CS61C - Machine Structures

Lecture 10 - Floating Point, Part II and Miscellaneous

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Review

- Floating Point numbers *approximate* values that we want to use.
- IEEE 754 Floating Point Standard is most widely accepted attempt to standardize interpretation of such numbers ($1T)
- New MIPS registers(`$f0`-`$f31`), instruct.:
  - Single Precision (32 bits, $2x10^{-38}$… $2x10^{38}$): `add.s, sub.s, mul.s, div.s`
  - Double Precision (64 bits , $2x10^{-308}$…$2x10^{308}$): `add.d, sub.d, mul.d, div.d`
- Type is not associated with data, bits have no meaning unless given in context

Overview

- Special Floating Point Numbers: NaN, Denorms
- IEEE Rounding modes
- Floating Point fallacies, hacks
- Catchup topics:
  - Representation of jump, jump and link
  - Reverse time travel:
    MIPS machine language
    -> MIPS assembly language
    -> C code
  - Logical, shift instructions (time permiting)

MIPS Floating Point Architecture (1/2)

- 1990 Solution: Make a completely separate chip that handles only FP.
- Coprocessor 1: FP chip
  - contains 32 32-bit registers: `$f0`, `$f1`, …
  - most registers specified in `.s` and `.d` instruction refer to this set
  - separate load and store: `lwc1` and `swc1` ("load word coprocessor 1", "store …")
  - Double Precision: by convention, even/odd pair contain one DP FP number: `$f0`/`$f1`, `$f2`/`$f3`, … , `$f30`/`$f31`

MIPS Floating Point Architecture (2/2)

- 1990 Computer actually contains multiple separate chips:
  - Processor: handles all the normal stuff
  - Coprocessor 1: handles FP and only FP;
  - more coprocessors?… Yes, later
  - Today, cheap chips may leave out FP HW
- Instructions to move data between main processor and coprocessors:
  - `¥mfc0`, `mtc0`, `mfc1`, `mtc1`, etc.
- Appendix pages A-70 to A-74 contain many, many more FP operations.

Special Numbers

- What have we defined so far? (Single Precision)

Exponent	Significand	Object
0	0	0
0	nonzero	???
1-254	anything	+/- fl. pt. #
255	0	+/- infinity
255	nonzero	???

- Professor Kahan had clever ideas; "Waste not, want not"

Representation for Not a Number

° What do I get if I calculate `sqrt(-4.0)` or `0/0`?
- If infinity is not an error, it may be useful not to crash program for these too.
- Called **N**ot **a** **N**umber (NaN)
- Exponent = 255, Significand nonzero

° Why is this useful?
- Hope NaNs help with debugging
- They contaminate: op(NaN,X) = NaN
- OK if calculate but don't use it
- Ask math majors

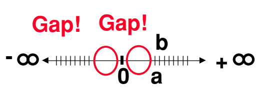
Special Numbers (cont'd)

° What have we defined so far? (Single Precision)?

Exponent	Significand	Object
0	0	0
0	nonzero	???
1-254	anything	+/- fl. pt. #
255	0	+/- infinity
255	nonzero	NaN

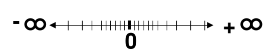
Representation for Denorms (1/2)

° Problem: There's a gap among representable FP numbers around 0
- Smallest representable pos num:
  - $a = 1.0\ldots_2 * 2^{-127} = 2^{-127}$
- Second smallest representable pos num:
  - $b = 1.000\ldots\ldots1_2 * 2^{-127} = 2^{-127} + 2^{-150}$
- $a - 0 = 2^{-127}$
- $b - a = 2^{-150}$



Representation for Denorms (2/2)

° Solution:
- We still haven't used Exponent = 0, Significand nonzero
- Denormalized number: no leading 1
- Smallest representable pos num:
  - $a = 2^{-150}$
- Second smallest representable pos num:
  - $b = 2^{-149}$



Rounding

° When we perform math on real numbers, we have to worry about rounding

° The actual math carries two extra bits of precision, and then round to get the proper value

° Rounding also occurs when converting a double to a single precision value, or converting a floating point number to an integer

4 IEEE Rounding Modes

° Round towards +infinity
- ALWAYS round "up": 2.001 -> 3
- -2.001 -> -2

° Round towards -infinity
- ALWAYS round "down": 1.999 -> 1,
- -1.999 -> -2

° Truncate: 2.001 -> 2, -2.001 -> -2
- Just drop the last bits (round towards 0)

