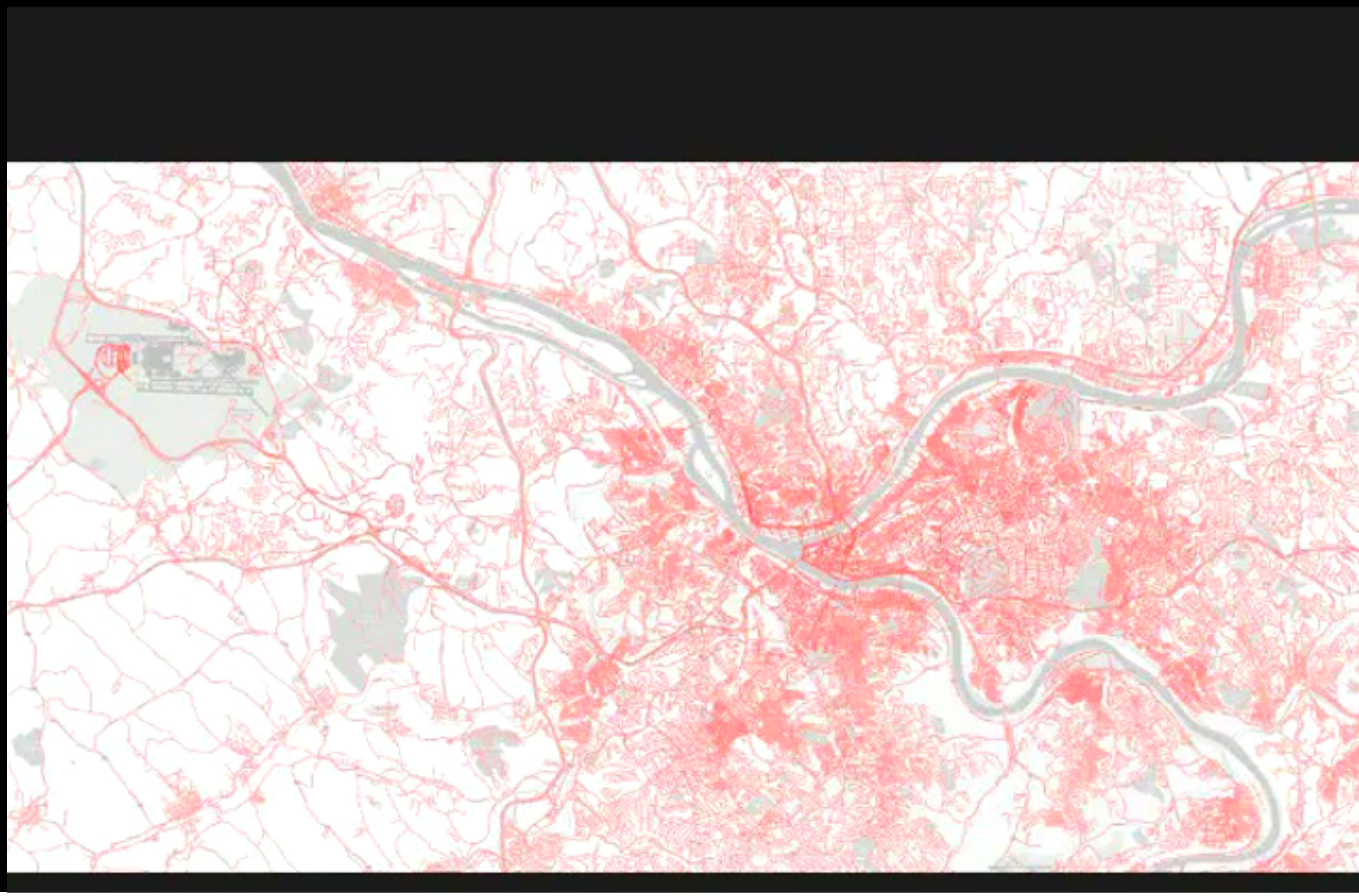
25 Taxi Drivers



Over 100,000 miles

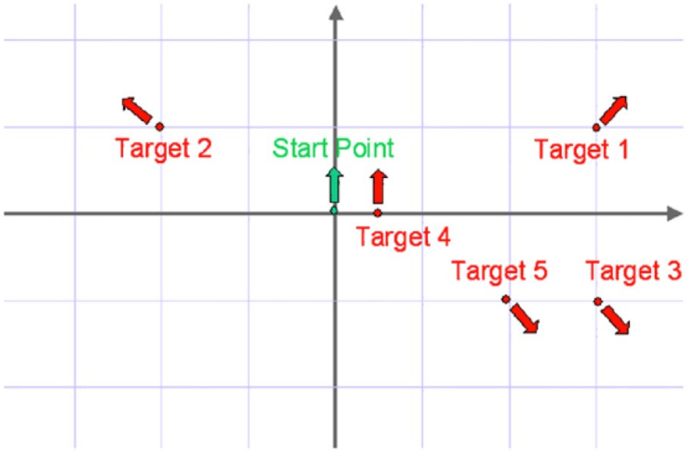
Ziebart+al, 2007/8/9

Destination Prediction



Human path planning

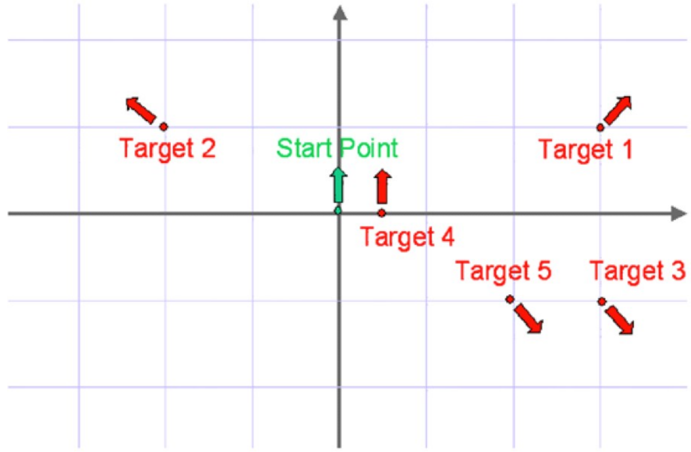
- Reward features:
  - Time to destination
  - (Forward acceleration)^2
  - (Sideways acceleration)^2
  - (Rotational acceleration)^2
  - Integral (angular error)^2



[Mombaur, Truong, Laumond, 2009]

