



Figure 13.4 (a) A node X is conditionally independent of its non-descendants (e.g., the Z_{ij} s) given its parents (the U_i s shown in the lavender area). (b) A node X is conditionally independent of all other nodes in the network given its Markov blanket (the lavender area).

For example, in Figure 13.2, the variable *JohnCalls* is independent of *Burglary*, *Earthquake*, and *MaryCalls* given the value of *Alarm*. The definition is illustrated in Figure 13.4(a).

It turns out that the non-descendants property combined with interpretation of the network parameters $\theta(X_i | \text{Parents}(X_i))$ as conditional probabilities $\mathbf{P}(X_i | \text{Parents}(X_i))$ suffices to reconstruct the full joint distribution given in Equation (13.2). In other words, one can view the semantics of Bayes nets in a different way: instead of defining the full joint distribution as the product of conditional distributions, the network defines a set of conditional independence properties. The full joint distribution can be derived from those properties.

Another important independence property is implied by the non-descendants property:

a variable is conditionally independent of all other nodes in the network, given its parents, children, and children's parents—that is, given its **Markov blanket**.

Markov blanket

(Exercise 13.MARB asks you to prove this.) For example, the variable *Burglary* is independent of *JohnCalls* and *MaryCalls*, given *Alarm* and *Earthquake*. This property is illustrated in Figure 13.4(b). The Markov blanket property makes possible inference algorithms that use completely local and distributed stochastic sampling processes, as explained in Section 13.4.2.

The most general conditional independence question one might ask in a Bayes net is whether a set of nodes \mathbf{X} is conditionally independent of another set \mathbf{Y} , given a third set \mathbf{Z} . This can be determined efficiently by examining the Bayes net to see whether \mathbf{Z} **d-separates** \mathbf{X} and \mathbf{Y} . The process works as follows:

D-separation

1. Consider just the **ancestral subgraph** consisting of \mathbf{X} , \mathbf{Y} , \mathbf{Z} , and their ancestors.
2. Add links between any unlinked pair of nodes that share a common child; now we have the so-called **moral graph**.
3. Replace all directed links by undirected links.
4. If \mathbf{Z} blocks all paths between \mathbf{X} and \mathbf{Y} in the resulting graph, then \mathbf{Z} d-separates \mathbf{X} and \mathbf{Y} . In that case, \mathbf{X} is conditionally independent of \mathbf{Y} , given \mathbf{Z} . Otherwise, the original Bayes net does not require conditional independence.

Ancestral subgraph

Moral graph