° Round to (nearest) even
- Normal rounding, almost

Round to Even

- Round like you learned in grade school
- Except if the value is right on the borderline, in which case we round to the nearest EVEN number
  - 2.5 -> 2
  - 3.5 -> 4
- Insures fairness on calculation
  - This way, half the time we round up on tie, the other half time we round down
  - Ask statistics majors
- Default C rounding mode; only Java mode

Floating Point Fallacy

- FP Add, subtract associative: FALSE!
  - x = – 1.5 x 10^{38}, y = 1.5 x 10^{38}, and z = 1.0
  - x + (y + z) = –1.5x10^{38} + (1.5x10^{38} + 1.0)
    = –1.5x10^{38} + (1.5x10^{38}) = 0.0
  - (x + y) + z = (–1.5x10^{38} + 1.5x10^{38}) + 1.0
    = (0.0) + 1.0 = 1.0
- Therefore, Floating Point add, subtract are not associative!
  - Why? FP result approximates real result!
  - This exampe: 1.5 x 10^{38} is so much larger than 1.0 that 1.5 x 10^{38} + 1.0 in floating point representation is still 1.5 x 10^{38}

Casting floats to ints and vice versa

- `(int) exp`
  - Coerces and converts it to the nearest integer
  - affected by rounding modes
  - `i = (int) (3.14159 * f);`
- `(float) exp`
  - converts integer to nearest floating point
  - `f = f + (float) i;`

int -> float -> int

```
if (i == (int)((float) i)) {
 printf( true );
}
```

- Will not always work
- Large values of integers don't have exact floating point representations
- Similarly, we may round to the wrong value

float -> int -> float

```
if (f == (float)((int) f)) {
 printf( true );
}
```

- Will not always work
- Small values of floating point don't have good integer representations
- Also rounding errors

Administrivia

- Need to catchup with Homework
- Reading assignment: Reading 4.8

J-Format Instructions (1/5)

- For branches, we assumed that we won't want to branch too far, so we can specify *change* in PC.
- For general jumps (`j` and `jal`), we may jump to *anywhere* in memory.
- Ideally, we could specify a 32-bit memory address to jump to.
- Unfortunately, we can't fit both a 6-bit opcode and a 32-bit address into a single 32-bit word, so we compromise.

J-Format Instructions (2/5)

- Define "fields" of the following number of bits each:

6 bits	26 bits

- As usual, each field has a name:

opcode	target address

- **Key Concepts**
  - Keep `opcode` field identical to R-format and I-format for consistency.
  - Combine all other fields to make room for target address.

J-Format Instructions (3/5)

- For now, we can specify 26 bits of the 32-bit bit address.
- Optimization:
  - Note that, just like with branches, jumps will only jump to word aligned addresses (since all instructions are one word long), so last two bits are always 00 (in binary).
  - So let's just take this for granted and not even specify them.
  - => 26 bits supplies a 28-bit byte address

J-Format Instructions (4/5)

- For now, we can specify 28 bits of the 32-bit address.
- Where do we get the other 4 bits?
  - By definition, take the 4 highest order bits from the PC.
  - Technically, this means that we cannot jump to *anywhere* in memory, but it's adequate 99.9999...% of the time, since programs rarely that long (> 2^{28} or 256 MB)
  - If we absolutely need to specify a 32-bit address, we can always put it in a register and use the `jr` instruction.

J-Format Instructions (5/5)

- Summary:
  - New PC = PC[31..28]
    || target address (26 bits)
    || 00
  - Note: II means concatenation
    4 bits || 26 bits || 2 bits = 32-bit address
- Understand where each part came from!

Decoding Machine Language

- How do we convert 1s and 0s to C code?
- For each 32 bits:
  - Look at `opcode` value: 0 means R-Format, 2 or 3 mean J-Format, otherwise I-Format.
  - Use instruction type to determine which fields exist and convert each field into the decimal equivalent.
  - Once we have decimal values, write out MIPS assembly code.
  - Logically convert this MIPS code into valid C code.