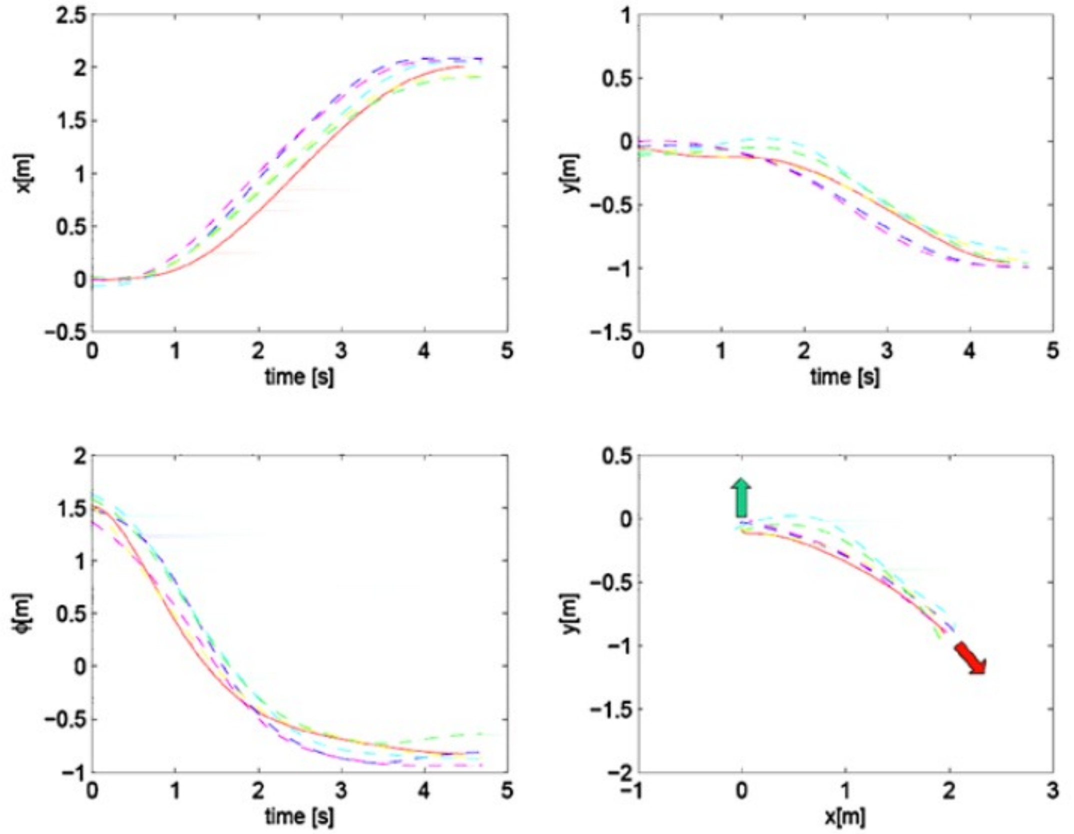
Human path planning

- Result:
  - Time to destination:
  - (Forward acceleration)^2 1.2
  - (Sideways acceleration)^2 1.7
  - (Rotational acceleration)^2 0.7
  - Integral (angular error)^2 5.2



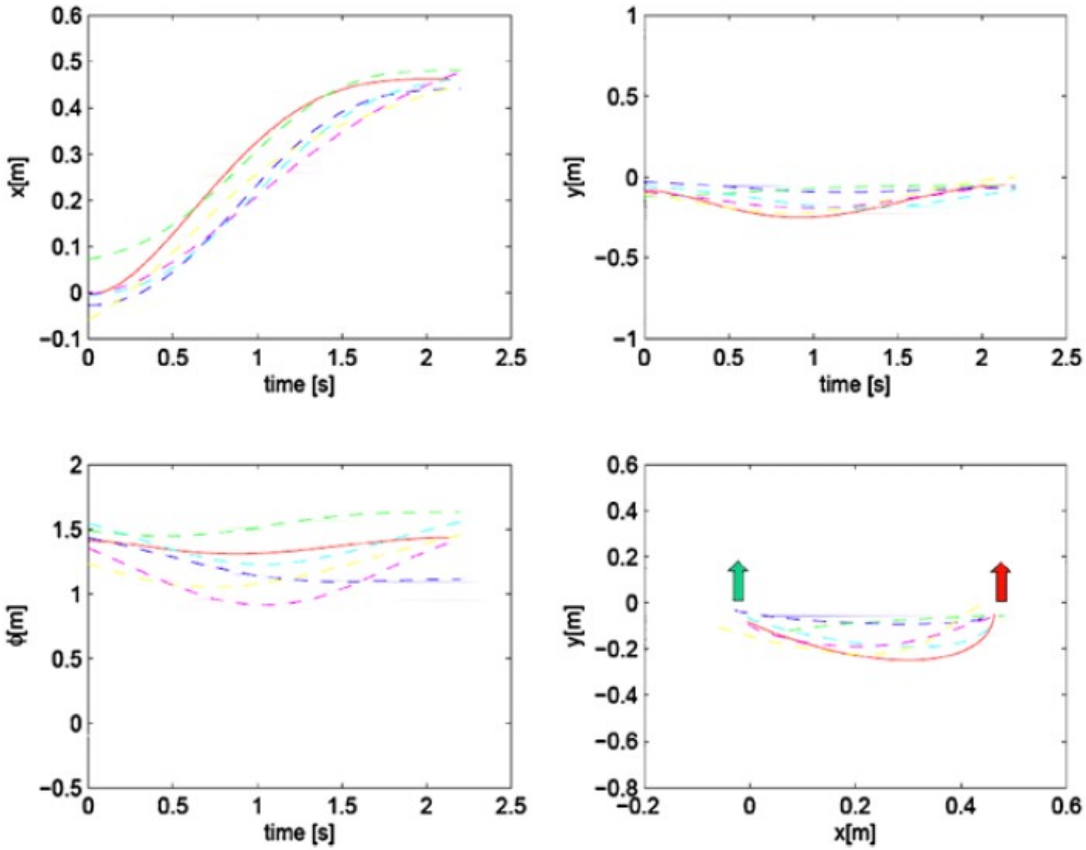
[Mombaur, Truong, Laumond, 2009]

