LOCAL SEARCH ALGORITHMS

CHAPTER 4, SECTIONS 3–4

Outline

- ♢ Hill-climbing
- ♢ Simulated annealing
- ♢ Genetic algorithms (briefly)
- ♢ Local search in continuous spaces (very briefly)

Iterative improvement algorithms

In many optimization problems, **path** is irrelevant;
the goal state itself is the solution

Then state space = set of "complete" configurations
(complete-state formulation vs. incremental formulation)

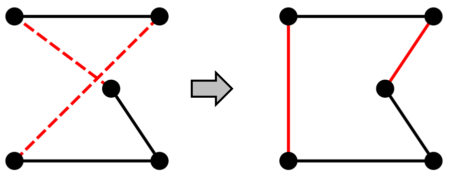
In such cases, can use iterative improvement algorithms;
keep a single "current" state, try to improve it

Constant space, suitable for online as well as offline search

Often want to find **optimal** configuration, e.g., TSP,
but also works for constraint satisfaction problems, e.g. $n$queens, timetabling

Example: Travelling Salesperson Problem

Start with any complete tour, perform pairwise exchanges

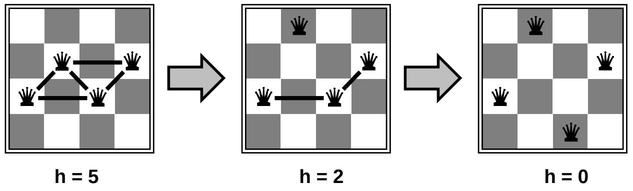


Variants of this approach get within 1% of optimal very quickly with thousands of cities

Example: n-queens

Put n queens on an $n \times n$ board with no two queens on the same row, column, or diagonal

Local search: start with all n, move a queen to reduce conflicts



Almost always solves n-queens problems almost instantaneously
for very large n, e.g., $n = 1$ *million*

(Why? Perhaps because choices for queen k are made wrt **all** others)

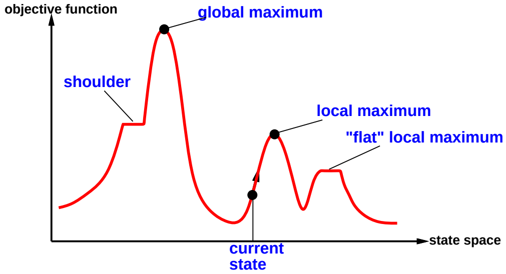
Hill-climbing (or gradient ascent/descent)

"Like climbing Everest in thick fog with amnesia"

```
function HILL-CLIMBING(problem) returns a state that is a local maximum
   inputs: problem, a problem
   local variables: current, a node
                    neighbor, a node
   current ← MAKE-NODE(INITIAL-STATE[problem])
   loop do
       neighbor ← a highest-valued successor of current
       if VALUE[neighbor] ≤ VALUE[current] then return STATE[current]
       current ← neighbor
   end
```

Hill-climbing contd.

Useful to consider state space landscape



Random-restart hill climbing overcomes local maxima—trivially complete

Random sideways moves ☺escape from shoulders ☹loop on flat maxima

Simulated annealing

Idea: escape local maxima by allowing some "bad" moves
but gradually decrease their size and frequency

```
function SIMULATED-ANNEALING(problem, schedule) returns a solution state
    inputs: problem, a problem
            schedule, a mapping from time to "temperature"
    local variables: current, a node
                     next, a node
                     T, a "temperature" controlling prob. of downward steps
    current ← MAKE-NODE(INITIAL-STATE[problem])
    for $t \leftarrow 1$ to $\infty$ do
        $T \leftarrow$ schedule[t]
        if $T = 0$ then return current
        next ← a randomly selected successor of current
        $\Delta E \leftarrow$ VALUE[next] – VALUE[current]
        if $\Delta E > 0$ then current ← next
        else current ← next only with probability $e^{\Delta E/T}$
```

Properties of simulated annealing

At fixed "temperature" T, state occupation probability reaches Boltzman distribution

$$p(x) = \alpha e^{\frac{E(x)}{kT}}$$

T decreased slowly enough $\Longrightarrow$ always reach best state x^*
because $e^{\frac{E(x^*)}{kT}}/e^{\frac{E(x)}{kT}} = e^{\frac{E(x^*)-E(x)}{kT}} \gg 1$ for small T

Is this necessarily an interesting guarantee??