Decoding Example (1/6)

- Here are six machine language instructions in hex:

```
00001025
0005402A
11000003
00441020
20A5FFFF
08100001
```

- Let the first instruction be at address 4,194,304$_{10}$ (0x00400000).
- Next step: convert to binary

Decoding Example (2/6)

- Here are the six machine language instructions in binary:

```
00000000000000000001000000100101
00000000000001010100000000101010
00010001000000000000000000000011
00000000010001000001000000100000
00100000101001011111111111111111
00001000000100000000000000000001
```

- Next step: separation of fields & convert each field to decimal
  - For all instructions, first 6 bits is opcode, so can easily determine format/instruction

Decoding Example (3/6)

- Decimal representation, in fields:

Format:						
R	0	0	0	2	0	37
R	0	0	5	8	0	42
I	4	8	0	+3		
R	0	2	4	2	0	32
R	8	5	5	-1		
J	2	1,048,577				

- Next step: translate to MIPS instructions

Decoding Example (4/6)

- MIPS Assembly (Part 1):

```
0x00400000  or    $2,$0,$0
0x00400004  slt   $8,$0,$5
0x00400008  beq   $8,$0,3
0x0040000c  add   $2,$2,$4
0x00400010  addi  $5,$5,-1
0x00400014  j     0x100001
```

- Next step: translate to more meaningful instructions (fix the branch/jump and add labels)
  - Remember: jump address add 00 to end

Decoding Example (5/6)

- MIPS Assembly (Part 2):

```
        or    $v0,$0,$0
Loop:   slt   $t0,$0,$a1
        beq   $t0,$0,Fin
        add   $v0,$v0,$a0
        addi  $a1,$a1,-1
        j     Loop
Fin:
```

- Next step: translate to C code (be creative!)

Decoding Example (6/6)

- C code:
  - Mapping: $v0: product
    $a0: mcand
    $a1: mplier

```
product = 0;
while (mplier > 0) {
    product += mcand;
    mplier -= 1;
}
```

Bitwise Operations (1/2)

- Up until now, we've done arithmetic (`add`, `sub`, `addi`) and memory access (`lw` and `sw`)
- All of these instructions view contents of register as a single quantity (such as a signed or unsigned integer)
- New Perspective: View contents of register as 32 bits rather than as a single 32-bit number

Bitwise Operations (2/2)

- Since registers are composed of 32 bits, we may want to access individual bits rather than the whole.
- Introduce two new classes of instructions:
  - Logical Operators
  - Shift Instructions

Logical Operators (1/4)

- How many of you have taken Math 55?
- Two basic logical operators:
  - AND: outputs 1 only if both inputs are 1
  - OR: outputs 1 if at least one input is 1
- In general, can define them to accept >2 inputs, but in the case of MIPS assembly, both of these accept exactly 2 inputs and produce 1 output
  - Again, rigid syntax, simpler hardware

Logical Operators (2/4)

- Truth Table: standard table listing all possible combinations of inputs and resultant output for each
- Truth Table for AND and OR

A	B	AND	OR
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	1

Logical Operators (3/4)

- Logical Instruction Syntax:

  1 2,3,4

  - where
    1) operation name
    2) register that will receive value
    3) first operand (register)
    4) second operand (register) or immediate (numerical constant)

Logical Operators (4/4)

- Instruction Names:
  - ¥`and`, `or`: Both of these expect the third argument to be a register
  - ¥`andi`, `ori`: Both of these expect the third argument to be an immediate
- MIPS Logical Operators are all bitwise, meaning that bit 0 of the output is produced by the respective bit 0's of the inputs, bit 1 by the bit 1's, etc.

Shift Instructions (1/4)

- Move (shift) all the bits in a word to the left or right by a number of bits, filling the emptied bits with 0s.
  - Example: shift right by 8 bits

0001 0010 0011 0100 0101 0110 0111 1000

0000 0000 0001 0010 0011 0100 0101 0110

  - Example: shift left by 8 bits

0001 0010 0011 0100 0101 0110 0111 1000

0011 0100 0101 0110 0111 1000 0000 0000

Shift Instructions (2/4)

- Shift Instruction Syntax:

1 2,3,4

  - where
    1) operation name
    2) register that will receive value
    3) first operand (register)
    4) second operand (register)

Shift Instructions (3/4)

- MIPS has three shift instructions:
  1. `sll` (shift left logical): shifts left and fills emptied bits with 0s
  2. `srl` (shift right logical): shifts right and fills emptied bits with 0s
  3. `sra` (shift right arithmetic): shifts right and fills emptied bits by sign extending

Shift Instructions (4/4)