Human path planning



[Mombaur, Truong, Laumond, 2009]

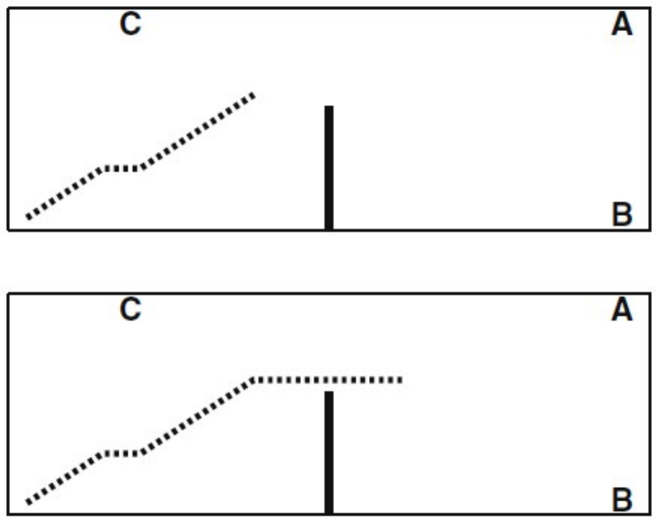
Human path planning



[Mombaur, Truong, Laumond, 2009]