Devised by Metropolis et al., 1953, for physical process modelling

Widely used in VLSI layout, airline scheduling, etc.

Local beam search

Idea: keep k states instead of 1; choose top k of all their successors

Not the same as k searches run in parallel!
Searches that find good states recruit other searches to join them

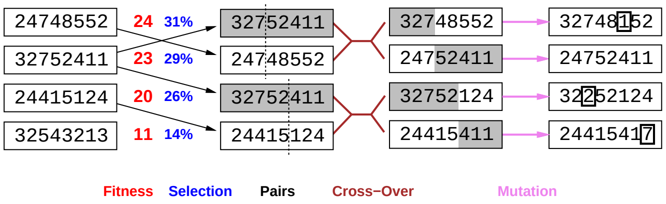
Problem: quite often, all k states end up on same local hill

Idea: choose k successors randomly, biased towards good ones

Observe the close analogy to natural selection!

Genetic algorithms

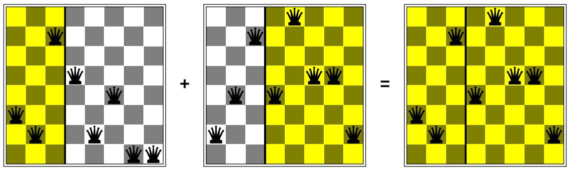
= stochastic local beam search + generate successors from **pairs** of states



Genetic algorithms contd.

GAs require states encoded as strings (GPs use programs)

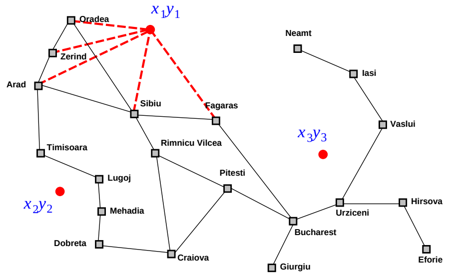
Crossover helps **iff substrings are meaningful components**



GAs $\neq$ evolution: e.g., real genes encode replication machinery!

Continuous state spaces

Suppose we want to site three airports in Romania:
- 6-D state space defined by (x_1, y_2), (x_2, y_2), (x_3, y_3)
- objective function $f(x_1, y_2, x_2, y_2, x_3, y_3) =$
  sum of squared distances from each city to nearest airport



Search methods

Discretization methods turn continuous space into discrete space,
e.g., empirical gradient considers $\pm\delta$ change in each coordinate

Gradient methods compute

$$\nabla f = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial y_1}, \frac{\partial f}{\partial x_2}, \frac{\partial f}{\partial y_2}, \frac{\partial f}{\partial x_3}, \frac{\partial f}{\partial y_3}\right)$$

to increase/reduce f, e.g., $\mathbf{x} \leftarrow \mathbf{x} + \alpha \nabla f(\mathbf{x})$

Locally, $f(x_1, y_2, x_2, y_2, x_3, y_3) = (x_1 - x_{\text{Arad}})^2 + (y_1 - y_{\text{Arad}})^2 + \cdots$, so

$$\frac{\partial f}{\partial x_1} = 2(x_1 - x_{\text{Sibiu}}) + 2(x_1 - x_{\text{Sibiu}}) + \cdots$$

Sometimes can solve for $\nabla f(\mathbf{x}) = 0$ exactly (e.g., with one airport).
Newton–Raphson (1664, 1690) iterates $\mathbf{x} \leftarrow \mathbf{x} - \mathbf{H}_f^{-1}(\mathbf{x})\nabla f(\mathbf{x})$
to solve $\nabla f(\mathbf{x}) = 0$, where $\mathbf{H}_{ij} = \partial^2 f / \partial x_i \partial x_j$