- Example: shift right arith by 8 bits

0001 0010 0011 0100 0101 0110 0111 1000

0000 0000 0001 0010 0011 0100 0101 0110

- Example: shift right arith by 8 bits

1001 0010 0011 0100 0101 0110 0111 1000

1111 1111 1001 0010 0011 0100 0101 0110

Uses for Logical Operators (1/3)

- Note that `and`ing a bit with 0 produces a 0 at the output while `and`ing a bit with 1 produces the original bit.
- This can be used to create a **mask**.
  - Example:

1011 0110 1010 0100 0011 1101 1001 1010

Mask: 0000 0000 0000 0000 0000 1111 1111 1111

  - The result of `and`ing these two is:

0000 0000 0000 0000 0000 1101 1001 1010

Uses for Logical Operators (2/3)

- The second bitstring in the example is called a **mask**. It is used to isolate the rightmost 12 bits of the first bitstring by masking out the rest of the string (e.g. setting it to all 0s).
- Thus, the `and` operator can be used to set certain portions of a bitstring to 0s, while leaving the rest alone.
  - In particular, if the first bitstring in the above example were in $t0, then the following instruction would mask it:

```
andi  $t0,$t0,0xFFF
```

Uses for Logical Operators (3/3)

- Similarly, note that `or`ing a bit with 1 produces a 1 at the output while `or`ing a bit with 0 produces the original bit.
- This can be used to force certain bits of a string to 1s.
  - For example, if $t0 contains 0x12345678, then after this instruction:

    ori $t0, $t0, 0xFFFF

  - … $t0 contains 0x1234FFFF (e.g. the high-order 16 bits are untouched, while the low-order 16 bits are forced to 1s).

Uses for Shift Instructions (1/5)

- Suppose we want to isolate byte 0 (rightmost 8 bits) of a word in $t0. Simply use:

```
andi    $t0,$t0,0xFF
```

- Suppose we want to isolate byte 1 (bit 15 to bit 8) of a word in $t0. We can use:

```
andi    $t0,$t0,0xFF00
```

but then we still need to shift to the right by 8 bits...

Uses for Shift Instructions (2/5)

- Instead, use:

```
sll    $t0,$t0,16
srl    $t0,$t0,24
```

0001 0010 0011 0100 0101 0110 0111 1000

0101 0110 0111 1000 0000 0000 0000 0000

0000 0000 0000 0000 0000 0000 0101 0110

Uses for Shift Instructions (3/5)

- In decimal:
  - Multiplying by 10 is same as shifting left by 1:
    - $714_{10} \times 10_{10} = 7140_{10}$
    - $56_{10} \times 10_{10} = 560_{10}$
  - Multiplying by 100 is same as shifting left by 2:
    - $714_{10} \times 100_{10} = 71400_{10}$
    - $56_{10} \times 100_{10} = 5600_{10}$
  - Multiplying by 10^n is same as shifting left by n

Uses for Shift Instructions (4/5)

- In binary:
  - Multiplying by 2 is same as shifting left by 1:
    - $11_2 \times 10_2 = 110_2$
    - $1010_2 \times 10_2 = 10100_2$
  - Multiplying by 4 is same as shifting left by 2:
    - $11_2 \times 100_2 = 1100_2$
    - $1010_2 \times 100_2 = 101000_2$
  - Multiplying by 2^n is same as shifting left by n

Uses for Shift Instructions (5/5)

- Since shifting is so much faster than multiplication (you can imagine how complicated multiplication is), a good compiler usually notices when C code multiplies by a power of 2 and compiles it to a shift instruction:

  `a *= 8;` (in C)

  would compile to:

  `sll    $s0,$s0,3` (in MIPS)

Things to Remember (1/3)

- **IEEE 754 Floating Point Standard: Kahan pack as much in as could get away with**
  - +/- infinity, Not-a-Number (Nan), Denorms
  - 4 rounding modes
- **Stored Program Concept: Both data and actual code (instructions) are stored in the same memory.**
- **Type is not associated with data, bits have no meaning unless given in context**

Things to Remember (2/3)

- **Machine Language Instruction: 32 bits representing a single MIPS instruction**

R	opcode	rs	rt	rd	shamt	funct
I	opcode	rs	rt	immediate		
J	opcode	target address				

- **Instructions formats are kept as similar as possible.**
- **Branches and Jumps were optimized for greater branch distance and hence strange, so clear these up in your mind now.**

Things to Remember (3/3)

- **New Instructions:**

```
and, andi, or, ori
sll, srl, sra
```