Goal inference

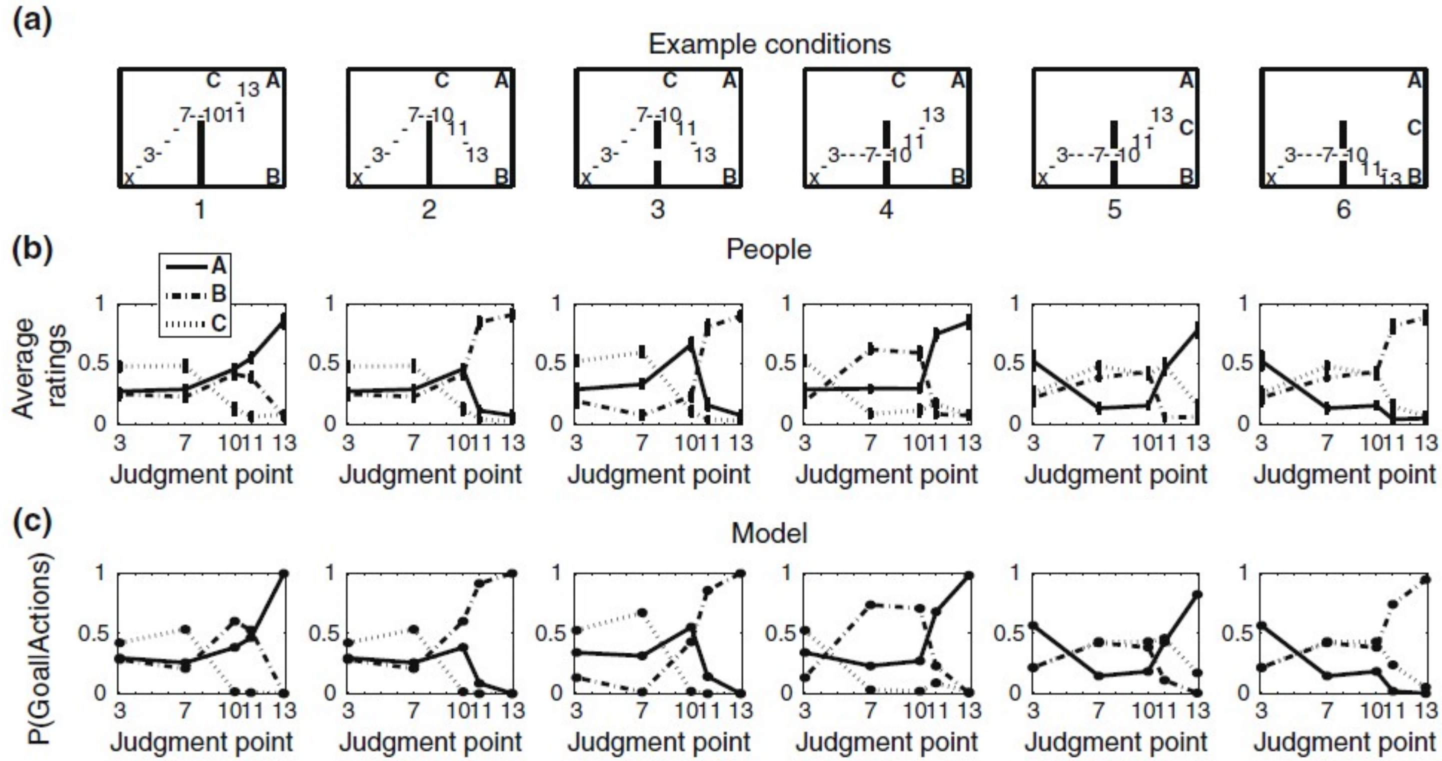
- Observe partial paths, predict goal. Goal could be A, B, or C.
- + HMM-like extension: goal can change (with some probability over time).



[Baker, Saxe, Tenenbaum, 2009]

Goal inference

- Observe partial paths, predict goal. Goal could be A, B, or C.



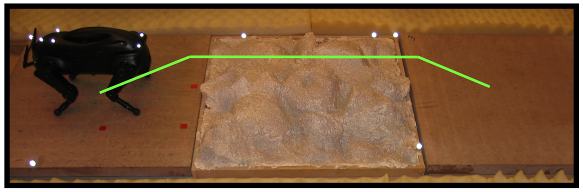
Quadruped

- Reward function trades off 25 features.

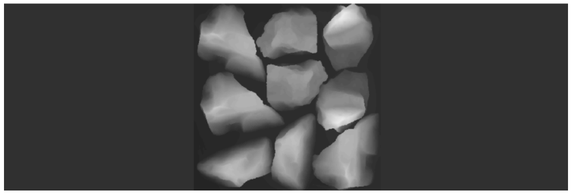
Hierarchical max margin [Kolter, Abbeel & Ng, 2008]

Experimental setup

- Demonstrate path across the "training terrain"



- Run our apprenticeship learning algorithm to find the reward function
- Receive "testing terrain"---height map.



- Find the optimal policy with respect to the *learned reward function* for crossing the testing terrain.

Without learning

With learned reward function

Quadruped: Ratliff + al, 2007

- Run footstep planner as expert (slow!)
- Run boosted max margin to find a reward function that explains the center of gravity path of the robot (smaller state space)

- At control time: use the learned reward function as a heuristic for A* search when performing footstep-level planning

Summary

- Example applications
- Inverse RL vs. behavioral cloning
- Sketch of history of inverse RL
- Mathematical formulations for inverse RL
- Case studies

- Open directions: Active inverse RL, Inverse RL w.r.t. minmax control, partial observability, learning stage (rather than observing optimal policy), … ?